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Statistical mechanics of two dimensional vesicles

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Abstract. — We have used a q-space method for calculating thermodynamic quantities of a two-dimensional vesicle introduced by Ostrowsky and Peyraud. This method has been used to calculate the radius of gyration, the area, and the shape of vesicles as a function of perimeter length $\ell$ and Helfrich curvature parameter $K$. It is found, in agreement with the hypothesis of Fisher, that all thermodynamic quantities can be plotted on universal curves as a function of $\ell/K$. In the small $\ell$, large $K$ (stiff), and large $\ell$, small $K$ (floppy or fractal) limits, the results are broadly consistent with real space computations of other authors.

1. Introduction.

Over the last few years, stimulated in particular by the work of Helfrich and collaborators [1-5], there has been considerable interest in the behaviour of lipid bilayer membranes in biophysical contexts. A particular aspect of membrane properties occurs when a membrane forms a closed manifold, or vesicle. There have been speculations that in some biological circumstances, such as, for instance, the behaviour of red blood cells [6, 7], vesicle shape change, driven only by simple changes in the physicochemical environment, can have profound effects on biological function.

The equilibrium shape of a closed membrane is governed by a number of factors. These include the intrinsic Hamiltonian of the membrane itself, as well as other factors such as directional forces and the pressure difference $\Delta p$ between the outside and the inside of the membrane. The intrinsic fluid membrane effective Hamiltonian (per unit area) may be written in terms of its principal curvatures $c_1$ and $c_2$:

$$H_c = \frac{1}{2} K (c_1 + c_2 - c_0)^2 + \frac{1}{2} \tilde{K} c_1 c_2$$

(1)

where the phenomenological parameter $c_0$ is the spontaneous curvature of the membrane, the quantities $\frac{1}{2} (c_1 + c_2)$ and $c_1 c_2$ are respectively the mean and Gaussian curvatures of the membrane, and $K, \tilde{K}$ are respectively the bending rigidity and Gaussian curvature elasticity.
constant. The total free energy is then obtained by combining this effective Hamiltonian with other terms that depend on, for example, the osmotic pressure difference $\Delta p$ between the outside and the inside of the vesicle, and yet other terms which couple with the membrane direction in more specific ways. If we ignore these latter terms, the full free energy is:

$$\mathcal{F} = \int H_c \, dS + \int \Delta p \, d\mathcal{V},$$

(2)

where the integrals over $S$ and $\mathcal{V}$ represent, respectively, the surface of the vesicle and the volume inside it.

The vesicle properties (primarily shape and size) can be calculated as a function of membrane area $S$, as well as of the control parameters $K, \tilde{K}$ and $\Delta p$, by minimising the free energy expression (2). Such calculations have been carried out by Deuling and Helfrich [3] and by Zhong-Can and Helfrich [8]. Calculations of this kind are essentially of the mean field type; i.e. they do not take into account explicitly the entropy due to elasticity modes.

To go beyond the mean field approximation, it turns out to be easier to consider a two dimensional version of this problem. More specifically one considers a closed one dimensional membrane embedded in a two dimensional space. Ostrowsky and Peyraud [9], and more recently Barker and Grimson [10], have discussed vesicle conformations of this model by evaluating the full partition function as a functional integral over all possible conformations. We note that this restricted problem is in fact equivalent to the closed two dimensional self-avoiding walk, as discussed, for example, by Family et al. [11]. Alternatively, one may consider vesicles made up from attached beads. This approach has been adopted by Leibler, Fisher, and collaborators [6, 12-14], who have carried out extensive (Metropolis) Monte Carlo simulations. These models do not in any way represent the true vesicle microscopic physics. In the long distance continuum limit, however, the Hamiltonian has the correct behaviour, and hence one expects that in the asymptotic large vesicle limit useful insight will be obtained. An important innovation in this context, first pointed out by Fisher [15], is the use of scaling forms to interpret the large amount of data on vesicle shape and size, and in particular to reduce the number of independent variables.

In this paper we use the method of Ostrowsky and Peyraud [9] to investigate aspects of the shape and size of two dimensional vesicles. We interpret the data using the scaling forms introduced by Fisher [15]. Where our results can be directly compared with the results of the bead simulations [6, 12-14], we find heartening agreement; this is consistent with general expectations based on ideas of universality.

