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Structure and optical behaviour of cholesteric soliton lattices

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Abstract. — We consider the elastic instability of a ferrocholesteric induced by a field. In a magnetic field acting perpendicular to twist axis, at low fields we get, a 2 $\pi$ soliton lattice. Above a threshold field, this will become unstable leading to a $\pi$ soliton lattice for positive diamagnetic anisotropy [$\chi_A > 0$] and a N-W soliton lattice for $\chi_A < 0$. We have also studied the optical reflection in such soliton lattices. At fields close to the nematic cholesteric transition point the higher order reflections are more intense than the primary order and each Bragg band splits into three sub bands. In these regions the electromagnetic waves are very different as regards their polarizations, and angle between $E$ and $H$. This is in contrast to the undistorted structure where these waves are locally linearly polarized with $E$ parallel $H$. In addition, optical diffraction pattern has been computed for propagation perpendicular to the twist axis. In the ferrocholesteric soliton lattice due to grain migration alone we predict a new type of diffraction. This diffraction pattern for N-W lattice is very different from that of a $\pi$ soliton lattice.

1. Introduction.

It is well known that when a magnetic field $H$ is applied perpendicular to the twist axis of cholesterics they undergo a transition to the nematic state at a critical field $H_{Ch-N}$ [1, 2]. It was also pointed out by earlier workers that for fields $0 < H < H_{Ch-N}$ the twist angle $\Theta$ varies non-linearly with distance along the twist axis [1, 2] leading to a soliton lattice of $\pi$ twist walls. This transition is purely due to the local diamagnetic anisotropy $\chi_A$ of the cholesterics. In 1970 Brochard and de Gennes [3] indicated the possibilities of obtaining stable, ferronematic (FN) and ferrocholesteric liquid crystals (FCh) — wherein the ferromagnetic grains are aligned with their magnetization $M_s$ along the local director. These authors also worked out the magnetic field effects by ignoring the intrinsic diamagnetic anisotropy of the host matrix. They predicted a soliton lattice of $2 \pi$ twist walls. Ferronematic systems have now been experimentally realised [4] and initial experiments in magnetic fields indicate that diamagnetic anisotropy of the host matrix cannot be ignored. Thus distortion of FCh in the presence of a magnetic field will be much more interesting when we consider the effect of both diamagnetic anisotropy $\chi_A$ and the magnetization $M_s$ of the grains. In fact, we find very different behaviours for $\chi_A < 0$ and $\chi_A > 0$ structures. We also consider the effects of grain segregation in such structures.
An optical study of these soliton lattices is one simple way of elucidating their structural details. With this in mind we have worked out optical reflection in the Bragg mode and diffraction in the phase grating mode. In many ways the optics of these soliton lattices is very different from that of normal cholesterics. In addition we have also undertaken an analysis of the polarization features of the electromagnetic waves in the Bragg bands.

2. Elastic instability in ferrocholesterics.

As pointed out earlier we incorporate in our analysis the diamagnetic anisotropy of the host matrix. Then the generalized Brochard-de Gennes free energy density of the FCh in a magnetic field (For the geometry shown in Fig. 1) is given by

\[ F_d = \frac{k_{22}}{2} \left( \left( \frac{\partial \Theta}{\partial z} \right)^2 - 2 q_0 \frac{\partial \Theta}{\partial z} \right) + \frac{x_a H^2}{2} \sin^2 \Theta - f M_s H \cos \Theta + \frac{f k_B T}{V} \ln f \]

where,
\[ k_{22} = \text{twist elastic constant,} \]
\[ x_a = \text{diamagnetic anisotropy,} \]
\[ M_s = \text{magnetization of the grains,} \]
\[ f = f(z) = \text{local volume fraction of the grains in the field,} \]
\[ V = \text{volume of the grain,} \]
\[ P = \text{pitch of the FCh} = 2 \pi/q_0, \]
\[ T = \text{temperature,} \]
\[ k_B = \text{Boltzmann constant.} \]

Under the influence of a magnetic field not only the director \( n \) gets distorted but also the magnetic grains will get redistributed from their initial uniform distribution. The distribution of grains is given by the quantity \( \psi(z) = f/\bar{f} \), where \( \bar{f} \) is the uniform volume fraction of the grains at \( H = 0 \).

