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Magnetic field induced transient periodic dissipative structures in nematics

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Abstract. — The linearized version of nonstochastic continuum theory is used with the rigid anchoring hypothesis to study the occurrence of transient periodic dissipative structures (DS) induced in a nematic sample by a magnetic field $\mathbf{H}$ of supercritical strength applied in a Freedericksz geometry ($\mathbf{H}$ normal to $\mathbf{n}_0$, the uniform initial director orientation with $\chi_a$ the diamagnetic susceptibility anisotropy being positive). The time constant and domain wavevector of DS are found to depend on the field strength $H$, the initial tilt of $\mathbf{n}_0$ relative to the sample planes and the oblique inclination of $\mathbf{H}$ in a plane normal to $\mathbf{n}_0$. In general, DS occurs at high $H$ while the occurrence of a nonperiodic mode is more favourable at low $H$. However, a nonperiodic mode endowed with certain properties might reappear at higher fields in a reentrant way when $\mathbf{n}_0$ is tilted and $\mathbf{H}$ oblique. The cases of negative $\chi_a$ and finite director anchoring energy are treated briefly. Based on the known experimental and theoretical results for the static stripe phase it is suggested that the formation of DS may be hampered by utilizing non-Freedericksz geometries (with $\mathbf{H}$ not normal to $\mathbf{n}_0$). By comparing energy dissipations of DS and the nonperiodic mode it is possible to arrive at the critical point separating the two in a simple geometry for short time regimes (immediately after the application of $\mathbf{H}$).

1. Introduction.

The anisotropic viscoelastic properties of nematics are well understood on the basis of the continuum theory [1, 2] in which the preferred direction of molecular orientation is described by the unit director vector $\mathbf{n}$. Nematic molecules can be uniformly aligned between parallel plates by suitable surface treatment of the sample boundaries. Due to $\chi_a$ of the material, the orientation of $\mathbf{n}$ can be influenced by torques produced by the application of a magnetic field $\mathbf{H}$. Under the action of $\mathbf{H}$, $\mathbf{n}$ changes from one static configuration to another with a lower free energy. The time rate of change of $\mathbf{n}$ occurring during this transition creates a viscous stress which induces transient viscous flow in the nematic sample. This transient flow, which can be understood to dissipate the difference in free energies between the initial and final states, is associated with velocity gradients which produce viscous torques. These transient viscous torques further influence the rate of change of $\mathbf{n}$ leading to many interesting phenomena which can be classified, roughly, into two categories both of which involve the Freedericksz geometry.
In the first case $H$ is impressed on a uniformly aligned sample. When $H$ is gradually increased above the Freedericksz threshold $H_F$, a homogeneous distortion sets in. If $H$ is now switched off the deformation relaxes to the original aligned state. Depending upon the geometry [3] the relaxation is accompanied by transient, nonconvective flow whose extent of coupling with the director field is again dependent on the initial configuration used. In particular, when $H$ is gradually raised well above $H_F$ and then switched off the relaxation of the highly deformed orientation may even cause the transient flow to reverse sign before damping out, leading to the "kickback effect" [4].

In the second case $H$ having several times the strength of $H_F$ is impressed on the uniformly aligned sample producing a strikingly different effect [5]. Optical observation shows that a striped texture (with wavelength $\sim$ sample thickness) appears for some time before $n$ assumes uniform orientation along $H$. This transient striped texture has been observed in different kinds of nematics and in various configurations [6-11]. A simple linear analysis has been developed on the basis of the nonstochastic continuum theory [5] to work out the wavevector of periodicity associated with the fastest growing periodic mode under the assumption that the perturbations are small. This method has led to a qualitative understanding of the variation of the stripe periodicity with different parameters [9-11]. Attempts at studying nonlinear perturbations have been fruitful in bringing greater accord between theoretical results and experimental observations [12], the stability of inversion walls in the context of transient flow has also been considered [13].

From a purely physical viewpoint the occurrence of transient stripes can be understood as being caused by $H$ coupling with director fluctuations and producing a high rate of reorientation with the system, suddenly removed far from its equilibrium state, tending to lower its free energy in the shortest possible time. This results in a correspondingly large viscous stress which cannot be relaxed by nonconvective flow (simply put, the fluid is pushed hard but cannot flow fast enough uniformly parallel to the sample planes). Aided by incompressibility in many cases this ultimately results in setting the fluid into transient convective flow with the magnitude and direction of periodicity being determined by various factors. Equivalently, it can also be said that under the impulse of a high reorienting field the rate of energy dissipation associated with convective flow is lower than that of the nonperiodic flow, in other words, the periodic mode grows faster than the nonperiodic one.

As the transient periodic dissipative structure (DS) appears, remains for some time and then disappears, the associated wavevector of periodicity must itself be a function of time. This fact does not emerge from the simple model [5] based on the nonstochastic theory. It has been shown that information about the time variation of periodicity of DS can be obtained on the basis of a stochastic theory [14-16] by explicitly assuming the form of the director fluctuations which initially couple with the field. Detailed comparison of these results with experiments is yet to be made.

Magnetic and electric field induced static periodic deformations have also been extensively studied [17-20]. It has been shown that the periodicity and even the very occurrence of the distortion can be controlled by varying different factors such as tilts of $H$ and $n_0$, surface anchoring strengths etc.; in some cases the deformation is not even described by a pure symmetry mode. It would be interesting to find out how these factors influence the formation of DS.

In many cases involving static deformations the Freedericksz transition can be regarded as of second order [17-20]. In particular, in cases where periodic and nonperiodic solutions are possible one can work out an expression for the critical point separating the two types of distortions. In a recent paper [21] a convenient method has been developed for studying the critical point in connection with the occurrence of the ripple phase in chiral nematic systems.
subjected to the action of a distorting field. It would be instructive to determine whether this technique can be applied with suitable modifications to the interface region between DS and the nonperiodic mode.

In this communication the nonstochastic continuum theory is used to study the effects of different parameters on the occurrence of DS. In section 2 equations and boundary conditions governing small perturbations are set up for a tilted \( n_0 \) configuration and the method of solution briefly explained. Sections 3 and 4 contain results for different configurations. Certain miscellaneous cases are treated in section 5 and section 6 concludes the discussion.

2. Mathematical model, boundary conditions.

Consider a nematic \((\chi_a > 0)\) aligned in the \(xz\) plane between plates \(z = \pm h\) making angle \(\theta_0\) with the \(x\) axis such that

\[
n_0 = (C, 0, S), \quad C = \cos (\theta_0); \quad S = \sin (\theta_0)
\]  

A magnetic field

\[
H = (-HC_\phi S, HS_\phi, HC_\phi C); \quad S_\phi = \sin \phi, \quad C_\phi = \cos \phi,
\]

is impressed on the sample in a plane normal to \(n_0\) making an angle \(\psi\) with the \(xz\) plane and having a strength

\[
H = RH_F, \quad H_F = (\pi/2 h)[f_2 f_7(f_a f_2 S_\phi^2 + f_7 C_\phi^2)]]^{1/2}, \quad R > 1
\]

where \(H_F\) is the Freedericksz threshold for the configuration (1)-(2) (see Appendix for definition of different symbols). Thus, when \(\theta_0\) is changed, the orientation of \(H\) also changes so that \(H\) is normal to \(n_0\). In the presence of fluctuations \(\theta, \phi\) the director field becomes

\[
n = [\cos (\theta_0 + \theta) \cos (\phi), \sin (\phi), \sin (\theta_0 + \theta) \cos (\phi)]
\]

\(H\) couples with \(\theta, \phi\) causing them to change in time and thus resulting in a viscous stress which produces a flow

\[
v = (v_{xx}, v_{yx}, v_{y})
\]

Following [5] we consider an instant of time soon after \(H\) is impressed so that the perturbations are small. Assuming dependence of the perturbations on \(x, y, z\) and \(t\) (time) and linearizing the governing equations one finds

\[
f_1 \theta_{xx} + f_2 \theta_{zz} + f_3 \theta_{xz} + f_4 \theta + f_5 \phi + K_2 \theta_{yy} + f_{10} \phi_{xy} + f_{11} \phi_{y} - \gamma_1 \theta + \eta_1 (v_{z} - v_{x}) - \eta_8 v_{x,z} = 0
\]

\[
f_6 \phi_{xx} + f_7 \phi_{xz} + f_8 \phi_{xz} + f_9 \phi + f_5 \theta + K_1 \phi_{yy} + f_{10} \theta_{xy} + f_{11} \theta_{y} - \gamma_1 \phi - \eta_1 (v_{y} - v_{x}) - \eta_8 v_{x,y} = 0
\]

\[
\eta_1 v_{xx} + \eta_2 v_{xz} + \eta_3 v_{x} + \eta_4 v_{z,x} + \eta_5 v_{z,x} + \eta_6 v_{z,z} + \eta_7 \phi_{,x} + \eta_8 \phi_{,y} - \rho v_{x} + \tau_1 v_{,y} + \tau_2 v_{,x} + \tau_3 v_{,y} + \tau_4 v_{,x} + \tau_5 \phi_{,y} = p_{xx}
\]

\[
\eta_9 v_{y,y} + \eta_10 v_{y,z} + \eta_11 v_{y,x} + \eta_12 \phi_{,x} + \eta_13 \phi_{,z} - \rho v_{y} + \tau_2 v_{x,x} + \tau_3 v_{x,y} + \tau_6 v_{z,z} + \tau_9 v_{y,y} + \tau_3 v_{z,x} = p_{yy}
\]
\[ \eta_4 v_{xx} + \eta_5 v_{xx} + \eta_{14} v_{xx} + \eta_{15} v_{xx} + \eta_{16} v_{xx} + \eta_{17} \theta_{x} - \\
- \eta_7 \theta_{x} - \rho \dot{v} + \tau_1 v_{xy} + \tau_3 v_{yy} + \tau_4 \dot{\theta} = p_{x} - \tau_{5} v_{xy} + \tau_{6} v_{yy} + \tau_{7} v_{yy} + \tau_{8} \phi_{y} = p_{x} \]  
(10)

where \( p \) is the fluctuation in pressure and (11) represents the condition of incompressibility, \( v_{n,y} = \dot{v}/\dot{y} \) etc., \( \phi = \phi_{x}/\dot{t} \) etc. The no slip conditions for \( v \) at the boundaries become

\[ (v_{x}, v_{y}, v_{z})(z = \pm h) = 0 \]  
(12)