2. Basic theory.

We briefly recapitulate on the salient points of the method of Ostrowsky and Peyraud [9]. A two dimensional vesicle is shown in figure 1. A point on the vesicle is parameterised by its distance $s$ from an arbitrary starting point, with $0 < s < \ell$, and with vesicle length $\ell$. The conformations of the vesicle are prescribed by $\{\psi(s)\}$, where $\psi(s)$ is the angle between the normal to the curve at $s$ and the normal at $s = 0$. Not every set $\{\Psi(s)\}$ prescribes an allowable conformation, however. The following conditions apply on $\{\psi(s)\}$:

(i) Continuity and Differentiability of $\Psi(s)$.

(ii) $\psi(0) = \psi(\ell)$; $\psi'(0) = \psi'(\ell)$ [periodic boundary conditions].

(iii) Physical closure of the vesicle. This may be imposed as follows. The Cartesian coordinate $r(s) = x(s) + i y(s)$ can be defined by:

$$r(s) = \int_0^s dt \exp i \psi(t).$$

(3)
Physical closure then demands that:

\[ r(\xi) = r(0) = 0. \]  

(iv) The vesicle must be physically allowable; i.e. the curve \( r(s) \) must not be self-intersecting.

The intrinsic membrane Hamiltonian, the analogue of equations (1) and (2), is now:

\[ H_c = \int_0^\xi ds \frac{1}{2} K \left( \frac{d\psi}{ds} \right)^2, \]  

where \( K \) is the bending rigidity of the curve, and where, as opposed to the three dimensional case, there is only one and not two elastic constants. All physical quantities follow from the partition function \( Z \), where:

\[ Z = \int \prod \psi(s) \exp - H_c \{ \psi(s) \}. \]  

In equation (6) and elsewhere in this paper, the inverse temperature \( \beta \) has been incorporated in the definition of the effective Hamiltonian. Thus \( K = \beta K' \), where \( K' \) is the physical rigidity with dimension energy \( \times \) length; \( K \) has the dimensions of a length. The functional integral in equation (6) is assumed only to be taken over allowed configurations.

A useful parameterisation of $\Psi(s)$ satisfying the continuity conditions is:

$$\psi(s) = 2 \pi \frac{s}{\mathcal{L}} + \psi_0 + \sum_{m=1}^{M} \left[ A_m \cos \frac{2 \pi m s}{\mathcal{L}} + B_m \sin \frac{2 \pi m s}{\mathcal{L}} \right].$$

Each allowable configuration $\{\psi(s)\}$ is now described by the set $\{\psi_0, A, B\}$, where the vectors $A, B$ represent the sets $\{A_m\}, \{B_m\}$ respectively. The quantity $M$ corresponds to a high $q$ cutoff, or equivalently a minimal length scale for the bending oscillations of the vesicle. Accordingly, as $\mathcal{L}$ is changed, $M$ is also changed, in such a way that

$$q_{\text{max}} = \frac{2 \pi M}{\mathcal{L}} \approx 2 \pi a^{-1},$$

where $a$ is the microscopic length scale, remains constant. In practice, in specific calculations, we let $\mathcal{L} = \frac{2 \pi}{5} M$; now $M$ parameterises the total perimeter length, and $a = \frac{2 \pi}{5}$.

This method concentrates on the amplitudes of the normal bending modes. Thus we are working in reciprocal space; the bead models, by contrast, concentrate on real space degrees of freedom.

The functional integral over $d\psi(s)$ in equation (6) now becomes a (constrained) multiple integral over the variables $d\psi_0, dA$ and $dB$. The orientational degree of freedom $\psi_0$ can be eliminated by prescribing, arbitrarily, that $\psi(0)$ be zero; this defines

$$\psi(0) = \psi(\mathcal{L}) = \psi_0 + \sum_{m} A_m = 0.$$  

The physical closure condition (3) eliminates two further degrees of freedom, which we take to be the variables $A_1$ and $B_1$, but these variables can only be eliminated at the cost of introducing a Jacobian term $J(A, B)$ into the statistical mechanical functional integrals. The precise details of the calculation of $J$ have been described elsewhere [9, 10].

