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Résumé. — Les phases smectiques B sont des phases ordonnées à 3D, mais ce ne sont pas des cristaux parfaits. Dans ces phases il existe des chaînes de molécules dont l'ordre est facilement perturbé par l'existence de lacunes. Dans des papiers précédents nous avons montré que la signature de ces défauts sur des clichés de RX est la présence d'une ligne blanche diffuse. La symétrie hexagonale du milieu engendre un champ de déformation anisotrope autour de chaque défaut. Nous relions la forme de la zone blanche diffuse aux constantes élastiques du matériau.

Abstract. — Smectic B phases are 3D ordered phases, but they are not perfect crystals. In these phases there exist strings of molecules, the order of which is easily disturbed by the existence of vacancies. We have shown in previous papers that a significant signature of these defects on X-ray diffraction patterns is the presence of a white diffuse line. The hexagonal symmetry of the medium generates an anisotropic deformation field around each defect. We relate the shape of the white diffuse zone to the elastic constants of the material.

1. Introduction.

Smectic liquid crystal [1] phases present an organization with layers more or less correlated. In smectic A phases the order within the layers is liquid-like. Smectic B (SmB) or G (SmG) phases show a crystalline 3-dimensional (3D) order. The layers are weakly correlated in the SmB phase (hexagonal symmetry) or in the SmG phase (monoclinic symmetry). One consequence is the presence of a large number of stacking faults. Another is the very low value of the shear elastic constant $C_{44}$ (related to the sliding mode of the layers over one another) compared to the other typical elastic constants. Contrary to the SmA phase, in both the SmB and SmG phases there exists an organization of the molecules in strings. In the SmB phase, the strings of molecules are perpendicular to the layers whereas in the SmG phase, they are tilted. X-ray diffraction patterns show white diffuse lines interpreted as induced by defects in these strings of molecules [2]. A vacancy is created when a molecule is expelled from one layer to

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the next (Fig. 1). This disturbs the string (defining the $z$ direction), over a distance $2L$ related to the vacancy lifetime. This length is the diffusion length of the compression wave along the string during this time. This time has been estimated in the case of TBBA [3] to be of the order of $10^{-11}$ s. The corresponding length concerns 5 to 10 molecules. The concentration of these defects has been evaluated to be of the order of $10^{-2}$ [2, 3]. In a previous paper [3] we analyzed this defect in a 1D asymmetric model explaining the presence of the white diffuse line at $q_z = 0$. We only took into account the elastic deformation along the string and computed the diffraction pattern of the 3D crystal in the presence of these non-correlated 1D defects [3]. Actually, for some compounds, experimental patterns reveal some broadening of the white zone for large $q_z$ [2] (Fig. 2). We show in this paper that the shape of the diffuse white line is related to the complete 3D elastic deformation field coupled to the defects considered symmetric this time. In section 3 we give an exact solution for this field in an infinite medium

![Fig. 1. Schematic representation of the defect. The ellipses symbolize the molecules which are displaced around the vacancy, along the three directions away from their positions in the ideal crystal.](image1)

![Fig. 2. X-ray diffraction pattern of the SmB phase of compound 40.8 (fixed crystal experiment, the arrow shows the direction of the $z$ axis). Notice the broadening (half angular width = $10^{-1}$ rad) of the white zone passing through the origin of the reciprocal space.](image2)
in terms of all the elastic constants in SmB phases: an important point is the very low value of $C_{44}$ compared to the other constants. Then, in section 5, we restrict the solution in the infinite medium to a domain defined by the finite range of the defect. Section 6 is devoted to the computation of the scattered intensity in the frame of small displacements in a continuous medium.

2. Diffuse scattering intensity.

The scattered amplitude $A(s)$ (where $s$ is the scattering vector of modulus $s = 2 \sin \theta/\lambda$ and $2 \theta$ is the scattering angle) is easily calculated with use of the following classical [4] expansion for small displacements $U_n \cdot s \cdot U_n \ll 1$:

$$A(s) = \sum_n f e^{i s \cdot r_n} \sum_n e^{i s \cdot r_n} (1 + 2 i s \cdot U_n)$$

where $r_n$ is the position of the $n$-th molecule in the perfect crystal. The diffuse amplitude is driven by the last term of equation (1). The continuous displacement $U(r)$ can be expanded in Fourier modes as:

$$U(r) = \int U_q e^{-2i\pi q \cdot r} d^3q,$$

which leads for the diffuse scattering intensity $I(s)$ in the continuous limit to:

$$I(s) \approx \sum f^2 |s \cdot U_q|^2$$

where $q = s - \ell c^*$. In the vicinity of a Bragg peak, only one term of this sum really contributes. Around the origin of reciprocal space, this gives for the diffuse intensity $I_0(s)$:

$$I_0(s) \approx f^2 |q \cdot U_q|^2, \quad q = s.$$

We shall now determine the elastic deformation field induced by the vacancy.