For rigid anchoring of the director at the sample planes

\[ (\theta, \phi)(z = \pm h) = 0 \]  
(13)

If, however, the surface energy per unit area takes the simple form [22]

\[ W_{s} = \left( B_{0} \theta^{2} + B_{\phi} \phi^{2}\right)/2 \]  
(14)

the boundary conditions on \( \theta \) and \( \phi \) take the form

\[ \Gamma_{\theta} + B_{\phi} \phi \]  
(12)

\[ \Gamma_{\phi} = f_{2} \theta_{x} + (f_{3}/2) \phi_{x} \]  
(13)

where \( K_{4} \) has been ignored; \( B_{\theta}, \dot{B}_{\theta} \) and \( B_{\phi}, \dot{B}_{\phi} \) are the splay and twist anchoring strengths at the boundaries \( z = + h, - h \) respectively. For the moment we shall assume rigid anchoring (13) and return to (15) at a later stage. The equations have been derived in a general form so that subsets of the expressions can be used for studying different special cases which arise in the discussion.

The first step is to find the growth rate for the nonperiodic Mode \( N \) in which the perturbations depend on only \( z \) and \( t \). It is convenient to determine the inverse rise time \( s_{N} \) (rather than the actual rise time \( T_{N} \)) such that the perturbations have a time dependence \( \sim \exp(s_{N} t) \). For Mode \( N \), \( v_{x} = 0 \) and (10) essentially defines \( p \) to within an arbitrary constant. In the most general case for Mode \( N \), (6)-(9) result in four coupled ordinary differential equations in which \( s_{N} \) now occurs as an additional parameter. By solving (6)-(9) with boundary conditions (12)-(13), \( s_{N} \) is determined for a given value of the reduced field \( R \). It must be remembered that, in general, \( s_{N} \rightarrow 0 \) as \( R \rightarrow 1 \), i.e. as \( H \rightarrow H_{F} \) the Freedericksz threshold.

For a solution periodic along \( x \) (Mode \( P \)) the ansatz \( \exp(s_{R} t + i q_{x} x) \) is made for the perturbations under the assumption that the lateral boundaries of the sample are far off, here, \( s_{R} = s_{R}(q_{x}) \) is the inverse rise time at wavevector \( q_{x} \). In the most general case (6)-(11) result in five coupled ordinary differential equations containing \( s_{R} \) and \( q_{x} \) as additional parameters and by using (12)-(13) \( s_{R} \) can be determined as a function of \( q_{x} \). When \( R \) is sufficiently large, \( s_{R}(q_{x}) \) generally increases with \( q_{x} \) and attains a maximum \( s_{P} = s_{R}(q_{P}) \) when \( q_{x} = q_{P} \). If \( s_{P} > s_{N} \), i.e. if Mode \( P \) grows faster than Mode \( N \), we say [5] that Mode \( P \) occurs with wavevector \( q_{P} \); this is equivalent to asserting that the pattern has periodicity with a wavelength \( \lambda_{P} = 2 \pi/q_{P} \) or a stripe width \( = \lambda_{P}/2 \). If \( s_{P} < s_{N} \), we conclude that Mode \( N \) is more favourable than Mode \( P \). It is usually found that for a given \( R \), \( s_{R}(q_{x}) \rightarrow s_{N} \) as \( q_{x} \rightarrow 0 \), thus Mode \( P \) can be generally regarded as a continuation or generalization of Mode \( N \).
Calculations are not always straightforward. In general, (6)-(9) support two independent nonperiodic modes — Modes $N_1$ and $N_2$. The inverse rise times $s_{N_1}$ and $s_{N_2}$ are calculated for both these modes and the higher of the two values is taken as $s_N$. A similar situation can arise for the periodic mode in certain cases, again, the higher of the two inverse rise times is taken as $s_p$ and the corresponding $q_p$ is regarded as the domain wavevector of DS.

Oblique patterns have also been observed [9] To study these one would assume the additional dependence of $\exp(iq_yy)$ while considering DS. This would lead to $s_k$ becoming a function of $q_x$ and $q_y$. Variation of $q_x$ and $q_y$ should result in a neutral stability surface showing an absolute maximum $s_p(q_{x,p}, q_{y,p})$ defining the characteristics of DS. In this communication these general periodic solutions are not studied.

The material parameters chosen are those for MBBA

$$ (K_1, K_2, K_3) = (6.66, 4.2, 8.61) \times 10^{-7} \text{ dyne} \ [23] , $$
$$ \chi_a = 1.15 \times 10^{-7} \text{ emu} \ [25] , \quad \rho = 1.088 \text{ cgs} \ [26] ; $$

$$ (\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6) = (-18.1, -110.4, -1.1, 82.6, 77.9, -33.6) \times 10^{-2} \text{ poise} \ [24] $$

The sample thickness $2h = 0.02 \text{ cm}$, angles are measured in radian and $H$ in emu. The inclusion or otherwise of the inertial term (proportional to $\rho$) makes little difference to the results which are presented in subsequent sections for different cases. It is, of course, known [5] that this term may be important in case DS is studied in very thick samples.

3. Results for homogeneous initial orientation.

For the sake of continuity and completeness, results are first presented briefly for the twist and splay geometries, these cases have been studied in some detail in earlier work [5, 7, 9]. Then the case of an oblique field is considered. Now $\mathbf{n}_0 = (1, 0, 0)$, i.e., $\theta_0 = 0$ with $S = 0$ and $C = 1$.

3.1 Twist Geometry, $\psi = \pi/2$. With $H$ along $y$, Mode $N$ is described by $\phi$ alone as $\theta$, $v_x$ and $v_y$ vanish. When $\phi$ is even w.r.t. the sample centre (Mode $N_1$) the lowest harmonic has the inverse rise time

$$ s_N = (\pi^2 K_2/4 \gamma_1 h^2)(R^2 - 1) \ [17] $$

while for DS (Mode $P_1$ with $\phi$ and $v_y$ even $w.r.t. z = 0$) [7] one finds

$$ s_R h^2/K_2 = [\alpha_1 Q_x^4 + \alpha_2 Q_x^2 + \alpha_3]/(\beta_1 Q_x^2 + \beta_2), \quad \alpha_1 = K_3 \eta_\phi/K_2 \eta_a, $$

$$ \alpha_2 = q_\xi^2[(\eta_\phi/\eta_a)(1 - R^2) + (K_3/K_2)], \quad \alpha_3 = (1 - R^2) q_\xi^2, \quad \beta_1 = (\mu_5^2 - \gamma_1 \eta_c)/\eta_a, $$

$$ \beta_2 = -\gamma_1 q_\xi^2, \quad q_\xi = \pi/2, \quad Q_x = q_x h \ [18] $$

from which the condition $\partial s_R/\partial Q_x = 0$ yields the (dimensionless) domain wavevector $Q_p$ and $s_p$ the corresponding (maximum) time constant for Mode $P_1$. Then $Q_P = 0$ yields the critical point $R_c$ given by

$$ R_c^2 = 1 + \gamma_1 K_3 \eta_\phi/K_2 \mu_5^2 \ [19] $$

such that DS can appear only for $R > R_c$, as $R \to R_c$, $Q_p \to 0$ and $s_p \to s_N$ showing that for $R < R_c$, DS cannot exist and only Mode $N_1$ is favourable. Figures 1a, 1b which illustrate the
variations of \( R_s = s_p/s_N \) and \( Q_p \) as functions of \( R \) bear out the above conclusions. Plotting the ratio \( R_s \) rather than the individual inverse times separately makes a compact presentation of results possible as will become clear especially in later sections. It must be noted that while Mode \( N_1 \) is associated with the single orientational degree of freedom \( \phi (z, t) \), Mode \( P_1 \) is described by both \( \phi (x, z, t) \) and the translational degree of freedom \( v_y (x, z, t) \). Hence it can be concluded that when \( Q_x \to 0 \), \( v_y \) must also vanish; this is indeed the case:
\[
v_y = v_{y0} \cos (q \xi z/h) \exp (s_p t), \quad v_{y0} = -Q_x (\beta_3 Q_x^2 + \beta_4)/(\beta_1 Q_x^2 + \beta_2); \quad \beta_3 = K_3 \mu_2/\eta_a h; \quad \beta_4 = K_2 \mu_2 q_x^2 (1 - R^2)/\eta_a h.
\]
(20)
\( v_x (x, z, t) \) which represents modulated flow along the sample planes can be understood to arise out of the stress \( \sigma_{xy} \) created primarily by the fluctuation \( \phi (x, z, t) \), the incompressibility condition (11) does not explicitly enter the picture. As described by figures 1a, 1b, DS can be regarded as appearing above a second order phase transition evolving continuously out of the initial configuration, \( Q_p \) can be looked upon as the (unscaled) order parameter of DS. Higher the \( H \) (or \( R \)), higher the \( Q_p \) and faster the growth rate of DS relative to Mode \( N \). Before concluding this section it must be remembered that for \( R < R_c \), Mode \( N_1 \) continues to exist with \( s_{N_1} \to 0 \) as \( R \to 1 \).