From the minimization of \( F_d \) with respect to \( f \), we get,

\[ \psi = \psi_0 \exp(\rho_0 H \cos \Theta) \]

where, \( \rho_0 = (M_s V/k_B T) \) and \( \psi_0 \) is the normalization constant so chosen that

\[ \int_0^P \psi \, dz = P. \]

Fig. 1. — Geometry representing a magnetic field applied perpendicular to the twist axis of a ferrocholesteric.
The Euler equation for $\Theta$ is

$$\frac{d^2 \Theta}{dz^2} = \frac{\sin \Theta \cos \Theta}{\xi_1^2} + \frac{\psi \sin \Theta}{\xi_2^2}$$

(3)

where,

$$\xi_1^2 = \frac{k_{22}}{x_a \mathcal{H}^2} \quad \text{and} \quad \xi_2^2 = \frac{k_{22}}{M_s \mathcal{H} f} = \frac{k_{22}}{M \mathcal{H}}.$$ 

Integrating equation (3) and using (1) we get

$$\frac{d \Theta}{dz} = \frac{1}{\delta_1} (a - \psi + \delta_2^2 \sin^2 \Theta)^{1/2}$$

(4)

where $a$ is a constant,

$$\delta_1^2 = \frac{\rho_0 \mathcal{H} \xi_2^2}{2} \quad \text{and} \quad \delta_2^2 = \frac{\rho_0 \mathcal{H} \xi_1^2}{2 \xi_1^2}.$$

From the minimization of the total free energy $\int_0^\rho F_0 \, dz$ we get [3],

$$2 \pi q_0 \delta_1 = \int_0^{2\pi} (a + \delta_2^2 \sin^2 \Theta - \psi)^{1/2} \, d\Theta.$$  

(5)

We can determine $\psi_0$, $a$ and $P$ by solving equations (2, 4, 5) simultaneously. These equations have been solved numerically.

When $x_a = 0$ we have two regimes. a) $q_0 \delta_1 < 1$, here a complete unwinding takes place at a critical field and b) $q_0 \delta_1 \gg 1$, where such a transition is not possible. However when $x_a \neq 0$ such regimes do not exist and a complete unwinding takes place for all values of $q_0 \delta_1$. It so happens that in our case $q_0 \delta_1 < 1$.

3. Structure of the soliton lattice.

For positive diamagnetic anisotropy $x_a > 0$ at fields lower than a threshold $\mathcal{H}_c = M \chi_a$ we get, the Brochard-de Gennes $2\pi$ soliton lattice — a periodic array of $2\pi$ twist walls. However above this threshold this become unstable with each $2\pi$ twist wall splitting into a double $\pi$ twist wall. This has been depicted in figure 2a (a similar behaviour was predicted by Hudák [5] in smectic C* in an electric field). In this process grains migrate out of the twisted regions to the weakly twisted regions where the director is oriented nearly along the field. Figure 2b, shows the grain concentration profile. For comparison we give the $\Theta$ profile and grain concentration profile for the Brochard-de Gennes case (i.e., $x_a = 0$) in figure 2.

For $x_a < 0$ materials, the behavior is very different. At low fields we get the usual $2\pi$ soliton lattice. For fields $\mathcal{H} > \mathcal{H}_c = (M/x_a)$ the equilibrium orientation of the director in a nematic makes an angle $\Theta_m = \cos^{-1}(M/x_a \mathcal{H})$ with the field [6, 7]. Then each $2\pi$ wall splits into a N twist wall and a W twist wall. In the N wall the director turns from $-\Theta_m$ to $+\Theta_m$ while in the W wall the director rotates from $+\Theta_m$ to $2\pi - \Theta_m$. It should be remarked that these are very similar to the N and W walls described by Belyakov and Dimitrienkov for SmC* [6] in an electric field. In figure 3a, is shown the $\Theta$ profile over one period of the N-W soliton lattice. In this case the segregation of magnetic grains has a different
Fig. 2. — The Θ profile (a) and ψ profile (b) of a double π lattice shown for one period (continuous line for \( \mathcal{H} < \mathcal{H}_c \), dashed line for \( \mathcal{H} = 0 \)), for the parameters \( \chi_3 = 5 \times 10^{-6} \), \( M = 1.8 \times 10^{-3} \) G, \( \mathcal{H} = 7.609 \) G, \( \mathcal{H}_c = M/\chi_3 = 360 \) G, and \( \rho_0 = 10^{-4} \). The same for \( \chi_3 = 0 \) is given (dotted line) for comparison.

profile from that of 2 π or double π soliton lattice. This is depicted in figure 3b. It should be noticed that unlike the previous case where the grains migrate to the weakly twisted regions here the grains go from the W wall to the N wall.