The total bending energy of the vesicle can be calculated from equations (5) and (7):

$$E_c = \frac{1}{2} \frac{K(4 \pi a)^2}{Ma} \left[ 1 + \frac{1}{2} \sum_{m} m^2(A_m^2 + B_m^2) \right].$$

The partition function (6) is now:

$$\mathcal{Z} = \int \prod_{m=2}^{M} dA_m \ dB_m \ J(A, B) \ \theta(A, B) \exp - E_c(A, B),$$

where $\theta(A, B) = 1$ or 0, depending on whether the self-avoidance criterion is satisfied or not. The average of a physical quantity $Q(A, B)$ is determined in the usual way from the weighted integral over the phase space given in equation (11).

The Ostrowsky-Peyraud method involves using a Monte Carlo method to evaluate the ratio of integrals used to calculate average quantities. We note that there is a strong distinction between this method and the classical Metropolis Monte Carlo algorithm, which evaluates average quantities by taking a specially constructed walk through the relevant phase space. In our case average quantities are calculated by walking at random through the space of $A$ and $B$, and then evaluating
\[ \langle Q \rangle = \frac{\sum Q(A, B) J(A, B) z(A, B)}{\sum z(A, B)} , \]

where \( z(A, B) \) is the integrand of the partition function (11). We have refined somewhat the random walking process used by previous authors [9, 10]; the details are relegated to an appendix.

4. Results.

We first concentrate on the properties of the vesicle in the absence of external forces. Our preliminary theoretical discussion follows that of Fisher [15]. The size of the vesicle can be described using the area \( A \) inside it, or by the radius of gyration \( R_G \), where:

\[ R_G^2 = \langle \langle r - \langle r \rangle \rangle^2 \rangle , \]

and where averages over individual vesicles are indicated by \( \langle \langle \rangle \rangle \), and over all vesicle configurations by \( \langle \rangle \). Some insight into the shape of the vesicle can be obtained from invariants of the gyration tensor \( R_{Gij} \) given by:

\[ R_{Gij} = \langle (r - \langle r \rangle)_i (r - \langle r \rangle)_j \rangle . \]

This tensor has eigenvalues \( R_{G1}^2 \) and \( R_{G2}^2 \), and the shape anisotropy is given by their ratio (which we take arbitrarily to be less than unity). The mean shape anisotropy is then given by:

\[ \Sigma = \left\{ \frac{R_{G1}^2}{R_{G2}^2} \right\} . \]

There are three length scales in this problem; the vesicle perimeter \( \ell \), the microscopic length scale \( a \), and the rigidity length \( \ell_k = K \) which gives the minimum length scale over which the vesicle can bend. For \( \ell_k \ll a \) the bending energy can be essentially ignored. For \( \ell \gg a \) and \( \ell \gg \ell_k \), the vesicle is floppy. We expect that \( A \sim R_G^2 \sim \ell^2 \nu \), where \( \nu = 0.75 \) is the Flory exponent which governs the analogous quantities for the open self-avoiding walk. By contrast, if \( \ell_k \gg \ell \), the vesicle will be stiff, and adopt its lowest energy conformation, which in this case is a circle. Now \( A \sim R_G^2 \sim \ell^2 \).

In between these limits (either at intermediate \( K \) or \( \ell \)) there will be a cross-over regime. Fisher [15] pointed out that one should expect:

\[ \langle A \rangle = \pi \ell^2 U(y) , \]
\[ \langle R_G^2 \rangle = \ell^2 V(y) , \]

with \( y = \ell/\ell_k \) the relevant scaling variable, with \( U(y) \) and \( V(y) \) scaling forms, and with \( U(0) = V(0) = 1 \). We plot \( U(y) \), for different length vesicles, in figure 2. We observe that there is a departure from the scaling form for sufficiently short \( \ell \), or sufficiently small \( K \). In this regime the limiting short length scale is \( a \) and not \( \ell_k \), and we therefore no longer expect scaling to hold. We show, for comparison, on the same graph, the analogous curve obtained in the real space simulations of Camacho et al. [14], and it will be seen that there is extremely close agreement. The exponent cross-over is best seen by plotting the « instantaneous »
Fig. 2. — Scaling of the area as a function of $y = L/K$, as described in equation (16). With \( \blacksquare \) $M = 2$, \( \oplus \) $M = 3$, \( \ast \) $M = 4$, \( \square \) $M = 6$, \( \times \) $M = 10$, \( \bigcirc \) $M = 20$, \( \triangle \) $M = 30$, \( \ominus \) LSF.

exponent:

$$\nu_{A, \text{eff}}(y) = \frac{1}{2} \left( \frac{\ln A}{\ln K} \right) (y); \quad (17)$$

this is plotted in figure 3. The stiff and floppy limits are clearly observable.