3.2 Splay Geometry, \( \psi = 0 \) --- \( H \) is along \( z \). Mode \( N \) is associated with the splay angle \( \theta (z, t) \) and the backflow \( v_x (z, t) \) [3]. The independent Mode \( N_1 \) (or \( N_2 \)) is described by even (or odd) \( \theta \) and odd (or even) \( v_x \). Thus Mode \( N_2 \) (or \( N_1 \)) involves (does not involve) net flow along \( x \). For reasons that become clear soon both modes will be studied. For Mode \( N_1 \) (ignoring inertial effects for the moment) one writes
\[
D \epsilon_\xi = -\delta_2 \frac{\partial \theta_1}{\partial t}; \quad D = d/d\xi, \quad q_\xi^2 = (\gamma h^2 + s_N \gamma^* h^2/K_1), \quad \gamma^* = -\gamma_1 + \mu_3^2/\eta_b, \quad \delta_1 = -h\mu_3/K_1, \quad \delta_2 = -\frac{h}{\eta_b}.
\]
(21)
where \( \sigma \) is proportional to the constant amplitude of the viscous stress \( \sigma_{xy} \) associated with the perturbations. From (12)-(13) it is seen that
\[
q_\xi^2 + (q_\xi - \tan q_\xi) \delta_1 \delta_2 = 0.
\]
(22)
As \( \delta_1 \delta_2 < 0 \) for \( s_N > 0 \), solutions for (22) exist only for \( q_\xi > \pi/2 \). Once the \( q_\xi = q_1 \) is determined from (22) the time constant for Mode \( N_1 \) is found from
\[
s_{N_1} = (K_1 q_1^2 - \chi a h^2 H^2)/h^2 \gamma^*_1.
\]
(23)
It is also clear from (21) that \( q_1 \) increases with \( H \). For Mode \( N_2 \) one writes (21) with \( \sigma = 0 \) and the time constant \( s_{N_2} \) is now found from (23) with \( q_1 = \pi/2 \).

For \( H > H_F \) the splay Freedericksz threshold, \( s_{N_1} > s_{N_2} \). When \( H \) becomes sufficiently high, however, \( s_{N_2} > s_{N_1} \); then Mode \( N_2 \) becomes the faster growing mode. This crossover point between Modes \( N_1, N_2 \) can be found from (21)-(23) to occur at
\[
H = H_N = 2 H_F (\gamma_1 \eta_b/\mu_3)^{1/2}.
\]
(24)
For MBBA parameters $H_N \sim 100 H_F$. As $R$ is restricted to about 10 in this work the crossover will not be encountered and $s_{N_1}$ will be taken as the time constant for Mode $N$ in splay geometry.

At this stage the following observations may be made though they are not of direct relevance in the present section (i) $\sigma$ of (21) is a constant of integration and cannot, therefore, be arbitrarily equated to zero. If this were done, it would be difficult to solve for $v_x$ while studying Mode $N_1$ though the crossover between the Modes $N$ would not now occur as $q_\xi$ for Modes $N_1, N_2$ would remain constant at $\pi/2, \pi$ respectively. Thus the crossover
between the Modes $N$ is not simply a mathematical result but follows necessarily when an
exact solution is sought (ii) In case $\mu_3 = 0$, there would not only be no coupling between flow
and orientation, $v_x$ would itself vanish identically as a consequence of the no slip condition.
Then again Mode $N_1$ would remain the faster growing mode (iii) In the opposite limit where
$\mu_3$ becomes large and positive (say close to the nematic-smectic A transition temperature [2])
$H_N$ may get lowered to roughly $2 H_F$ (iv) The amplitudes of the Mode $N_1$ perturbations are
proportional to $\delta J \sigma$ while those of Mode $N_2$ are proportional to the amplitude of $\tau$ which is
again an arbitrary constant. This makes comparison of the two modes difficult $w \cdot r \cdot t$
quantities like free energy and energy dissipation rate (v) the unique nature of Mode
$N_2$ must be appreciated. While Mode $N_1$ is associated with a stress $\sigma_{xx}$ there is no net flow
along $x$ as $v_x$ is odd $w \cdot r \cdot t$ Mode $N_2$ has the opposite attributes, it is associated with
zero stress $\sigma_{xx}$ and a net flow along $x$ as $v_x$ is even $w \cdot r \cdot t$ the sample centre. Such solutions
have been encountered in other flow situations [27] and can be regarded as being peculiar to a
structured fluid like a nematic which has an additional (orientational) degree of freedom. At
first sight it seems strange that there should occur a flow along $x$ even when $\sigma_{xx}$ is zero (the
force along $x$ is given by $\sigma_{xx}$, ), indeed, such a solution would be difficult to accept in the case
of an isotropic fluid In the case of a nematic this is still possible because both linear and
angular momenta have to be balanced for a complete solution. In the present situation the
viscous torque $\sim \sigma_{xx} - \sigma_{xz}$ does not vanish as $\sigma_{xz}$ is nonzero. The torque balance results in
one relationship between $v_x$ and $\theta$. The balance of linear momentum ($\sigma_{xx} = 0$) gives another
relationship between these quantities, these two can be solved to get the Mode
$N_2$ solution If an a priori assumption were to be made that either $\theta$ or $v_x$ were zero, the
remaining quantity also would vanish and Mode $N_2$ would not exist as a solution. As stated
earlier, such an assumption may be difficult to make while using a phenomenological theory

For the splay geometry DS is associated with $\theta, v_x, v_z$ which depend on $x, z, t$. The presence of $v_z$
is necessitated by the incompressibility condition (11) One again encounters two uncoupled modes — Mode $P_1$ (or Mode $P_2$) with $\theta, v_x$ even (or odd) and $v_z$ odd (or even).
Mode $P_1$, which is the faster of the two, is studied by solving (6), (8), (10)-(13) to estimate
$s_p$ and $Q_p$. Figures 1c, 1d depict plots of $R_q$ and $Q_p$ as functions of $R$ for MBBA. It is seen that
when $R \to a$ lower limit $R_q(=2.4), R_q \to 1$ For $R < R_q$, $R_q < 1$ showing that Mode
$P_1$ exists as a solution but does not grow faster than Mode $N_1$. In addition $Q_p$ does not
become zero in this limit. For $R < 2.3$, it appears not possible to find Mode $P_1$ even as a
solution. It must be stressed that Mode $N_1$ will continue to exist for $1 < R < 2.4$ such that
$s_{N_1} \to 0$ as $R \to 1$. This also means that $s_{Q_p}(Q_p) \to s_{N_1}$ as $Q_p \to 0$ for any value of $R$, but for
$R < 2.4$, $s_{Q_p}$ cannot show a maximum at any value of $Q_p$. This qualitatively differentiates the results for splay and twist geometries. If $Q_p$ is regarded as the order parameter of DS then it is clear that DS cannot be understood to evolve
continuously from the initial configuration as $R$ is increased. This can be intuitively appreciated by observing that Mode $P_1$ is described by the additional translational degree of freedom $v_x$, a convective flow against the sample planes which comes into the picture mainly
due to the condition of incompressibility. A qualitative difference between the two geometries is that for a given value of the reduced field $R$, the stripes are narrower in splay
graphy but the growth rate of Mode $P$ relative to that of Mode $N$ is higher in twist
geometry.

3.3 Oblique Field $H$, $0 < \psi < \pi/2$ — $H$ is now impressed in the $yz$ plane making angle $\psi$
with the $z$ axis so that

$$H_F = (\pi/2 \ h)[K_2 K_1 / \{\chi_a (K_1 S_0^z + K_2 C_0^z)\}]^{1/2}$$
Clearly the two configurations already considered are special cases of the present one. When \( H \) is oblique, the cross terms \( f_5 \phi \) and \( f_5 \theta \) in (6), (7) couple the \( \theta, \phi \), \( v_x \), \( v_y \), \( v_z \) and the \( \phi, v_y \) modes so that DS is now determined by all the five perturbations regarded as functions of \( x, z, t \). In the limits \( \psi \to 0 \) and \( \psi \to \pi/2 \) the results of this section reduce to those of sections 3.2 and 3.1, respectively. It is also clear that the extent of « mixing » of the two solutions will be maximum when \( \psi \) is close to \( \pi/4 \).

The nonperiodic solution is now determined by \( \theta, \phi, v_x \) and breaks up again into two independent modes — Mode \( N_1 \) (or \( N_2 \)) with \( \theta, \phi \) even (or odd) and \( v_x \) odd (or even). As the effective order of the equations now becomes six, it is not straightforward to get analytical expressions for the time constants of the Modes \( N \) and to determine whether there will be a crossover. It is found numerically that over the values of \( \theta \) range considered Mode \( N_1 \) grows faster than Mode \( N_2 \) so that the former is compared with Mode \( P \) (DS).

Mode \( P \) also dissociates into two uncoupled modes — Mode \( P_1 \) (or \( P_2 \)) with \( \theta, \phi, v_y, v_z \) even (or odd) and \( v_x \) odd (or even). Mode \( P_1 \) is found to grow faster than Mode \( P_2 \). It is to be noted that \( \theta \) the field ratio, \( Q_p \) the wavevector of Mode \( P_1 \), as well as \( \theta_p \) the ratio of the time constants of DS and Mode \( N \) are all functions of the additional parameter \( \psi \), the field angle. Figures 1e, 1f containing plots of \( \theta \) and \( Q_p \) as functions of \( \theta \) for different magnetic tilt angles are in qualitative accord with figures 1a-1d. The \( \theta \) range of existence of DS becomes narrower as \( H \) is rotated away from the \( y \) axis.