4. Bragg reflections at normal incidence.

The dielectric tensor \( \varepsilon \) of cholesterics and FCh's and the soliton lattices depend only on \( z \). In the case of a plane monochromatic wave incident on the medium through the boundary at \( z = 0 \) only four out of the six components of the \( \mathbf{E}, \mathbf{H} \) are coupled. In the Berreman formalism
Fig. 3. — The $\Theta$ profile (a) and $\psi$ profile (b) of N-W lattice shown for one period. $\chi_d = -10^{-6}$, $M = 1.8 \times 10^{-7}$ G, $\mu = 22065$ G and $\rho_0 = 10^{-4}$.

[8] the $x$ and $y$ components of $E$ i.e., $(E_x, E_y)$ and $H$ i.e., $(H_x, H_y)$ are independent and the Maxwell equations take the form:

$$\frac{d\Psi(z)}{dz} = ikD(z) \Psi(z)$$  \hspace{1cm} (6)$$

where, $k = \omega/c$, $D$ is a $4 \times 4$ matrix and $\Psi = (E_x, H_x, E_y, -H_y)^T$. Integrating equation (6) we get

$$\Psi(z) = F(0, z) \Psi(0)$$  \hspace{1cm} (7)$$

where $F$ is a $4 \times 4$ transfer matrix. The eigen vectors of the matrix $F(0, P)$ gives the net electromagnetic field for the proper modes. In the reflection band these modes correspond to the waves formed by the superposition of the waves propagating along the forward and
backward directions. The electromagnetic field vector components at any arbitrary location $(nP + h)$ (for any integer $n$ and $0 < h < P$) are determined by multiplying the eigen vector of $F(0, nP)$ by $F(nP, h)$. Using this method we study the polarization structure of the eigen waves.

In an experimental situation the sample is usually confined between two homogeneous transparent isotropic media. To analyze such cases we express the Berreman vector $\Psi (z)$ as a superposition of the waves in the confining media through a transformation [9]

$$\Psi = B \Phi .$$

Here $B$ is a $4 \times 4$ matrix with columns representing the proper modes in the isotropic media (the eigenvectors of $D$) and $\Phi = [t_e t_m r_e r_m]'$ with $t_e$, $t_m$ representing the strengths of $E_x$, $E_y$ modes propagating in the $+z$ direction and $r_e$, $r_m$ in the $-z$ direction. Thus when $[i_1, i_2]$, $[r_1, r_2]$ and $[t_1, t_2]$ are the strengths of the incident, reflected and transmitted $E_i$ and $E_j$ modes respectively, from equation (7) we get

$$\begin{bmatrix} i_1 \\
 i_2 \\
 r_1 \\
 r_2 \end{bmatrix} = G \begin{bmatrix} t_1 \\
 t_2 \\
 0 \\
 0 \end{bmatrix} \tag{8}$$

where $G = B^{-1} F^{-1} B$. From equation (8) we obtain $2 \times 2$ transmission matrix $F_t$ and reflection matrix $F_r$ to be such that:

$$\begin{bmatrix} i_1 \\
 i_2 \end{bmatrix} = F_t \begin{bmatrix} t_1 \\
 t_2 \end{bmatrix}$$

and

$$\begin{bmatrix} r_1 \\
 r_2 \end{bmatrix} = F_r \begin{bmatrix} i_1 \\
 i_2 \end{bmatrix}.$$

The eigenvectors $\Gamma$ of the matrix $G$ give the proper modes of the multilayer in terms of the modes in the bounding isotropic media. This allows us to analyze the attenuated waves in the reflection band in terms of the forward and backward propagating waves. For example in the case of normal cholesterics we see that a standing wave is formed by the circular waves propagating in the $+z$ and $-z$ directions. Thus, in this case we have, $\Gamma(1) = j \Gamma(2)$ and $\Gamma(3) = -j \Gamma(4)$.