Another indication of the fractal dimension of the vesicle is given by the ratio

$$\Pi(y) = \frac{\langle A \rangle}{\langle R^2 \rangle}. \quad (18)$$

This is known from other studies to be about 2.4 in the floppy limit, and of course in the stiff
limit is must necessarily be π. In figure 4, we show the crossover between these two limits, once again plotted using the scaling variable y. Although these results are obtained numerically, we note that the low y, almost stiff, limit can be obtained analytically [14].

The quantity \( \Sigma \), defined in equation (15), describes the anisotropy of the vesicles. We plot \( \Sigma(y) \) in figure 5. This quantity is the ratio of the width to the length of an average vesicle which is taken to be ellipsoidal. This simple measure misses more complicated features such as lobing, which involve taking means of higher harmonic quantities. We show also the results of Camacho et al. [14] for the analogous quantity. It is not, in fact, immediately obvious whether one should measure \( \Sigma \) or \( \Sigma' \), where:

\[
\Sigma' = \frac{\langle R_{G1}^2 \rangle}{\langle R_{G2}^2 \rangle};
\]
Fig. 4. — The shape parameter $\Pi(y)$, defined in equation (18). With (■) $M = 2$, (+) $M = 4$, (*) $M = 6$, (□) $M = 10$, (×) $M = 20$, (◇) $M = 40$, (Δ) $M = 60$, (□) $M = 80$.

however, explicit calculation shows that they give almost identical results. The important limits are $y = 0$ (stiff), for which the equilibrium conformation is circular, and hence $\Sigma = 1$, and $y = \infty$ (floppy) for which it is known that $\Sigma \approx 0.4$. We observe, however, that $\Sigma(y)$ does not seem to be a monotonically decreasing function of $y$, and there is an intermediate region of lengths (for given $K$) over which this ratio seems to dip below 0.4, before recovering again. We do not have an intuitive explanation of this phenomenon, which seems a priori unlikely. However, Camacho et al. [14] in their bead simulations find the same phenomenon (although their data is less unambiguous than ours), and it seems likely, therefore, that this is not a numerical artefact.

We turn now to a consideration of the effect of extra terms in the Hamiltonian. The simplest, already considered by Leibler et al. [6], involves the effect of an osmotic pressure
difference $\Delta p$ between the outside and the inside of the vesicle. The Hamiltonian is now given by equation (2). A positive $\Delta p$ favours squashed vesicles; conversely, a negative $\Delta p$ favours expanded vesicles. The most interesting limit is the low $K$, small negative $\Delta p$ regime. Here there is competition between the intrinsic floppiness and the effect of the externally applied osmotic pressure. Leibler et al. [6] have proposed that the relevant scaling variable is now $x$, with:

$$x = |\Delta p| \ell^{2.5}$$  \hspace{1cm} (20)

Now we expect:

$$A = \ell^{2.5} f(x)$$  \hspace{1cm} (21)
in the low $x$ regime, with departures from scaling in the high $x$ regime when the area saturates. We show these results in figure 6, together with a comparison with the results of Leibler et al. [6]. We have also verified the result of Maggs et al. [13] that, for intermediate values of $x$, $A \sim |\Delta p|$. Leibler et al. [6] and Maggs et al. [13] have discussed the effect of stretching vesicles with a force $f$, and propose, in the low $K$ regime, a scaling variable $z = fL^\nu$. Barker and Grimson [10] consider a similar phenomenon, in which the particles making up the vesicle are subject to an external harmonic force around a central axis; they find, unsurprisingly, that in these
circumstances the vesicle is squashed around that axis. Plausible circumstances in which such phenomena would occur are: i) the vesicle is placed in a non-uniform flow field, or ii) the particles in the vesicle are subject to nematic orientational forces. An effective Hamiltonian is now:

\[ H = H_c + \int_0^c \mathcal{U} \cos 2\psi(s) \, ds. \]  