Figures 2a, 2b contain plots of \( \theta \) and \( Q_p \) as functions of \( \psi \) at different field strengths \( H \). It is found that \( \theta \) increases but \( Q_p \) diminishes as \( \psi \) is changed from 0 (splay geometry) to \( \pi/2 \) (twist geometry). In other words, as \( H \) is rotated from \( z \) to \( y \) in the \( yz \) plane, the domain width of DS should increase. The fact that this is accompanied by an increase in the growth rate of DS relative to that of Mode \( N \) can be tentatively understood by noting that, when \( \psi \) is increased from 0 at constant \( H, \theta \) is also being enhanced as \( H_{\theta} \) decreases.

So far only the ratio \( \theta_p \) has been studied. Given below are the actual values of the growth rates calculated at three different field angles and \( H = 2 \) 000 emu:

\[
\begin{align*}
\psi &= 0.00, \quad s_p \text{ (sec}^{-1}) = 0.88, \quad s_N \text{ (sec}^{-1}) = 0.406, \quad R_s = 2.16, \quad Q_p = 3.93; \quad R = 5.29. \\
\psi &= 0.75; \quad s_p \text{ (sec}^{-1}) = 1.10, \quad s_N \text{ (sec}^{-1}) = 0.408, \quad R_s = 2.68, \quad Q_p = 3.51, \quad R = 5.97. \\
\psi &= 1.55, \quad s_p \text{ (sec}^{-1}) = 1.42, \quad s_N \text{ (sec}^{-1}) = 0.411; \quad R_s = 3.45, \quad Q_p = 3.13, \quad R = 6.66
\end{align*}
\]

(25)

It is found that \( s_N \) changes very little when \( \psi \) is changed. This is mainly due to the weak coupling between flow and orientation. This can be visualized from the fact that the effective viscosity associated with Mode \( N \) changes from \( \gamma_1 \) for \( \psi = \pi/2 \) where there is no secondary flow to \( \gamma_1 - \mu \psi^2/\eta_0 \) for \( \psi = 0 \).

4. Results for tilted initial director orientation; \( \theta_0 \neq 0 \).

Here \( \eta_0 \) is assumed to be tilted making angle \( \theta_0 \) with the \( x \) axis in the \( xz \) plane so that results of the previous section can be regarded as those for the special case \( \theta_0 = 0 \). With \( \theta_0 \) coming in as an additional parameter it should be interesting to find out in what way this entity affects the different solutions. It is again convenient to split the discussion into three parts depending upon the tilt of \( H \).

4.1 « TWIST GEOMETRY » WITH PRETILT, \( \psi = \pi/2 \) — An immediate consequence of the director pretilt is that Mode \( N \) is now described by the twist \( \phi(z,t) \) and also the transverse flow \( v_y(z,t) \). Interestingly, the modal structure here follows that of the modes \( N \) of section 3.2 and this facilitates the study. The solution for the two independent modes — Mode
Fig 2 — $R$, and $Q_p$ versus $\psi$ the angle made by $H$ with the $xz$ plane (a, b) $n_0$ is along $x$ (homogeneous initial orientation) Curves are drawn for $H = (1) 1500 (2) 2000 (3) 2500$ emu When $H$ is rotated away from $z$ axis, the DS domain width should increase and DS should grow faster $w.r.t$ Mode $N$ (Sect. 3.3, Eq (25)) (c, d, e, f) The initial orientation is in the $xz$ plane and tilted at angle $\theta_0$ with $x$ axis so that $n_0 = (C, 0, S)$ $H$ is normal to $n_0$ and makes angle $\psi$ with the $xz$ plane (Eq (2)). One value of $H = 2000$ emu is used Curves are drawn for $\theta_0 = (1) 0.01 (2) 0.25 (3) 0.5$ rad in (c, d) and for $\theta_0 = (1) 0.75 (2) 1.0$ rad in (e, f). While the nonperiodic solution breaks up into two uncoupled modes $N_1$, $N_2$, the periodic solution does not, DS is described by perturbations without definite spacial symmetry $R_s = R_{s1}$ or $R_{s2}$ which ever is lower, $R_{s1} = s_p/s_{N1}$ and $R_{s2} = s_p/s_{N2}$ where $s_{N1}$, $s_{N2}$ are the inverse growth rates of the nonperiodic modes $N_1$, $N_2$, respectively In (e, f), $R_s = R_{s2}$ as Mode $N_2$ is faster than Mode $N_1$ over the entire $\psi$ range In (c, d), $R_s = R_{s1}$ for curve 1 For curves 2, 3, $R_s = R_{s2}$ to the right of the cusp and $R_s = R_{s1}$ to the left of the cusp. For all values of initial tilt chosen, the width of DS domains increases as $H$ is rotated away from the $xz$ plane The variation of the relative growth rates of the periodic and nonperiodic modes is, however, strongly dependent on the initial tilt (Sect 4.3)

$N_1$ (or $N_2$) associated with $\phi$ even (or odd) and $v_y$ odd (or even) — follows exactly as set out in (21)-(24) except that we put

$$\theta \rightarrow \phi ; \ v_x \rightarrow v_y ; \ K_1 \rightarrow f_7 ; \ \mu_3 \rightarrow \eta_{13} ; \ \eta_b \rightarrow \eta_{10}$$ \hspace{1cm} (26)

$\sigma_{y_2} \sim \sigma \exp(s_{N} t)$ is the viscous stress associated with the perturbations but $\sigma = 0$ for Mode $N_2$. Mode $N_2$ grows faster than Mode $N_1$ provided that

$$H > H_N(\theta_0) = 2 HF(\gamma_1 \eta_{10}/\eta_{13})^{1/2}, \ HF = (\pi/2 h)(f_7/x_0)^{1/2}$$ \hspace{1cm} (27)

When $\theta_0$ is high enough, $H_N$ is quite low, for instance, $H_N = 2.2 HF$ for $\theta_0 = \pi/2$. When $\theta_0 \rightarrow 0$, $H_N \rightarrow \infty$ as it should, Mode $N_2$ is always lower than Mode $N_1$ in the twist geometry. Using (27) one can also arrive at a $\theta_N$ for a given value of $H$ such that for $\theta_0 = \theta_N$, Mode $N_2$ grows faster than Mode $N_1$. By solving for $v_y$ one finds that $v_y \rightarrow 0$ as $\theta_0 \rightarrow 0$ again in qualitative agreement with results of section 3.1. The above conclusions are borne out by figures 3a, 3b which present variation of growth rates for Modes $N_1$, $N_2$ as functions of $\theta_0$ and $R$.
With $\mathbf{H}$ along $y$, the $\theta, v_x, v_z$ mode gets damped out leaving only the $\phi, v_y$ mode in the case of DS as for the twist geometry of section 3.1 Interestingly, however, now there exists no pure modal structure. Mode $P$ is a superposition of Modes $P_1$ and $P_2$ of section 3.1 with a phase difference of $\pi/2$ along $x$. This «mode mixing» occurs due to the pretilt of $n_0$, which brings in additional elastic and viscous couplings which are absent in the twist geometry where pretilt is zero. Noting that the governing equations go over to those of twist geometry in the limit $\theta_0 \to 0$, one can intuitively discern that as $\theta_0$ is gradually increased from zero, a progressively greater amount of Mode $P_2$ will get «mixed» with Mode $P_1$. Mode $N$, on the other hand, is still described by one of two uncoupled modes As Mode $P_2$ is generally the slower mode, this mode mixing can be expected to affect the growth rate of Mode $P$ relative to that of Mode $N$.

Figures 3c, 3d show the variation of the relative growth rates of the periodic and nonperiodic modes as functions of $H$ (for a given $\theta_0$) and as functions of $\theta_0$ (for a given $H$).

![Fig. 3](image-url)

**Fig. 3** — (a, b) Plots of inverse rise times $s_{N_1}$ (odd numbered curves) and $s_{N_2}$ (even numbered curves) for the nonperiodic Modes $N_1$ and $N_2$, respectively, as functions of the reduced field $R$ and initial director tilt $\theta_0$. $\mathbf{H}$ is along $y$ and $n_0 = (C, 0, S)$ in the $xz$ plane $R = H/H_E$ where $H_E = (\pi/2)h/(\kappa_i)\frac{1}{2}$ is the Freedenncksz threshold Mode $N_2$, but not Mode $N_1$, is associated with a net flow along $y$. In (a), $\theta_0 = 0.5$ radian $H_N$ corresponds to $R = 3$. For $R > 3$ (or $R < 3$) Mode $N_1$ (or Mode $N_2$) is the faster growing mode. In (b), $H = 2000, 2500, 3000$ emu for curves (1, 2), (3, 4), (5, 6), respectively. For a given $H$ there exists a $\theta_N$ such that for $\theta_0 > \theta_N$, Mode $N_2$ grows faster than Mode $N_1$. Values of $H_N$ and $\theta_N$ are in good agreement with (27). This is helpful in finding out the faster growing Mode $N$ whose time constant can be compared with that of Mode $P$ (c, d) Plots of $R_1$ and $R_2$ as functions of $R$ and $\theta_0$. $R_1 = s_p/s_{N_1}$ and $R_2 = s_p/s_{N_2}$ where $s_p$ is the inverse rise time of Mode $P$ which is described by perturbations having no modal purity. In (c), $\theta_0 = 0.25$ radian In (d), $H = 2000$ emu. The dashed parts of curves (which are higher) correspond to mode $P$ being compared with the slower Mode $N$, these parts of the curves are considered to be of academic interest. The solid lines of the curves intersect in a cusp and serve to illustrate ranges of $H$ or $\theta_0$ over which $N_1$ or $N_2$ is the faster growing mode $N$. This way of illustration also indicates into which Mode $N$, Mode $P$ will merge when $Q_p \to 0$. Thus, for low $H$ (c), Mode $P$ merges into Mode $N_1$. When $\theta_0$ is sufficiently high (d), Mode $P$ approaches Mode $N_2$ (Sect. 4.1).
respectively. The fact that the formation of Mode $P$ becomes unfavourable when $R$ becomes sufficiently small (Fig 3c) is in qualitative agreement with the results of figures 1a, 1b. The conclusion that Mode $P$ can be suppressed for a given $H$ when $\theta_0$ exceeds a certain limit (Fig. 3d) is specific to the present case. This can be understood as being caused by two factors. (i) Increase of $\theta_0$ at constant $H$ has the same result as decreasing $R = H/H_0(\theta_0)$; but a decrease in $R$ will diminish the time constants of all the three modes ($N_1, N_2, P$), hence this cannot be regarded as the major cause (ii) The occurrence of mode mixing must be borne in mind with the slower component (Mode $P_2$) determining a greater portion of the total solution (DS) as $\theta_0$ is enhanced from zero. This can be expected to decrease the growth rate of Mode $P$ below that of Mode $N$ (pure mode) when the director pretilt exceeds a critical value.