It should also be mentioned that the vector $\Psi (P)$ is linearly related to $\Psi (0)$ through the Berreman matrix $F$. We also see that

$$\Psi (P)' S \Psi (P) = \Psi (0)' S \Psi (0) = 0 \tag{9}$$

where

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 \\
 -1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 \\
 0 & 0 & -1 & 0 \end{bmatrix}$$

since

$$F(0, P) \Psi (0) = \Psi (P)$$

we get from 9

$$\Psi (0)' F' SF \Psi (0) = \Psi (0)' S \Psi (0)$$
or

\[ F^* S F = S. \]  \tag{10} 

Equation (10) shows that the matrix \( F \) (0, P) is symplectic [10]. Then from the properties of symplectic matrices we get that if \( \lambda \) is an eigen value then so are \( \lambda^* \), \( 1/\lambda \) and \( 1/\lambda^* \) \(^{(1)}\). Where \( \lambda^* \) is the complex conjugate of \( \lambda \). This property of the simplectic matrices results in the coupling of the eigenvalues. For example far away from the reflection band we have all the \( \lambda \)'s complex with \( \lambda \lambda^* = 1 \). Here we have only two independent eigenvalues. Let this be \( \lambda_1 \) and \( \lambda_2 \) then the other two are fixed as \( \lambda_1^* = 1/\lambda_1 \) and \( \lambda_2^* = 1/\lambda_2 \). In the Bragg band we have two real eigenvalues with one the inverse of the other. The other two eigenvalues are complex with unit modulus and are complex conjugates of each other. On the other hand in the non-Bragg band we have complex eigenvalues with the modulus not equal to one; Thus here all the eigenvalues are coupled and are given as \( (\lambda_1, 1/\lambda_1, \lambda_1^*, 1/\lambda_1^*) \). Therefore, in the non-Bragg regions there cannot be any propagating modes since the medium is non absorbing.

5. Phase grating mode.

Raman and Nath [RN] in 1935 [11] developed a theory of isotropic gratings with phase modulation. This theory in its simplest form has been applied to anisotropic dielectric gratings like that of twisted liquid crystals [12, 13]. Here we make the following assumptions: i) the sample is thin; ii) the birefringence is small and iii) the periodicity of the medium is large compared to the wavelength of light.

In the RN theory the emergent wavefront is described by

\[ U(z) = A_0 \exp(j2\pi n_z t/\lambda) \]

where \( t \) is the thickness of the sample, \( A_0 \) is the amplitude of the incident plane wavefront and \( n_z \) is the refractive index for the vibration perpendicular to the twist axis at any point \( z \) given by

\[ \frac{1}{n^2} = \frac{\sin^2 \Theta}{n_e^2} + \frac{\cos^2 \Theta}{n_0^2} \]

\( n_e \) and \( n_0 \) being the principle refractive indices parallel and perpendicular to the director, respectively. The diffraction pattern is then given by the Fourier transform of \( U(z) \)

\[ \sum (K) = \int_{-\infty}^{\infty} U(z) \exp(-jKz) \, dz . \]

Where

\[ K = \frac{2\pi \sin \theta}{\lambda} , \]

\( \theta \) being the angle of diffraction.

The above assumptions are not always valid in usual experimental situations. In such cases a multiple beam analysis [14, 15] is very useful. In this paper we follow the method of

\(^{(1)}\) We are thankful to J. Samuel for discussions on the symplectic nature of the matrix \( F \).
Rokushima and Yamakita [RY] [15] and compare the results with those obtained through Raman and Nath theory.

In the RY theory we Fourier expand the dielectric tensor and electromagnetic field components as

$$\varepsilon_{ij}(z) = \sum_{l=-m}^{m} \varepsilon_{ijl} \exp(j q_{l}z)$$

$$E_i = \frac{1}{\sqrt{\varepsilon_0}} \sum_{m=-m}^{m} \varepsilon_{im}(x) \exp(-j (s_0 + mq) z)$$

where

$$q = \frac{2 \pi}{P} \quad \text{and} \quad s_0 = n_i \sin \beta.$$

Here $n_i$ is the refractive index of the incident medium and $\beta$ is the angle of incidence. Thus we can express the Maxwell equations for the tangential field components in the $zx$ plane in the form,

$$\frac{d\eta}{dx} = jC \eta \quad (11)$$

where

$$\eta = \begin{bmatrix} e_1 \\ h_z \\ e_z \\ h_y \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 0 & -1 & 0 & 0 \\ \varepsilon_{\|\|^{-1}} \varepsilon_{\|\|} - \varepsilon_{\perp\perp} + q^2 & 0 & 0 & -\varepsilon_{\|\|^{-1}} q \\ q \varepsilon_{\perp\perp}^{-1} \varepsilon_{\|\|} & 0 & 0 & -q \varepsilon_{\perp\perp}^{-1} q + 1 \\ 0 & 0 & \varepsilon_{\perp\perp} & 0 \end{bmatrix}.$$