(22)

This vesicle has a nematic orienting field of size \( \mathcal{U} \) which prefers the arc perimeter to point in the \( y \) rather than the \( x \) direction. This can be compared to the case of the stretching force in which the Hamiltonian is:

\[ H = H_c - f \cdot r, \]  

(23)

---

Fig. 7. Dependence of the \( x-y \) shape anisotropy \( \Sigma^* \) on nematic potential \( \mathcal{U} \). With (\( \ast \)) \( M = 4 \), (\( \square \)) \( M = 5 \), (\( \times \)) \( M = 6 \), (\( \circ \)) \( M = 10 \).
with \( r \) the end-to-end distance. However it turns out that this case has no true scaling behaviour. Rather the crucial variable is \( \mathcal{U} a = a/\xi_N \), where \( \xi_N \) is the nematic coherence length. The physical significance of \( \xi_N \) is that length of membrane which, if rigid, would be more or less well-ordered in the presence of the field \( \mathcal{U} \).

The vesicle is stretched in the \( y \) direction. A suitable measure of this stretching is the order parameter

\[
\Sigma^* = \frac{\langle R_x^2 \rangle}{\langle R_y^2 \rangle},
\]

where \( R_x \) and \( R_y \) are respectively radii of gyration of the vesicle measured in the \( x \) and \( y \) directions. The quantity \( \Sigma^* \) is defined by analogy with the anisotropy \( \Sigma \) in equation (15). It is related to the two dimensional nematic order parameter \( Q \) for the vesicle shape:

\[
Q = \frac{(1 - \Sigma^*)}{(1 + \Sigma^*)}.
\]

For an ensemble of vesicles with no special anisotropy direction \( Q = 0 \), \( \Sigma^* = 1 \), whereas for vesicles highly stretched in the \( y \) direction \( Q = 1 \), \( \Sigma = 0 \). In figure 7 we plot the behaviour of \( \Sigma^*(\mathcal{U}) \), for a number of different vesicle lengths. Except for very short vesicles (the results of which are not shown), we find that all the curves fall on top of each other to a good approximation.

An interesting feature of these results, whose origin is not immediately apparent, is the apparent saturation of the order parameter \( \Sigma^* \) at approximately 0.1 for large nematic potentials \( \mathcal{U} \). This contrasts with a naive intuition, in which the vesicle is string-like, with two ends each of arc length \( D \sim a \). In this case \( \Sigma^* \) would tend to zero for large \( \mathcal{U} \) and large perimeter length. It seems that there is extra repulsion between the two « sides » of the string, presumably caused by self-avoidance effects. These and related effects are currently under study.

5. Conclusions.

We have investigated the properties of a model two dimensional vesicle. The vesicle consists of a closed membrane subject to a bending rigidity modulus, and extra terms which couple to the area inside it and the orientation of the local arc perimeter. The model is the continuum version of a bead model whose properties were investigated by other authors [6, 12-15] using the Metropolis Monte Carlo method. We have used a method essentially due to Ostrowsky and Peyraud [9] in order to carry out the functional integrals necessary in order to obtain average quantities for this model. We have found that the model is rather more versatile than its authors had originally anticipated; in particular we have been able to verify scaling laws, first found in the bead simulations, for the behaviour of the vesicle as a function of the various control parameters. Despite the encouraging results that we have obtained in this calculation, however, we are not optimistic about using an analogue of this method to investigate vesicles embedded in three dimensional space. In two dimensions the differential geometry is trivial; in three dimensions, however, difficulties emerge which we are not able to overcome at this stage.

Acknowledgements.

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Appendix.

In this appendix we discuss in greater detail the method of integration over phase space used in the Ostrowsky-Peyraud technique. One can set up a configuration vector space defined by the quantities \( X = \{ l A_1, l B_1 ; l = 2, \ldots, M \} \). We can then define a norm on this space by the usual criterion of \( X = \| X \| = \sum \{ l^2 [A_1^2 + B_1^2] \} \). It should be apparent from equation (10) that the norm \( X \) is closely related to the elastic energy of the configuration \( X \). Hence large \( X \) configurations are unlikely on energetic grounds (although how unlikely depends on the value of \( K \)). The direction of \( X \) in this vector space is determined by the ratios of the \( \{ A_1, B_1 \} \). Note however that \( A_1 \) and \( B_1 \) for any \( X \) are determined from the physical closure condition equation (4), and thus the ratios of \( A_1 \) and \( B_1 \) to the other \( \{ A_i, B_i \} \) are not fixed by the direction of \( X \), but depend on its norm as well.