It must be stressed that in figures 3c, 3d both $R_s = s_p/s_N$ and $R_s = s_p/s_{N_2}$ vary smoothly with $R$ and $\theta_0$. However, as Mode $N_1$ is faster than Mode $N_2$ for small $\theta_0$ or $H$, $R_s < R_s$ in these ranges. When $H$ or $\theta_0$ is sufficiently high the reverse holds good with Mode $N_2$ becoming faster than Mode $N_1$. As we are interested in comparing the growth rate of Mode $P$ with that of the faster growing Mode $N$, only the solid parts of the curves in figures 3c, 3d are meaningful. If we define $R_s$ as the smaller of $R_s$ and $R_s$, only the solid part of the curve will be obtained. The cusp in the curve will help demarcate the range of variable over which Mode $N_1$ or Mode $N_2$ is the faster growing Mode $N$. Secondly, such a procedure will also indicate into which Mode $N$, Mode $P$ will merge (or towards which Mode $N$, Mode $P$ will approach) when $R_s \to 1$. Thus, for instance, when $H$ is sufficiently small (Fig 3c) and $Q_p \to 0$, $s_p \to s_N$ showing that Mode $P$ approaches Mode $N_1$ at this point. Similarly (Fig 3d) when $\theta_0 \to 12$ and $Q_p \to 0$, $s_p \to s_{N_2}$ so that it is clear that Mode $P$ will merge into Mode $N_2$ in this limit. Lastly, this will also help present results in a compact manner, otherwise it will be necessary to plot $s_p$, $s_{N_1}$, $s_{N_2}$ as functions of different variables and this will lead to a cluttering up of the curves. The above comments must be borne in mind when examining results of figures 4a, 4b and 5a, 5b. It is clear that the $\theta_0$ range of existence of DS broadens when $H$ is increased, similarly, the $R$ range of occurrence of DS shrinks when $\theta_0$ is enhanced, i.e. when the initial director orientation is removed from the homogeneous. At a given $H$, increase of $\theta_0$ should be accompanied by a broadening of the DS domains.

So far only the relative growth rate has been discussed. From the experimental view point it may be interesting to know how the actual growth rate of DS will change when $\theta_0$ is increased at a given $H$, say 2000 emu, this is contained in (28) below.

\[
\begin{align*}
\theta_0 &= 0.01, \quad s_p = 1.420 \ (\text{sec}^{-1}), \quad s_{N_1} = 0.41 \ (\text{sec}^{-1}), \quad s_{N_2} = 0.38 \ (\text{sec}^{-1}), \quad Q_p = 3.13 \\
\theta_0 &= 0.60, \quad s_p = 1.423 \ (\text{sec}^{-1}), \quad s_{N_1} = 0.64 \ (\text{sec}^{-1}), \quad s_{N_2} = 0.74 \ (\text{sec}^{-1}), \quad Q_p = 2.37 \\
\theta_0 &= 1.19, \quad s_p = 1.621 \ (\text{sec}^{-1}), \quad s_{N_1} = 1.28 \ (\text{sec}^{-1}), \quad s_{N_2} = 1.62 \ (\text{sec}^{-1}), \quad Q_p = 0.25
\end{align*}
\]

It is seen that $s_p$ increases markedly only when $\theta_0$ is enhanced sufficiently. When $\theta_0$ is high enough, $s_p$ is overtaken by $s_{N_2}$ which increases very rapidly in that range of pretilt.

4.2 « SPLAY GEOMETRY » WITH PRETILT, $\psi = 0$ — $H$ is now applied in the $xz$ plane normal to $n_0$. The nonperiodic solution is again associated with $\theta(z, t)$ and $v_x(z, t)$, as in section 3.2, except that $\theta_0$ enters as an additional parameter through the different elastic and viscous couplings. The modal structure is the same as in section 3.2 except that the crossover field $H_N$ is now given by

\[
H_N(\theta_0) = 2 H_F(\eta_2 \gamma_1/\eta_2^*)^{1/2}; \quad H_F = (\pi/2h)(f_2/x_a)^{1/2}.
\]
Fig 4 — Variations of $R_s$ and $Q_p$ as functions of $R$ for different initial director pretilt angles $\theta_0$ and magnetic tilt angles $\psi$. $R_s$ is $R_{s1}$ or $R_{s2}$ whichever is less (a, b) $\psi = \pi/2$ (« twist geometry » with pretilt) $H$ is along $y$ and $n_0$ is tilted in the $xz$ plane. Curves are drawn for $\theta_0 = (1)$ 0.01 (2) 0.5 (3) 0.75 (4) 1.0 radian. In curve 1, $R_s = R_{s1}$ (Mode $N_2$ is throughout slower than Mode $N_1$.) Similarly, $R_s = R_{s2}$ in curve 4. In curves 2, 3, $R_s = R_{s2}$ (or $R_s = R_{s1}$) to the right (or to the left) of the cusp. The $R$ range of existence of DS becomes narrower as $\theta_0$ is increased away from the homogeneous. When $H$ becomes low enough, Mode $P \rightarrow$ Mode $N_1$ (Sect 4.1) (c, d) $\psi = 0$ (« splay geometry » with pretilt) $H$ is in the $xz$ plane normal to $n_0$ which is also tilted in the $xz$ plane. Curves are drawn for $\theta_0 = (1)$ 0.05 (2) 0.5 (3) 1.0 radian. Curves 1, 3 correspond, respectively, to $R_s = R_{s1}$ and $R_s = R_{s2}$. In curve 2, $R_s = R_{s1}$ and $R_s = R_{s2}$ to the right and to the left of the cusp, respectively. Curve 3 appears to indicate a propensity towards reentrance (Sect 4.2) (e, f) $\psi = 0.75$ radian $H$ is oblique in a plane normal to $n_0$ which is tilted in the $xz$ plane. Curves are drawn for $\theta_0 = (1)$ 0.05 (2) 0.5 (3) 1.0 radian. In curve 1, $R_s = R_{s1}$, in curves 2, 3, $R_s = R_{s1}$ (or $R_s = R_{s2}$) to the left (or to the right) of the cusp. Curve 3 shows reentrance (dashed part) where the growth rate of Mode $N_2$ is higher than that of DS, see also figures 6 (Sect 4.3).

At low enough $H$ or tilt angle $\theta_0$, Mode $N_1$ grows faster than Mode $N_2$. It must be noted that the flow alignment angle $\theta_f$ is a solution of the equation $\eta_8 = 0$. This has a bearing on the variation of $H_N$ with $\theta_0$. At $\theta_0 = \pi/2$, $H_N = 2H_F$ as in section 4.1 (this is equivalent to saying that when the initial orientation is homeotropic, $H$ applied along $x$ or $y$ has the same effect for Mode $N$). When $\theta_0$ is diminished from $\pi/2$, $H_N$ increases and diverges as $\theta_0 \rightarrow \theta_f$. In the range $0 < \theta_0 < \theta_f$, $H_N$ takes high positive values before diminishing to about 100 $H_F$ when $\theta_0 \rightarrow 0$.

As in section 4.1, owing to the additional elastic and viscous couplings introduced by the director pretilt, DS is again a superposition of Modes $P_1$ and $P_2$ (of Sect. 3.2). The plots of $R_s$ and $Q_p$ as functions of $R$ and $\theta_0$, shown in figures 4c, 4d and 5c, 5d, are self explanatory. A comparison with figures 4a, 4b and 5a, 5b shows that in the present case the $R$ range of existence of Mode $P$ gets curtailed while its $\theta_0$ range of occurrence gets slightly broadened with the pretilted twist geometry. It may be noted that though $R_s \rightarrow 1$, $Q_p$ does not vanish. Thus in the limit $Q_p \rightarrow 0$, one does not expect to find the critical point between DS and Mode $N_1$ (for $R$ variation) or between DS and Mode $N_2$ (for $\theta_0$ variation). In this respect the present results differ from those of section 4.1 just as results for splay geometry differ from those for twist geometry.
Fig. 5. — Plots of \( R_s \) and \( Q_P \) versus \( \theta_0 \) for different \( H \) and \( \psi \). \( R_s = R_{c1} \) or \( R_{c2} \), whichever is less (a, b) \( \psi = \pi/2 \) (twist geometry) with pretilt. Curves are drawn for \( H = (1) \ 2 \ 000 \ (2) \ 2 \ 500 \ (3) \ 3 \ 000 \) emu \( R_s = R_{c1} \) (or \( R_{c2} \)) to the left (or to the right) of the cusp Stronger the \( H \), broader the \( \theta_0 \) range of occurrence of Mode \( P \). When \( \theta_0 \) is large enough, DS merges into Mode \( N_2 \) (Sect 4.1) (c, d) \( \psi = 0 \) (splay geometry) with pretilt. Curves are drawn for the same values of \( H \) as in (a, b) The conclusions are similar to those of (a, b) (Sect. 4.2) (e, f) \( 0 < \psi < \pi/2 \), \( H \) is oblique in a plane normal to \( n_0 \). \( H = 2 \ 000 \) emu and curves are for \( \psi = (1) \ 0 \ 75 \ (2) \ 1.2 \ (3) \ 1.55 \) radian. The \( \theta_0 \) range of existence of Mode \( P \) becomes narrower as \( H \) is moved away from the \( xz \) plane (Sect. 4.3).