3. Deformation field in an infinite medium.

We first consider the displacement field induced in an infinite medium by an anisotropic local dilatation in the $z$ direction, parallel to the molecular direction. The elastic forces satisfy the following equations:

$$\begin{cases} F_x = 0 \\ F_y = 0 \\ F_z = W \delta_z \delta_z' \end{cases}$$

A dilatation corresponds to a negative value of $W$.

In a hexagonal phase the elastic forces [5] read:

$$F_x = \frac{1}{2} (C_{11} - C_{12}) \Delta U_x + \frac{1}{2} (C_{11} + C_{12}) \frac{\partial}{\partial x} \text{div} U$$

$$+ (C_{13} - C_{12}) \frac{\partial^2 U_x}{\partial x \partial z} + (C_{44} - C_{66}) \left( \frac{\partial^2 U_x}{\partial z^2} + \frac{\partial^2 U_z}{\partial x \partial z} \right),$$

where $\Delta U_x = U_x - \langle U_x \rangle$.
\[ F_y = \frac{1}{2} (C_{11} - C_{12}) \Delta U_y + \frac{1}{2} (C_{11} + C_{12}) \frac{\partial}{\partial y} \text{div} \ U \]
\[ + (C_{13} - C_{12}) \frac{\partial^2 U_z}{\partial y \partial z} + (C_{44} - C_{66}) \left( \frac{\partial^2 U_y}{\partial x^2} + \frac{\partial^2 U_z}{\partial y \partial z} \right) \]  
\[ (7) \]

and

\[ F_z = \frac{1}{2} (C_{11} - C_{12}) \Delta U_z + \frac{1}{2} (C_{11} + C_{12}) \frac{\partial}{\partial z} \text{div} \ U \]
\[ + (C_{44} - C_{66}) \left( \frac{\partial^2 U_x}{\partial x \partial z} + \frac{\partial^2 U_y}{\partial y \partial z} \right) \]
\[ + (C_{13} - C_{12}) \left( \frac{\partial^2 U_x}{\partial x \partial z} + \frac{\partial^2 U_y}{\partial y \partial z} \right) + (C_{33} - C_{11}) \frac{\partial^2 U_z}{\partial z^2} \]  
\[ (8) \]

with \( C_{66} = \frac{1}{2} (C_{11} - C_{12}) \).

Calculations can be performed with the classical method of the Green functions [6], but, as we are essentially interested in the X-ray scattering, equations (6), (7) and (8) can be directly solved in Fourier space with:

\[ \mathbf{q} = q_{\perp} \mathbf{\xi} + q_{z} \mathbf{\zeta} \]  
\[ (9) \]
\[ \mathbf{\tilde{U}} = \tilde{U}_{q_{\perp}} \mathbf{\xi} + \tilde{U}_{q_{z}} \mathbf{\zeta} \]  
\[ (10) \]

where \( \mathbf{\xi} \) and \( \mathbf{\zeta} \), are the unit vectors in the radial and \( z \) directions of the Fourier space. The Fourier components of the deformation are:

\[ \tilde{U}_{q_{\perp}} = + W \frac{i (C_{11} + C_{44}) q_{z}^2 q_{\perp}}{2 \pi D} \]  
\[ (11) \]
\[ \tilde{U}_{q_{z}} = - W \frac{i q_{z} (C_{11} q_{\perp}^2 + C_{44} q_{z}^2)}{2 \pi D} \]  
\[ (12) \]

with

\[ D = (C_{11} C_{44}) q_{\perp}^4 + (C_{11} C_{33} - C_{13}^2 - 2 C_{13} C_{44}) q_{\perp}^2 q_{z}^2 + C_{33} C_{44} q_{z}^4 \] and \( q_{\perp}^2 = q_{x}^2 + q_{y}^2 \).