Now in the original paper of Ostrowsky and Peyraud [9] the procedure was to sample uniformly over the configuration space inside a \( (2M) \) dimensional sphere \( R_0 \). By ensuring that \( R_0 \) is sufficiently large, one can guarantee an upper bound to the error induced by sampling inside the sphere rather than over the whole configuration space. A practical difficulty in this approach is that in the floppy limit it is necessary to take \( R_0 \rightarrow \infty \); it is thus only possible to obtain realistic results for ensembles with non-zero curvature constants. However, large \( X \) configurations also are likely to be self-crossing, and self-crossing trajectories do not contribute to any phase space sums. Thus an estimate of the \( R_0 \) necessary to obtain a given degree of accuracy based on elastic energy criteria will in fact be too large because of the added effect of self-crossing.

We have used a sampling procedure which takes account of the self-crossing effects for large \( X \). In order to do this we first investigated the relationship between \( X \) and the vesicle configuration. A given choice of unit configuration vector \( \hat{X} \) (i.e. a set of \( \{ A_i, B_i \} \) subject to the condition that \( |\hat{X}| = 1 \)) generates a one parameter family of vesicles \( \{ X = \alpha \hat{X} \} \). The configuration \( \alpha = 0 \) corresponds to a circle; increasing \( \alpha \) causes the configuration to become increasingly convoluted until, for some critical value of \( \alpha = \alpha_{\text{crit}} \), the configuration finally crosses over itself.

The procedure to check whether a given configuration crosses itself works in the following way. The Cartesian trajectory \( r(s) \) of the vesicle is calculated using equation (3). The trajectory is divided into 60 segments of equal length. No two of these segments may cross. This is determined either trivially (the ends may be so far away from each other that there is no possibility of intersection), or by solving for the intersection of the two lines tangential to each of the segments, and determining whether they intersect within or outside the segment intervals. In all cases of interest 60 segments were sufficient; in a number of cases of possible ambiguity we tested with 120 and 200 segments with no apparent change.

In fact, we find that this one-parameter set of configurations is partitioned into two subsets: \( \alpha < \alpha_{\text{crit}} \), for which the configurations do not cross, and \( \alpha > \alpha_{\text{crit}} \), for which they do. The region of allowed integration is said to be star-shaped.

The value of \( \alpha_{\text{crit}}(\hat{X}) \) is not know a priori. It depends both on \( \hat{X} \) and on \( M \). We have calculated frequency diagrams of \( \alpha_{\text{crit}} \) for various different values of \( M \). Empirically we find that \( 2.4 \leq \alpha_{\text{crit}} \leq 2M \). One could then use the original random sampling procedure of Ostrowsky and Peyraud [9], with \( R_0 \sim 2M \). However, the procedure we actually adopted, which uses the star shaped property of the integration space, allows further optimisation. A unit vector (or ray) \( \hat{X} \) is chosen at random in the configuration space. The value of \( \alpha_{\text{crit}}(\hat{X}) \) is then determined using a binary search method. One may then integrate over the allowed region \( 0 < \alpha < \alpha_{\text{crit}} \). Mean values of thermodynamic quantities are found by
repeating this process over many different rays. In practice we find that a few thousand rays are sufficient to obtain one per cent accuracy in thermodynamic quantities.