As in the previous section it is found that the growth rate of Mode \( P \) increases with \( \theta_0 \). Typically at \( H = 2 \ 000 \) emu one finds the following values:

\[
\begin{align*}
\theta_0 &= 0.01, \ s_P = 0.88 \text{ (sec}^{-1}) \ ; \ s_{N_1} = 0.41 \text{ (sec}^{-1}) \ ; \ s_{N_2} = 0.36 \text{ (sec}^{-1}) \ ; \ Q_P = 3.93 \\
\theta_0 &= 0.60, \ s_P = 0.94 \text{ (sec}^{-1}) \ ; \ s_{N_1} = 0.43 \text{ (sec}^{-1}) \ ; \ s_{N_2} = 0.44 \text{ (sec}^{-1}) \ ; \ Q_P = 3.05 \\
\theta_0 &= 1.18, \ s_P = 1.13 \text{ (sec}^{-1}) \ ; \ s_{N_1} = 0.89 \text{ (sec}^{-1}) \ ; \ s_{N_2} = 1.13 \text{ (sec}^{-1}) \ ; \ Q_P = 1.31
\end{align*}
\]

It is found that curve 3 (Fig. 4c) for \( \theta_0 = 1.0 \) radian is rather different from the other two curves drawn for lower director tilt. \( R_s \) decreases to less than one, reaches a minimum and subsequently increases to unity as \( Q_P \rightarrow 0 \). As has been made clear in the figure caption, \( s_{N_2} \) is higher than \( s_{N_1} \) almost throughout the \( R \) range so that \( R_s = R_{c1} \), the quantity which has been plotted. In the next section it will be seen that under certain conditions this variation of \( R_s \) can become more pronounced and lead to a new kind of behaviour.

4.3 Tilted Alignment, Oblique Magnetic Field, \( 0 < \psi < \pi/2 \) — \( H \) is assumed to be applied normal to \( n_0 \) making angle \( \psi \) with the \( xz \) plane. The nonperiodic solution is now associated with \( \theta, \phi, \nu_x, \nu_y \) which are functions of \( x \) and \( t \). Mode \( N_1 \) (or \( N_2 \)) is described by even (or odd) \( \theta, \phi \) and odd (or even) \( \nu_x, \nu_y \). It is not straightforward to solve analytically for the growth rates of these two Modes \( N \). It can be shown numerically that when \( \theta_0 \) and \( \psi \) are sufficiently removed from zero Mode \( N_2 \) grows faster than Mode \( N_1 \); Mode \( N_1 \) generally dominates when \( H \) is sufficiently low.

In this case Mode \( P \) involves all five perturbations \( \theta, \phi, \nu_x, \nu_y, \nu_z \) which are functions of \( x, z, t \); as in the previous section Mode \( P \) is described by a single mode without any specific
Plots of $R_s$ and $Q_p$ as functions of the field angle $\psi$ are shown in figures 2c-2f for one value of $H$ and different director tilt angles $\theta_0$. It is found that the qualitative nature of the variation of domain width remains the same as in the case of homogeneous initial alignment (Figs. 2a, 2b), the domain width increases as $H$ is rotated away from the $xz$ plane. The behaviour of $R_s$ does change at higher tilts $\theta_0$ (Figs. 2e, 2f).

The variation of $R_s$ as a function of $\theta_0$ for different field angles $\psi$ and one field strength $H$ is shown in figures 5e, 5f. It is seen that the $\theta_0$ range of existence of DS becomes narrower as $\psi$ is increased i.e. as $H$ is rotated away from the $xz$ plane. The dependence of $R_s$ on $R$ for different $\theta_0$ becomes more interesting (Figs. 4e, 4f) when $H$ is tilted sufficiently away from the $xz$ plane (say $\psi = 0.75$). In general, the $R$ range of occurrence of DS is found to shrink as the initial director tilt is moved away from the homogeneous alignment. When $\theta_0$ is sufficiently high ($= 10$; curve 3, Fig. 4e) there occurs a new development. Though Mode $P$ occurs as a solution in the entire range of $R$, the growth rate of Mode $P$ becomes less than that of Mode $N_2$ in the range $2.5 < R < 4$. Thus the occurrence of Mode $P$ is favourable in $2 < R < 4$ and $R > 4$ but not in the intermediate range; in other words, Mode $N$ reappears over a range of $R$ where it is not expected to occur. This is reminiscent of the phase diagram a reentrant phenomenon with the more symmetric Mode $N$ occurring over a short range of $R$, sandwiched on either side by the less symmetric Mode $P$.

To illustrate this more clearly, the inverse rise times for the individual modes have been plotted as functions of $R$ in figure 6 for three different director tilts at the same magnetic angle $\psi = 0.75$. It is found that reentrance does not occur for $\theta_0 = 0.9$ (Figs 6a, 6b) though there is a strong propensity with $s_{N_2}$ approaching $s_P$ quite closely. Reentrance occurs for $\theta_0 = 1.0$.

---

Fig 6 — Variations of the inverse rise times of Modes $N_1$, $N_2$, $P$ and $Q_p$ as functions of the reduced field $R$ for magnetic tilt angle $\psi = 0.75$ radian. The director tilt angle $\theta_0 = (a, b) 0.9$ (c, d) 1.0 (e, f) 1.1 radian. A range of $R$ close to the possible range of reentrance is chosen. Curves (1, 2, 3) correspond to $s_{N_1}$, $s_{N_2}$, $s_p$, respectively (in sec$^{-1}$). At $\theta_0 = 0.9$, reentrance does not occur at all (a). Reentrance can be seen for $\theta_0 = 1.0$ (c) and 1.1 (e). The points $P_1$, $P_2$ demarcate the range of reentrance over which $s_{N_2} > s_p (\geq s_{N_1})$ so that Mode $N_2$ grows faster than Mode $P$. This has been shown using dashed lines only on the $Q_p$ curves to avoid confusion. The occurrence of reentrance is strongly dependent on the existence of Mode $N_2$ (Sect 4.3).
(Figs 6c, 6d) and the range over which Mode $N_2$ eclipses Mode $P$ broadens for $\theta_0 = 1.1$ (Figs 6e, 6f).

Calculations for $\psi = 0.05$ and 1.5 have not been shown at $\theta_0 = 1.0$. In the former case there seems to be only a tendency towards reentrance while in the latter case reentrance does not occur at all. It must be remembered that for $\psi$ close to zero Mode $P$ will strongly resemble the $\theta, v_x, v_z$ DS of section 4.2 while for $\psi$ close to $\pi/2$ Mode $P$ will closely approach the $\phi, v_y$ DS of section 4.1. At intermediate values of $\psi$ Mode $P$ of this section will be an admixture of the above two solutions. It may thus be concluded that this behaviour is strongest when $\psi$ is close to $\pi/4$ but weakens when $\psi$ is changed away from this value.

5. Miscellaneous cases.

5.1 Negative diamagnetic anisotropy. — When $\chi_\theta < 0$ (this assumption is made only in this subsection) the only Fredericksz configuration is with $\mathbf{H}$ along $n_0$. The governing equations can be written down from (6)-(10) with the changes

$$f_5 = 0, \quad f_4 = f_9 = |\chi_\theta| H^2.$$  \hspace{1cm} (31)

For any initial director tilt $0 \leq \theta_0 \leq \pi/2$ it is found that $\mathbf{H}$ couples with both $\theta$ and $\phi$ perturbations but in all cases the $\phi, v_y$ and the $\theta, v_x, v_z$ modes get uncoupled. Hence for any given set of values of $\theta_0$ and $H$, both the uncoupled solutions must be studied, their time constants determined and compared. As the complete set of material constants for a $\chi_\theta < 0$ nematic are not available, the parameters for MBBA (16) have been chosen with the sign of $\chi_\theta$ reversed. Calculations have been performed for a number of different director tilts including the homogeneous. It is found that the $\phi, v_y$ mode is generally the faster growing one. Thus, for the material parameters used, the DS mode involving modulated flow $v_y$ parallel to the sample planes is found to be faster growing than the DS mode involving convective flow $v_z$ against the sample boundaries.

Most of the experimental work on $\chi_\theta < 0$ nematics [6, 7, 8, 10], which has been done in the twist geometry, appears to agree fairly well with theoretical predictions especially in the short time regime (immediately after application of $\mathbf{H}$). There is certainly need to investigate more complex geometries with initial pretilt of $n_0$. The possibility of crossover between the $\phi, v_y$ and the $\theta, v_x, v_z$ modes cannot be entirely ruled out.

5.2 Finite director anchoring energy at the sample planes. — It must be borne in mind that all the above results have been obtained for the case of rigid anchoring of the molecules at the sample walls, this corresponds to the anchoring energy being very high at the sample planes. The case of finite anchoring energy is certainly more realistic but also more involved as the anchoring strengths come into reckoning as additional parameters not only by determining the Fredericksz threshold but also by influencing the boundary conditions imposed on the splay and twist fluctuations. Though detailed results cannot be presented here, an attempt can still be made to give a brief qualitative discussion of the possible effects of weakening the anchoring at the boundaries especially in the context of DS having only one direction of spatial periodicity and with the surface energy density taking the rather simple form (14). Interestingly, Fincher [7] has commented that it might be important to take account of finiteness of anchoring energy in the case of polymer nematics.