Equations (11) and (12) may be transformed with use of the relation:

\[ D = (C_{11} C_{44})(q_{z}^2 + a_2 q_{z}^2)(q_{\perp}^2 + a_3 q_{\perp}^2) , \]  
\[ (13) \]

where \( a_2 \) and \( a_3 \) are the roots of:

\[ a^2 C_{11} C_{44} + (C_{13}^2 + 2 C_{13} C_{44} - C_{11} C_{33}) a + C_{33} C_{44} = 0 . \]  
\[ (14) \]

One obtains:

\[ \tilde{U}_{q_{\perp}} = - W \frac{i q_{z} (C_{11} + C_{44})}{2 \pi C_{11} C_{44} (a_2 - a_3)} \left[ \frac{1}{a_2 q_{z}^2 + q_{\perp}^2} - \frac{1}{a_3 q_{z}^2 + q_{\perp}^2} \right] \]  
\[ (15) \]
\[ \tilde{U}_{q_{z}} = - W \frac{i q_{z}}{2 \pi C_{11} C_{44} (a_2 - a_3)} \left[ \frac{a_2 C_{11} - C_{44}}{a_2 q_{z}^2 + q_{\perp}^2} - \frac{a_3 C_{11} - C_{44}}{a_3 q_{z}^2 + q_{\perp}^2} \right] . \]  
\[ (16) \]
The Fourier transforms \( F \) of equations (15) and (16) are obtained with use of the property:

\[
\frac{\partial}{\partial x_i} F \left[ g(q) \right] = - F \left[ i \frac{\partial}{\partial q_i} g(q) \right]
\]

and of the following result:

\[
F \left( \frac{4 \pi}{(2 \pi)^2 (aq_1^2 + q_2^2)} \right) = \frac{1}{(z^2 + ap^2)^{1/2}}
\]

where \( r = \rho \rho^0 + zz^0 \), \( \rho^0 \) and \( z^0 \) are the unit vectors in the radial and \( z \) directions of the direct space and \( p^2 = x^2 + y^2 \). This Fourier transform is easily deduced from \( F \left[ 1/q'^2 \right] \) with \( q'^2 = q_{2\perp}^2 + q_{2z}^2 \) after a rescaling \( q_{2z}^2 = aq_{2\perp}^2 \).

The resulting deformation field which also has a cylindrical symmetry:

\[
U = U_\rho \rho^0 + U_z z^0
\]

then reads:

\[
U_\rho = + W \frac{(C_{13} + C_{44}) \rho}{4 \pi C_{11} C_{44}(a_3 - a_2)} \left( \frac{a_2}{(z^2 + a_2 \rho^2)^{3/2}} - \frac{a_3}{(z^2 + a_3 \rho^2)^{3/2}} \right)
\]  

(17)

and

\[
U_z = + W \frac{z}{4 \pi C_{11} C_{44}(a_3 - a_2)} \left( \frac{a_2 C_{11} - C_{44}}{(z^2 + a_2 \rho^2)^{3/2}} - \frac{a_3 C_{11} - C_{44}}{(z^2 + a_3 \rho^2)^{3/2}} \right).
\]  

(18)

Before exploiting these results in order to compute the diffuse scattering intensity, we shall first discuss the order of magnitude of the elastic constants.

4. Elastic constants.

In SmB phases the layers can glide more or less easily over each other. This property is characterized by the elastic constant \( C_{44} \), which is much smaller than the other elastic constants. The coefficient \( C_{44} \) has been measured by different techniques in several SmB samples.

For the SmB phase of compound 40.8, results obtained from mechanical shear experiments [7] give \( C_{44} \approx 5 \times 10^{+5} \) cgs. This result is in agreement with estimations from dislocation motions [8] and ultrasonic studies [9]. The value of the other elastic constants are much larger, typically of the order of \( 10^9-10^{10} \) cgs [7, 10]. Precise measurements of these elastic constants have been performed [11] but for another related compound (50.8). Far from the \( S_A \)-\( S_B \) transition, the authors give: \( C_{11} \approx 1.63 \times 10^{10} \) cgs, \( C_{13} \approx 1.52 \times 10^{10} \) cgs, \( C_{33} \approx 1.61 \times 10^{10} \) cgs. The parameter which will appear as pertinent is the ratio \( \alpha = C_{44}/C_{11} \). This ratio \( \alpha \) has a very low value \( \approx 10^{-3} \) and will be used as a small expansion parameter in the following. More precisely, we can obtain an estimation of the coefficients \( a_2 \) and \( a_3 \) in terms of \( \alpha \). To the lowest order in \( \alpha \), one has:

\[
\begin{align*}
a_2 & \approx \frac{C_{11}}{C_{44}} \left( \frac{C_{11} C_{33} - C_{13}^2}{C_{11}^2} \right) = \frac{1}{\alpha} \left( \frac{C_{11} C_{33} - C_{13}^2}{C_{11}^2} \right) \\
a_3 & \approx \frac{C_{44}}{C_{11}} \left( \frac{C_{33} C_{11}}{C_{11} C_{33} - C_{13}^2} \right) = \alpha \left( \frac{C_{33} C_{11}}{C_{11} C_{33} - C_{13}^2} \right).
\end{align*}
\]  

(19)
X-ray diffraction patterns have been mainly performed on the SmB phase of 40.8 for which the coefficient \( \alpha \) has been estimated. On the contrary, the other elastic constants appearing in the expressions of \( a_2 \) and \( a_3 \) (Eq. (19)) have not yet been measured. In order to get an estimation, we have used the ratios \( \left( \frac{C_{11} C_{33} - C_{13}^2}{C_{11}^2} \right) \approx 0.1 \), and \( \left( \frac{C_{33} C_{11}}{C_{11} C_{33} - C_{13}^2} \right) \approx 8 \) as evaluated from the measurements in the SmB phase of 50.8 [11]. Then the order of magnitude of \( a_2 \) and \( a_3 \) are \( a_2 \approx 10^{-2} \) and \( a_3 \approx 10^{-1} \cdot 10^{-2} \).

5. Defects of finite size.

As the vacancy has a finite lifetime the elastic deformation field only extends over a finite range. Let \( 2L \) be the characteristic length in the \( z \) direction which can be estimated, as explained in [3], to a few molecular lengths from coherent neutron scattering experiments [12]. Then the boundary condition taking this feature into account reads:

\[
U_z(\rho = 0, z = L) = 0. \tag{20}
\]

A solution satisfying both the elastic equations (5-8), and the above boundary condition (20) is:

\[
U_z = A W_z \left[ \frac{a_2 C_{11} - C_{44}}{(z^2 + a_2 \rho^2)^{3/2}} - \frac{a_2 C_{11} - C_{44}}{L^3} \right] - \left( \frac{a_3 C_{11} - C_{44}}{(z^2 + a_3 \rho^2)^{3/2}} - \frac{a_3 C_{11} - C_{44}}{L^3} \right) \tag{21}
\]

where

\[
A = \frac{1}{4 \pi C_{11} C_{44}} \frac{1}{(a_3 - a_2)}
\]

In order to determine the radial extension of the distortion, we sketch the plot of the curve \( U_z = 0 \) in the \((\rho, z)\) plane (Fig. 3). In the limit of small \( \alpha \), the region between the curve and the plane \( z = 0 \) is very small. Then we can neglect this zone and approximate the volume to an ellipsoid, the surface of which is given by:

\[
z^2 + a_2 \rho^2 = L^2 \tag{22}
\]

Fig. 3. — Curve \( U_z \sim 0 \) defining the extension of the deformation field. Its global shape is ellipsoidal except near the plane \( z = 0 \). The coordinates of point \( E \) are \( \rho(E) \sim L/\sqrt{a_2} \) and \( z(E) \sim L/a_2^{4/3} \). In the SmB case where \( a_2^{-1} \) is small, the ellipsoid is elongated in the \( z \) direction.
Here it is important to stress that, due to the 3 dimensional nature of the elasticity, a distortion extending over a distance \( L \) in the \( z \) direction also extends in the transverse direction over a distance \( \sqrt{a_2} \). Then the deformation field is limited to the ellipsoidal volume scaled by the vacancy lifetime. Therefore we shall restrict the displacement \( U(\rho, z) \) to this volume and take:

\[
U_{\rho} = W \frac{(C_{13} + C_{44} \rho)}{4 \pi C_{11} C_{44}(a_3 - a_2)} \left( \left( \frac{a_2}{(z^2 + a_2 \rho^2)^{3/2}} \right) \frac{a_2}{L^3} - \left( \frac{a_3}{(z^2 + a_3 \rho^2)^{3/2}} \right) \frac{a_3}{L^3} \right).
\]