References

Revue de livres

Large ion beams. Fundamentals of generation and propagation
A. Th. FORRESTER

Ce texte distribué en 10 chapitres est essentiellement un traité consacré aux sources d’ions. On sait le succès technologique actuel des sources ioniques, utilisant la méthodologie plasma. A cet égard, on peut mentionner les sources faisant usage de la résonance cyclotron (ECR), et les diodes, produisant des faisceaux intenses d’ions légers pour la fusion inertielle. Curieusement, malgré son sous-titre, cet ouvrage exhaustif, essentiellement consacré aux sources d’ions, ignore de telles réussites ! Une explication partielle de cette attitude est donnée par l’auteur dans une seconde préface, que je laisse au lecteur éventuel, le soin de savourer ! Les 3 premiers chapitres sont consacrés à une présentation des plasmas sans collisions. La loi de Child-Langmuir sur les courants intenses y est présentée de plusieurs manières différentes, ainsi que les effets de charge d’espace et de gaine. Cette redondance est d’une incontestable efficacité pédagogique. Les effets collisionnels ou photoabsorption, capture électronique sur états excités, ionisation d’atomes ou d’ions par collisions électroniques, photons, et effet Auger. Les mécanismes atomiques sont clairement discutés, y compris la recombinaison électronique. Les approches alternatives sont traitées au chapitre 6. Elles incluent la génération d’harmonique et le mélange de fréquence (3 ondes) basés sur la polarisation induite non linéaire. Le laser à électron libre (en mode atomique) est ensuite brièvement discuté. Cette partie s’achève sur une bonne mise au point sur le laser gamma. Le chapitre 7 termine le texte sur une prospective pour les lasers du futur, et les nombreuses applications potentielles : lithographie, radiologie médicale, diagnostics des plasmas de fusion, réseaux optiques à très forte résolution, métallurgie... Le texte est très clair et bien organisé. Il est éclairé par de nombreuses figures. Les calculs sont maintenus au minimum indispensable. Un bon équilibre est maintenu entre la discussion des schémas initiaux de pompage, les estimations de gain, les études expérimentales de faisabilité, les modélisations détaillées, les expériences complètes visant à optimiser le gain et les études de développement. L’auteur produit une discussion serrée des configurations ioniques susceptibles d’émission laser, en commençant bien sûr, par la fameuse génre-Ne 3s-3p. Le texte ainsi produit représente une remarquable illustration de la spectroscopie des plasmas chauds rencontrés en fusion thermonucléaire. Chaque chapitre introduit une abondante littérature. Une liste détaillée des symboles utilisés facilite l’entrée dans l’ouvrage. Ce livre peut être abordé avec profit par des tricylistes en Astrophysique, Plasma, Optique, Physique atomique, Spectroscopie... Au niveau des réserves, il faut signaler l’impasse quasi-totale sur les schémas de pompage par faisceaux ioniques intenses, qui semblent très prometteurs. Quelques coquilles sont à éradiquer. En conclusion, il s’agit d’un ouvrage très réussi et qui vient à son public au bon moment.

C. DEUTSCH.

X-Ray Lasers
R. C. ELTON

Il s’agit de la première monographie entièrement consacrée à tous les aspects de la physique du laser X. L’ouvrage est réparti en 7 chapitres. Le premier rappelle les concepts de base de la physique du laser. Le second, le plus important, articule l’ensemble du texte sur les contraintes de l’action laser à courte longueur d’onde. A la suite d’une discussion serrée du gain, apparaissent les nécessités spécifiques de l’émission spontanée amplifiée (ESA) en un seul passage dans le milieu plasma. Les conditions optimales de densité électronique et température sont discutées en détail, en fonction de la longueur
J. BELLET
et l'instrumentation numérique. Aspects plus modernes

L'apex du texte se trouve au chapitre 8 avec la taxinomie des sources d'ions positifs. Ici, on trouve une présentation descriptive et exhaustive de nombreux dispositifs vénérables ou récents tels les sources Penning, Calutron et autres Duoplasmatron. Les sources ECR y sont finalement très brièvement introduites en une page. Le chapitre 9 discute des sources à ionisation superficielle. C'est l'occasion de présenter un certain nombre de dispositifs utilisés en Physique spatiale tels les ioniseurs Sastrugi. L'ouvrage se termine par le stimulant chapitre 10 dédié aux sources d'ions négatifs utilisant la production en volume de ces derniers à l'aide des états excités vibro-rotationnels de la molécule d'hydrogène en milieu plasma. La première motivation de cette contribution est la réaction de double changement de phase qui permet le chauffage additionnel des machines à confinement magnétique. Les deux tiers des chapitres sont suivis de texte de problème, avec corrigés donnés à la fin du texte. Celui-ci est éclairé par de nombreuses figures. A l'exclusion des 4 chapitres d'introduction, le formalisme mathématique est soit inexistant, soit réduit à un très modeste minimum. Au total, malgré l'impasse volontaire sur les dispositifs les plus récents et les plus performants (rien notamment sur les sources LIS au laser), ce texte offre une présentation bien introduite, systématique et pédagogiquement valable de la plupart des sources d'ions. Il peut être abordé avec profit par les étudiants 3e cycle en Physique des plasmas, Physique atomique, Grands instruments et les élèves de nombreuses écoles d'ingénieurs.