As a uniaxial nematic is endowed with two independent orientational degrees of freedom it is possible to define two independent anchoring strengths $B_\theta$ and $B_\phi$ at an interface [22] corresponding to the splay ($\theta$) and twist ($\phi$) deformations, respectively. Depending upon the configuration studied, one or both of the anchoring strengths will come into the picture. For
instance, when \( \psi = 0 \) (\( \mathbf{H} \) in the \( xz \) plane) the Freedericksz threshold \( H_F \) as also the boundary value of the perturbation \( \theta \) will be determined by \( B_\theta \). When \( \psi = \pi/2 \) (\( \mathbf{H} \) along \( y \)) \( B_\theta \) will determine \( H_F \) as well as the boundary condition on \( \phi \). In the general case of oblique \( \mathbf{H} \), both anchoring strengths will determine \( H_F \) as well as the boundary conditions for \( \theta \) and \( \phi \).

It is, however, not straightforward to qualitatively deduce the effects of weak anchoring on DS. To see this, consider what happens when \( \mathbf{H} \) is fixed in a given configuration and the anchoring is weakened. On the one hand, \( H_F \) decreases so that \( R = H/H_F \) increases. Increase of \( R \) should be accompanied by corresponding enhancements in \( s_P, s_{N_1}, s_{N_2} \) and also in \( Q_P \) as per calculations already presented. On the other hand, investigations on static periodic deformations has shown [19] that weakening of director anchoring tends to reduce the domain wave vector, in some cases, the very occurrence of the periodic distortion is prevented. It is, therefore, clear that an exact solution of the problem will alone indicate how precisely diminution of anchoring energy at the sample planes will affect the growth rate, periodicity and domain of existence of DS in any given configuration.

Another aspect of anchoring which is worth noting is that (as indicated rather generally in (14)-(15)) the anchoring strengths at the two boundaries can be different. When the corresponding anchoring strengths are identical at the boundaries, it will be possible to obtain solutions of the same level of modal purity as in the case of rigid anchoring; for instance, Modes \( N_1 \) and \( N_2 \) will remain uncoupled; it will be possible to get independent Modes \( P_1 \) and \( P_2 \) in splay and twist geometries. When the corresponding anchoring strengths are unequal at the sample planes (\( B_\theta \neq \mathbf{B}_\theta, B_\phi \neq \mathbf{B}_\phi \)) modes which are uncoupled for rigid anchoring become mixed and this will hold not only for the Modes \( P \) but also for the Modes \( N \). Imparting different surface treatments to the sample planes will thus presents itself as an important factor in controlling the domain of existence of DS as also the periodicity. Only exact calculations will be able to shed light on this matter.

5.3 Role of Energy Dissipation — In the case of static deformations the total free energy can be used for determining the relative stability of different solutions; that distortion is regarded as stable if it has lower free energy than the other possible deformations. Cohen and Hornreich [21] have devised a convenient way of calculating the free energy as a power series in the domain wavevector for periodic distortions. In the limit of vanishing periodicity, this method enables a determination of the critical point between the periodic and nonperiodic distortion thresholds. The question that arises is whether a similar procedure can be used for deducing the critical point in the case of DS and the Modes \( N \). At this stage it appears possible to extend this only for the twist geometry (Sect. 3.1) as briefly described below.

The free energy by itself will have little or no bearing on the existence of otherwise of the different modes in the short time regime. As compared to the free energy of Mode \( N \), that of Mode \( P \) is always higher due to the positive definite term \( K_3 \phi_x^2 \) in the free energy density. Hence, the free energy of DS can always be expected to remain higher than that of Mode \( P \). On the other hand, the rate of energy dissipation per unit volume (\( E \)) is given by

\[
E \sim \gamma_1 \phi^2 + [\eta_c v_{y, x} v_x^2 + \mu_2 \phi v_{y, x} + \eta_4 v_{y, x}^2] \sim E_0 \exp(2st)
\]

(32)

where the terms [ ] are unique to DS and \( E_0 \) is a function of \( x, z \). For Mode \( N \), \( E_0 \) is a function of \( z \) alone and the terms [ ] are absent. In the light of results of section 3.1 (Figs. 1a, 1b) it is clear that under certain circumstances the coupling between \( v_y \) and \( \phi \) associated with DS may be able to reduce \( E_0 \) of DS \( w.r.t. t \) that of Mode \( N \). We are interested in finding out the condition for the two to be just equal. It must also be noted that closed solutions have been obtained by assuming a dependence of \( \exp(st) \) for all perturbations so that the dependence of \( E \) on \( \exp(2st) \) follows naturally. By using this ansatz and by
introducing \( s \) as a parameter into the problem, time has been removed as a variable. This is equivalent to asserting that the equations so derived are valid at each instant of time With

\[
\phi = g(z)[\cos (q_x x)] \exp(st), \quad v_y = m(z)[\sin (q_x x)] \exp(st)
\]

(7) and (9) reduce to

\[
D^2 g + g (\xi_1 + Q_x^2 \xi_2) + \xi_3 Q_x m = 0; \quad D^2 m + \xi_4 Q_x^2 m + \xi_5 Q_x g = 0,
\]

\[
\xi_1 = (x_4 H^2 - \gamma_1 s) h/K_2, \quad \xi_2 = -K_3/K_2, \quad \xi_3 = -\mu_2 h/K_2;
\]

\[
\xi_4 = -\eta_2/\eta_a, \quad \xi_5 = -\mu_2 hs/\eta_a.
\]

(34)

For small \( Q_x \) the functions \( g \) and \( m \) are expanded in powers of \( Q_x \)

\[
(g, m) = \sum_{r=0}^{\infty} (g_r, m_r) Q_x^r
\]

(35)

and substituted into (34) to result in the following sets of coupled equations which split into two independent sets:

Set 1: \( D^2 g_0 + \xi_1 g_0 = 0; \quad D^2 m_1 + \xi_5 g_0 = 0; \quad D^2 g_2 + \xi_1 g_2 + \xi_2 g_0 + \xi_3 m_1 = 0; \)

Set 2: \( D^2 m_0 = 0, \quad D^2 g_1 + \xi_1 g_1 + \xi_3 m_0 = 0; \quad D^2 m_2 + \xi_4 m_0 + \xi_5 g_1 = 0, \)

(36)

the remaining functions being determined from

\[
D^2 g_{r+2} + \xi_1 g_{r+2} + \xi_2 g_r + \xi_3 m_{r+1} = 0, \quad D^2 m_{r+2} + \xi_4 m_r + \xi_5 g_{r+1} = 0, \quad r = 1, 2, ...
\]

(37)

As the two sets are uncoupled, it is reasonable to assume that the functions of one set vanish identically when those of the other set are studied. If this is done it becomes possible to write \( E_0 \) (32) as a series in ascending even powers of \( q_x \) (or \( Q_x \)). Integration of \( E_0 \) w.r.t. \( y \) makes \( \ell_y \), the lateral width of the sample along \( y \), a multiplicative parameter. Integration of \( \sin^2 q_x x \) and \( \cos^2 q_x x \) w.r.t. \( x \) also gives \( \ell_x \), the lateral width of the sample along \( x \), as a multiplier provided that \( Q_x \ell_x \gg 1 \) (i.e., if the lateral width along \( x \) \( \gg \) the domain width). To second order in \( Q_x \) one gets

\[
h^2 E_x/\ell_x \ell_y \eta_a \sim [\xi_6 g_0^2 + (Dm_0)^2] + Q_x^2 [-\xi_4 m_0^2 + \xi_6 g_1^2 - \xi_5 m_0 g_1 + 2(Dm_0)(Dm_2)] +
\]

\[
+ Q_x^2 [2 \xi_6 g_0 g_2 - \xi_5 m_1 g_0 + (Dm_1)^2] + \cdots, \quad \xi_6 = \gamma_1 h^2 s^2/\eta_a
\]

(38)

where \( E_x \) is a function of \( \xi \) is \( E_0 \) integrated over \( x \) and \( y \). Because of the no slip condition (12), \( m_0 = 0 \) (this also means that Mode \( N \) is not associated with flow). This leaves only the purely positive term \( \sim Q_x^2 \xi_6 g_1^2 \) for Set 2 showing that this periodic solution cannot dissipate less energy than Mode \( N \) to lowest order in the wave vector. It will, therefore, be sufficient if we concentrate on the second \( Q_x^2 \) term containing only the Set 1 contributions Writing \( g_0 = \bar{g} \cos (\pi \xi/2) \) (with \( \bar{g} = \) constant and \( \xi_1 = \pi^2/4 \)), solving for \( m_1 \) and \( g_2 \) from (36), substituting in (38) and integrating over \( \xi \) one finds a measure of the rate of energy dissipation \( (E_x) \) by the fluid to be

\[
E_x h^2/\ell_x \ell_y \eta_a \sim \xi_6 \bar{g}^2 - (2 \xi_6 \bar{g}^2 h/\pi^2)(\xi_2 + 4 \xi_3 \xi_5/\pi^2) Q_x^2
\]

(39)
The first term on the right is the energy dissipation rate for Mode $N$ alone. The total expression on the right corresponds to the energy dissipation rate for DS upto quadratic order in $Q_x$. If DS is to grow faster than Mode $N$, the coefficient of $Q_x^2$ must be negative; if this coefficient vanishes we get the critical point between DS and Mode $N$. Substitution from (34) and (38) leads to precisely the condition (19).