(23)

6. X-ray intensity scattered by the defects.

As explained in section 2, the diffuse scattering intensity (Eq. 4) is related to the Fourier transform of \( U \) which we shall limit to the above ellipsoid. The exact computation of this Fourier transform is difficult, but its behaviour can be deduced from that of a specific displacement field inside a spherical domain, with use of an anisotropic linear rescaling. Indeed, each part of both components of the displacement field can be deduced from a spherical displacement field:

\[
U^0(r) = \frac{r}{r^3} - \frac{r}{R^3}
\]

by the rescaling:

\[
x^2 \to a_2 x^2 \quad \text{resp} \ a_3, \quad y^2 \to a_2 y^2 \quad \text{resp} \ a_3, \quad z^2 \to z^2
\]

(24)

Let us take this field limited to a spherical domain of radius \( R \). Its Fourier transform reads:

\[
\int_0^R U^0(r) e^{2i\pi q \cdot r} \, d^3r = \frac{2i\mathbf{q}}{q^2} (1 - E(2\pi q R))
\]

(25)

\( E(qR) \), the Fourier transform of a homogeneous sphere of radius \( R \), is:

\[
E(2\pi q R) = 3 \frac{\sin(2\pi q R) - (2\pi q R) \cos(2\pi q R)}{(2\pi q R)^3}
\]

(26)

One recognizes in expression (25) two contributions. The first one \( \frac{2i\mathbf{q}}{q^2} \) is the Fourier transform of \( \frac{r}{r^3} \) in an infinite medium. The second one corresponds to the correction induced by the finite size of the perturbation.

In the SmB case, the anisotropic size effect can be deduced from equation (26) with use of the following rescaling:

\[
q_x^2 \to \frac{q_x^2}{a_2}, \quad q_y^2 \to \frac{q_y^2}{a_2}, \quad q_z^2 \to q_z^2.
\]

(27)

We assume that the intensity, as in the spherical case, is still given by the product of the Fourier transform of the field in an infinite medium with the form factor of the finite domain. This assumption is strictly valid for \( a_3 = 0 \). After some straightforward calculation one obtains the diffuse scattering intensity:

\[
I_0(q) = f^2 \left\{ \frac{W q_z^2 ((C_{11} - C_{13} - C_{44}) q_{z}^2 + C_{44} q_{z}^2)}{2 \pi (C_{11} C_{44})(q_{z}^2 + a_2 q_z^2)(q_{z}^2 + a_3 q_z^2)} \right\}^2 (1 - E(2\pi q R))^2
\]

(28)

where \( \bar{q}^2 = q_z^2 + q_{z}^2 / a_2 \).
Fig. 4. — Behaviour of $I_0(q_z)$ for $a_z = 100$, $L = 5$ and for different values of $q_\perp$. $I_0(q_z)$ is given in arbitrary unit. Starting from zero at $q_z = 0$, it reaches a plateau for larger $q_z$. The limit of the white region corresponds to point B determined by $I_{\text{max}}/2$. 
The diffuse scattering intensity vanishes for \( q_z = 0 \), this gives the white line on the diffraction pattern. The white diffuse zone is therefore localized around the plane \( q_z = 0 \), but for the measurements on 40.8, it gets broader for larger \( q_\perp \). This can be interpreted using equation (28). The behaviour of \( I_0(q_z) \) for different values of \( q_\perp \) is shown in figure 4. The intensity vanishes at \( q_z = 0 \). Then it increases for increasing \( q_z \) reaching a plateau with some oscillations. The border of the white zone corresponds to a strong variation of the scattered intensity. It may be characterized by the point B in figure 4 at half value of the maximum intensity. The position of this point in the reciprocal space gives the relation \( q_z(q_\perp) \) defining the border of the white zone. A typical variation of \( q_z(q_\perp) \) is given in figure 5, where one notices the cross over between the two following regimes:

for low values of \( q_\perp \).

\[ q_z \sim L^{-1} \]  

(29)

for large values of \( q_\perp \).