C. DEUTSCH.

Courses de Mécanique Générale. Problèmes de Mécanique Rationnelle
D. BELLET


Ce cours au contenu classique, délivré à l'ENSAE (Toulouse) et destiné à la formation des élèves ingénieurs, vise à réaliser un équilibre entre approche fondamentale et applications, rigueur et sens physique. Une première partie est consacrée aux principes fondamentaux, aux formulations newtonienne et lagrangienne, à leur application aux petites oscillations. La seconde partie traite du mouvement des solides en général, des gyroscopes en particulier. Les 42 problèmes traités dans le second volume sont tous composés d'un énoncé complet et d'une mise en équation soignée et d'une solution détaillée.

Aspects théoriques et numériques de la dynamique des structures
J. DONÉA, H. LAVAL, Y. BAMBERGER, R. P. SHAW et J. PLANCHARD


Deuxième volume consacré aux cours CEA-EDF-INRIA délivrés en 1986 à l'Ecole d'été d'Analyse Numérique. Y sont développées les applications de la méthode des éléments finis en dynamique des fluides, les méthodes d'équations intégrales en élasto-dynamique (cours en anglais), l'étude de la
relation entre forme et vibrations et son application au contrôle non destructif, l’analyse des interactions fluide-structure (e.g. faisceaux de tubes immergés). Un ensemble plutôt spécialisé, à réserver aux ingénieurs.

Exercices de mécanique
J. P. DEDONDER, M. MOUCHET, D. SCHMAUS et L. VALENTIN

Questions, exercices et problèmes de 1er cycle et classes préparatoires. L’approche met l’accent sur la signification physique des résultats (ordres de grandeur notamment).

Mécanique des systèmes de solides
M. BAUSSET

Ce livre de cours présente les applications des principes de la dynamique au mouvement d’ensembles des solides sans déformation liés les uns aux autres. Il considère les mécanismes, des plus simples aux plus complexes (coulisse, rotule, engrenage élémentaire, différentiel, chaînes robotiques...), sous l’angle de la dynamique newtonienne et lagrangienne, prend en compte les chocs et percussions, traite le cas des petites oscillations. La mécanique céleste est abordée en vue de l’application aux satellites artificiels.

Mécanique des vibrations linéaires
M. LALANNE, P. BERTHIER et J. DER HAGOPIAN

Ce cours de niveau « école d’ingénieur » développé à l’INSA de Lyon est la 2e édition d’un livre initialement publié en 1980. Il passe en revue les systèmes discrets à nombre croissant de degrés de liberté (1, puis 2, puis n) avant d’introduire les milieux continus (poutres et plaques). Les méthodes numériques (éléments finis) et expérimentales (capteurs, excitateurs) sont au programme.

Mécanique expérimentale des fluides, t. III, recueil de problèmes (4e édition)
R. COMOLET et J. BONNIN

Complément des deux volumes de Mécanique Expérimentale des Fluides, cet ouvrage vise à familiariser l’abord des problèmes qui se posent à l’ingénieur dans sa pratique industrielle. Les nombreux exercices résolus, organisés en 12 chapitres, sont tirés des situations réelles et introduits par de brefs rappels théoriques, de la simple hydrostatique à des études de cas complexes (ventilation d’un tunnel routier...).
Dynamique
J. P. LARRALDE


Ce livre d'introduction, résultat d'un travail collectif, est destiné aux techniciens supérieurs et aux ingénieurs. Chaque chapitre commence par un énoncé d'intentions pédagogiques et se termine par un résumé des éléments essentiels. 14 chapitres de mécanique classique dans une formulation essentiellement newtonienne plutôt élémentaire.