Here $\varepsilon_{ij}$ are $(2m+1) \times (2m+1)$ submatrices with elements $\varepsilon_{ijl} = \varepsilon_{ijl-m}, \quad q = \delta_{nl} q_t$ and $q_t = q_j + s_0$. Equation (11) is solved numerically for $\eta$. We consider the case of normal incidence only.

6. Results.

6.1 Bragg Reflection Mode. — It is known [16] that soliton lattices exhibit multiple Bragg reflections. In addition we find that when $\mathcal{H} = \mathcal{H}_{\text{Ch-N}}$ some of the higher orders are more

Fig. 4. — The first three orders in the reflection spectra of a cholesteric $\pi$ soliton lattice for normal incidence at $\mathcal{H} = 22 065$ G, and $\chi_s = 10^{-6}$. Continuous line is the $(E_i - E_i)$ reflection. Broken line represents the $(E_i - E_i)$ reflection. The $(E_i - E_i)$ and $(E_i - E_i)$ reflections are represented by the dotted line. $n_c = 1.595$. and $n_0 = 1.505$. 
intense than the first order. In figure 4, we show the reflection spectra of a \( \pi \) soliton lattice. It should be remarked that this kind of multiple Bragg reflections are possible in the case of discreet system of thick birefringent plates also [17]. However, in the case of soliton lattices we also have each Bragg band splitting into three peaks.

By computing the eigenvalues of the transmission matrix \( F \) for one period we can get the nature of the dispersion curve for an infinite sample. In figure 5 we show, this dispersion curve in the first reflection band of the same \( \pi \) soliton lattice. It has three sub-bands. The eigen modes of the transmission matrix \( F \) are linear in all the three regions. For regions I and III we have a propagating mode and an attenuated mode. The attenuated mode experiences reflection. As shown in figure 4 the reflected light in these regions are of the same polarization state as the incident light. The net vibration at any point is obtained by adding the forward and backward propagating waves of the same polarization. These vibrations have \( E \) perpendicular to \( H \) with a relative phase of \( \pi/2 \) between them. The corresponding modes in I and III are orthogonal to

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**Fig. 5.** The imaginary part of the wavevector \( W \) (the eigenvalues of the matrix \( F(0, P) \)) in the first reflection band of the \( \pi \) soliton lattice.

**Fig. 6.** The variation of \( |E|^2 \) with \( z \) of the attenuated modes in regions corresponding to I and III (broken line) and in region II (doted line). The same in the reflection band of normal cholesteric (continuous line) is given for comparison.
each other. In region II we have non-Bragg reflection where both eigen modes are attenuated. The reflected light here is of orthogonal polarization to that of the incident light. The net waves here (which is a superposition of the forward and backward propagating waves of orthogonal polarizations) are elliptical with there ellipticities varying along the twist axis.

On the other hand, in the reflection band of a normal cholesteric the net attenuated mode is locally linear and is coupled to the local director. The E and H vectors of this mode are parallel to each other with a phase difference of $\pi/2$ between them.

In figure 6, we compare the variation of $|E|^2$ as a function of these attenuated modes in the soliton lattices with that of a usual cholesteric.

The reflection and transmission spectra of cholesteric soliton lattices get altered considerably when we include linear dichroism. This is shown in figure 7 for two incident linear polarizations. It should be noticed that on the shorter wave length side, the $E_y$ mode (marked YY) is found to be strongly reflected as well as transmitted compared to the $E_x$ mode (marked XX) i.e., we get, an anomalous transmission.