\[ q_z = \tau q_\perp / \sqrt{a_2} \]  

(30)

The cross over between the two regimes (29, 30) happens for \( q_\perp = \tau^{-1} L^{-1} \sqrt{a_2} \). The interesting result is that we predict a broadening of the white zone as observed experimentally and that the strength of this effect is directly related to the elastic constants. The value of \( \tau (\tau = 1.3) \) is deduced from the theoretical curves plotted for different values of the parameter \( a_2 \). Let us compare this theoretical result to the experimental diffraction pattern performed on 40.8. In figure 5 we also show the experimental results (half intensity width) obtained from microdensitometric recordings of the X-ray diffraction patterns. The half angular spread of the white zone is evaluated to about \( 10^{-1} \text{rad} \) which gives, with use of equation (30),

Fig. 5. — Calculated coordinates \( q_z(q_\perp) \) of the border of the white diffuse zone for \( L = 5 \) \( c \) and 10 \( c \) and for \( a_2 = 20 \) and 100. One fits on these curves the parameter \( \tau \sim 1.3 \) defining the regime for large values of \( q_\perp \). \( q_z = \tau q_\perp / \sqrt{a_2} \).
$a_2 \sim 100$. This leads to the estimated ratio:

$$\frac{C_{44}}{C_{11}} \sim 10^{-2} \left( \frac{C_{11} C_{33} - C_{13}^2}{C_{11}^2} \right).$$

If one uses the values of $C_{11}$, $C_{33}$ and $C_{13}$ of 50.8, one obtains $\frac{C_{44}}{C_{11}} \sim 10^{-3}$ which is in good agreement with the experimental estimation [7-10]. There is no experimental points for low enough $q_\perp$ which prevents a precise determination of the length of the defects to be performed. Nevertheless their total length $2L$ can be estimated as concerning at least 10 molecules. Besides one can evaluate the elastic energy of the defect with use of the deformation field given in equations (21) and (23). This energy depends on the strength $W$ of the vacancy. The dilatation $V_0$ induced by the defect scales as $W = V_0 \left( \frac{C_{11} C_{33} - C_{13}^2}{C_{11}} \right)$. With the assumption that $V_0$ is of the order of a molecular volume one finds that the elastic energy of one defect is of the order of $kT$. This confirms the thermal origin of the defect.

7. Conclusion.

We have given a theoretical interpretation of a very peculiar feature of the experimental X-ray diffraction pattern of some SmB phases: the existence of a white diffuse line close to the origin of the reciprocal space. It is interpreted as due to the presence of uncorrelated transient defects linked to a disturbance of the packing in the molecule strings. Although this defect mainly concerns a finite portion of length $2L$ in the $z$ direction, it also induces a distortion in the other directions through the 3D elasticity of the hexagonal phase. This brings about the broadening of the white zone for large $q_\perp$. The resulting anisotropic elastic deformation field leads to the observed diffusion shape. Although our calculation is done in the frame of small, continuous and static deformations, it still reveals the importance of the elastic anisotropy of the medium.

We have shown that the shape of the white zone depends on the two ratios, $\alpha = \frac{C_{44}}{C_{11}}$ and $\beta = \left( \frac{C_{11} C_{33} - C_{13}^2}{C_{11}^2} \right)$ (in the assumption $\alpha$ small compared to $\beta$). $\beta$ is known from [11] in 50.8 and we assume that this term is strongly related to the crystalline structure and therefore should be nearly the same for all compounds exhibiting a SmB phase with hexagonal compact array. Then the comparison of theoretical results and experimental data in SmB phases provides an evaluation of $\frac{C_{44}}{C_{11}}$ in good agreement with the mechanical measurements. It is worth noticing that X-ray experiments thus give information on the elastic constants via the distortion generated by the defects. However one has to keep in mind that, in a general case, the assumption of $\alpha$ much smaller than $\beta$ may be ruled out preventing a straightforward derivation of $C_{11}/C_{44}$. This is certainly the case for the SmG phase of several compounds, such as TBBA, where $C_{11}/C_{44}$ is one or two orders of magnitude larger than in 40.8: the previous model would then predict a large broadening of the white zone. However, experimentally this zone remains of constant thickness and thus, the 1-dimensional treatment of [3] seems more appropriate.
DIFFUSE SCATTERING BY VACANCIES IN SMECTICS B

References