![Figure 7](image_url)

**Fig. 7.** — The transmission coefficient $\mathcal{G}$ (a) and reflection coefficient $\mathcal{R}$ (b) of a $\pi$ soliton lattice in the presence of linear dichroism. $n_\parallel = 1.595 + 0.025 \, j$, $n_\perp = 1.505 + 0.001 \, j$, other parameters being the same as in figure 4.
6.2 Phase Grating Mode. — In the simple theory of RN as the sample thickness increases the number of diffraction orders increases. While in the RY theory the number of diffraction orders increases up to a certain thickness above which it decreases. We compare in figure 8 the RN and RY theories. The intensities of the central and second orders are given as a function of the sample thickness. We see that for the parameters used RN theory is valid only for thickness less than 7 \( \mu \text{m} \).

When the incident beam is in the plane perpendicular to the twist axis making an angle \( \Omega \) with respect to the field direction such that \( 0 < \Omega < \frac{\pi}{2} \), the diffraction pattern is asymmetric. This is shown in figure 9. This feature is typical of a soliton lattice. When the structure is absorbing we have also linear dichroism. Then the development of the soliton lattice can be easily recognized. Here we find for \( \Omega = 0 \) all the orders to be weak for positive dichroism. This implies that the weakly twisted regions are nearly parallel to the field.

![Figure 8](image.png)

**Fig. 8.** — A comparison of the intensities in the central and second order of the phase grating diffraction pattern computed using RN theory (continuous line) and RY theory (broken line) as a function of the sample thickness \( d \) for a \( \pi \) soliton lattice at \( H = 22 060 \) with other parameters same as in figure 4.

![Figure 9](image.png)

**Fig. 9.** — The asymmetry in the diffraction pattern of the \( \pi \) soliton lattice same as in figure 8 with \( \Omega = \pi/16 \).
We shall now consider the peculiar features associated with the ferrocholesteric soliton lattices. One important feature is the migration of the magnetic grains. These grains can be made up of either transparent materials (e.g., garnets) or absorbing materials (e.g., ferrites). When these grains are of the later type, then due to migration of the grains, we will have a nonuniform absorption along $z$. This nonuniformity alone results in diffraction when the incident polarization is parallel to the twist axis. This diffraction is peculiar to these systems. This is depicted in figure 10. In figure 10a we show the diffraction pattern due to N-W soliton lattice. This is very different from that of the soliton lattice in $\chi_{a} > 0$ systems whose diffraction pattern is shown in figure 10b. The peculiar feature of the diffraction in N-W soliton lattice is that intensity fluctuates from order to order and we find the even orders to be less intense than the odd orders, where as for the double $\pi$ lattice the intensity decreases monotonically. This feature in the pattern could be used to distinguish N-W lattices from soliton lattices in $\chi_{a} > 0$ materials.

6.3 TEST FOR TWIST INDUCED BIAXIALITY. — It is known that cholesteric liquid crystals are in principle biaxial [16]. The inherent chirality and the hindered rotation of the molecule due to
the twist are the two mechanisms proposed \[18\] to explain this biaxiality. It has been argued that the induced biaxiality increases with increase in twist. To our knowledge, no direct optical measurements have been proposed to verify this. Due to twist induced biaxiality in the soliton lattice the biaxiality will vary as function of $z$. As a result, in cholesteric soliton lattice for the incident light with polarization parallel to the twist axis, these variation will give a diffraction pattern. In figure 11, we show one such diffraction pattern computed by assuming the biaxiality to vary linearly with twist. The pattern appears even for $\Delta n_2$ as small as $5 \times 10^{-3}$. Thus, the method appears to be quite sensitive to twist induced biaxiality.

7. Conclusions.

The structure of ferrocholesteric in a magnetic field is worked out for an infinite sample taking into account the effect of the diamagnetic anisotropy of the host, the grain magnetization and the grain segregation. We find the $2\pi$ soliton to be unstable above a threshold field. The optical properties of these structures are studied for both Bragg mode and the phase grating mode. The soliton lattices exhibits multiple Bragg reflection. The net waves inside the Bragg band is shown to have interesting polarization features. When the grains are absorbing soliton lattices exhibit anomalous transmission.

In the phase grating mode we give a comparison of the results obtained using the RN theory and RY theory. The soliton lattice exhibits asymmetric diffraction when angle of incidence $\Omega$ with respect to the field direction is between 0 and $\pi/2$. It is shown that the segregation of the grains gives rise to different diffraction patterns in the double $\pi$ and N-W lattice. It is also possible to test the twist induced biaxiality using the phase grating.

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