In principle this procedure can be repeated for the other geometries too. As the equations become rather complicated, no attempt is made to solve them here. The following conclusions can, however, be anticipated: (i) For splay geometry the coefficient of $Q_x^2$ in $E_v$ must be positive definite (there is no critical point in the limit $Q_x \to 0$, Figs. 1c, 1d). (ii) For the pretilted twist geometry the critical point, at given $\theta_0$, should be between DS and Mode $N_1$ for $H$ variation (Figs 4a, 4b); the critical point for $\theta_0$ variation at given $H$ should be between DS and Mode $N_2$ (Figs 5a, 5b).

6. Conclusions: limitations of the mathematical model used.

Using the approach of [5] the linearized governing equations of the nonstochastic continuum theory have been solved numerically for studying the relative occurrences of the periodic mode (DS) and the nonperiodic mode ($N$) in a $\chi_u > 0$ nematic by comparing the growth rates in the limit of small perturbations. As only Freedericksz geometries are chosen in all cases (H normal to $n_0$), this essentially involves solving eigenvalue problems to determine the highest possible time constants for DS and Mode $N$. As the actual amplitudes of the perturbations are not known, information about the orientation and velocity fields of the fully developed pattern cannot be obtained. The rigid anchoring hypothesis has been employed but there are definite indications that a study of weak anchoring should be fruitful especially because it represents a realistic situation. The periodicity of DS has been assumed to occur along one direction parallel to the sample planes, thus oblique patterns have not been considered.

A method [21], recently developed for studying the relative energetics of static periodic and nonperiodic distortions, has been adapted to the simplest case (twist geometry) to show that the condition for the occurrence of DS being more favourable than that of Mode $N$ (immediately after application of $H$) follows as a natural consequence of minimization of energy dissipation rate. The free energy does not appear to play a decisive role in this matter. It must be remembered, however, that once sufficient time has elapsed and DS has weakened, it is the free energy minimization which becomes important in determining the final alignment of the director $w.r.t. H$. This regime is beyond the scope of this work.

Changing the tilt of $H$ in a plane normal to $n_0$ affects the growth rate and the periodicity of DS. For a given $H$, changing the orientation of $n_0$ away from the homogeneous has a deleterious effect on DS accompanied by a diminution of the wavevector (or broadening of the domains). When $n_0$ is sufficiently close to the homeotropic, DS can get completely suppressed. This result is in qualitative agreement with similar conclusions arrived at for static periodic deformations [19, 20].

Though DS may or may not occur as a solution with pure modal symmetry, Mode $N$ always manifests itself as one of two independent modes $N_1$, $N_2$. Mode $N_1$ generally grows faster than Mode $N_2$ when $H$ is low enough and the tilt of $n_0$ is sufficiently close to the homogeneous, in this region the growth rate of Mode $N_1$ is compared with that of DS. In the opposite range, however, Mode $N_2$ is the faster growing Mode $N$ and is therefore compared with DS. Mode $N_2$ is a solution which is peculiar to a complex fluid, such as a nematic, and is associated with no stress in the principal shear plane but is still accompanied by a net flow. Such solutions have been encountered before [27] and appear to stem as much from the law of balance of linear momentum as from that of angular momentum. For certain tilts of
$\mathbf{n}_0$ when $\mathbf{H}$ is oblique Mode $N$ may appear in a reentrant way, with increasing $H$, one would find Mode $N_1$, DS, Mode $N_2$ and finally DS again as the fastest growing mode.

The above results can be put to experimental test. Of especial interest may be experiments on $\chi_a < 0$ nematics with tilted $\mathbf{n}_0$. As the Freedencksz geometry for this case is $\mathbf{H}$ along $\mathbf{n}_0$, $\mathbf{H}$ can couple with both orientational fluctuations. One cannot rule out the possibility that there might occur a cross over from a twist — bend dominated DS to a splay — bend dominated DS when the tilt of $\mathbf{n}_0$ is varied relative to the sample planes.

Certain conclusions become inevitable not only from the purely physical view point but also in the context of previous results. These relate to non-Freedencksz geometries without static threshold with $\mathbf{H}$ not normal to $\mathbf{n}_0$. It is known [17, 20], experimentally and theoretically, that in a $\chi_a > 0$ nematic slight deviations of $\mathbf{H}$ from its Freedencksz position can suppress the static stripe phase leaving only a homogeneous distortion. A similar conclusion is inevitable for DS also as the following discussion will show.

Noting that the configuration of $\mathbf{H}$ along $\mathbf{n}_0$ is one of stable equilibrium it is intuitively clear that a tilt $\kappa$ of $\mathbf{H}$ towards $\mathbf{n}_0$ (or a tilt of $\mathbf{n}_0$ towards $\mathbf{H}$) in the ($\mathbf{n}_0, \mathbf{H}$) plane should weaken the principal destabilizing mechanism responsible for transient flow, viz. the torques exerted by $\mathbf{H}$ on the director perturbations. This should not only bring down the growth rates of DS and Mode $N$ but also broaden the DS domains. While Mode $N$ can exist (however weakly) as long as $\mathbf{n}_0$ and $\mathbf{H}$ are not exactly parallel, the same may not be true of DS. Starting with the Freedencksz geometry ($\kappa = \pi/2$) and decreasing $\kappa$ one can envisage the existence of a critical $\kappa_a$ such that for $\kappa < \kappa_a$ only Mode $N$ is possible and DS cannot exist. The mathematical model developed in this work cannot be easily extended to the non-Freedencksz case mainly because, instead of zero, driving terms proportional to $H^2$ and independent of perturbations will appear on the right side of the torque equations (6), (7) making justification of small perturbation amplitudes difficult. Experimentally, however, there should be no special difficulty as even the simple homogeneous alignment can be used to check the above hypothesis. Interestingly, there exists on experimental observation [10] on a $\chi_a < 0$ lyotropic system according to which the tilting of the sample towards $\mathbf{H}$ weakens the stripes. More systematic work in this direction is, therefore, called for.

Appendix

$K_1$, $K_2$, $K_3$ are the splay, twist and bend elastic constants, respectively. The $\mu_i$ ($i = 1$ to 6) are the viscosity coefficients of a nematic connected by the Parodi relation [1]

$$\mu_6 = \mu_2 + \mu_3 + \mu_5$$

(A1)

The Miesowicz viscosity coefficients are

$$\eta_a = \mu_4/2; \quad \eta_b = (\mu_3 + \mu_4 + \mu_5)/2, \quad \eta_c = (\mu_4 + \mu_5 - \mu_2)/2$$

(A2)

The twist viscosity $\gamma_1$ and the elongational viscosity $\nu_1$ are

$$\gamma_1 = \mu_2 - \mu_3, \quad 2\nu_1 = \mu_1 + \mu_4 + \mu_5 + \mu_6$$

(A3)

Using (A1)-(A3) the five independent $\mu_i$ ($i = 1$ to 5) can be expressed as linear combinations of $\eta_a$, $\eta_b$, $\eta_c$, $\gamma_1$ and $\nu_1$. The other definitions are given below

$$f_1 = K_1 S^2 + K_3 C^2, \quad f_2 = f_1(\theta_0^2), \quad \theta_0^2 = \theta_0 + \pi/2, \quad f_3 = df_2/d\theta_0;$$

$$f_4 = \chi_a H^2 C_\phi, \quad f_5 = \chi_a H^2 S_\phi C_\psi, \quad f_6 = K_3 S^2 + K_3 C^2, \quad f_7 = f_6(\theta_0^2);$$

$$f_8 = df_7/d\theta_0, \quad f_9 = f_4(\psi + \pi/2), \quad f_{10} = (K_2 - K_1) S, \quad f_{11} = (K_1 - K_2) C$$

(A4)
\[ \eta_1 = \mu_1 C^4 + \mu_4 + (\mu_5 + \mu_6) C^2, \]
\[ 2 \eta_2 = 2 \mu_1 S C^2 + \mu_4 + (\mu_5 - \mu_2) S^2 + (\mu_3 + \mu_6) C^2; \]
\[ \eta_3 = 2 SC(\mu_1 C^2 + \mu_5), \quad \eta_4 = SC(\mu_1 C^2 + \mu_6), \quad \eta_5 = -\eta_4(\theta_0^a), \]
\[ 2 \eta_6 = 2 \mu_1 S C^2 + \mu_2 + \mu_4 + \mu_5, \quad \eta_7 = -(\mu_2 + \mu_3) SC, \quad \eta_8 = \mu_3 C^2 - \mu_2 S^2, \]
\[ 2 \eta_9 = \mu_4 + (\mu_5 - \mu_2) C^2, \quad \eta_{10} = \eta_9(\theta_0^a), \quad \eta_{11} = (\mu_5 - \mu_2) SC, \quad \eta_{12} = \mu_2 C; \]
\[ \eta_{13} = -\eta_{12}(\theta_0^a), \quad \eta_{14} = \mu_1 S C^2, \quad \eta_{15} = \eta_3(\theta_0^a), \quad \eta_{16} = \eta_1(\theta_0^a), \]
\[ \eta_{17} = -\eta_3(\theta_0^a), \quad \eta_{18} = -\eta_8(\theta_0^a) \quad (A5) \]
\[ 2 \tau_1 = \mu_4 + (\mu_3 + \mu_6) C^2, \quad 2 \tau_2 = \mu_4 + (\mu_2 + \mu_5) C^2; \quad 2 \tau_3 = SC(\mu_2 + \mu_5), \]
\[ 2 \tau_4 = SC(\mu_3 + \mu_6), \quad \tau_5 = \mu_3 C, \quad \tau_6 = \tau_2(\theta_0^a); \quad \tau_7 = \tau_1(\theta_0^a), \quad \tau_8 = -\tau_3(\theta_0^a), \quad \tau_9 = 2 \eta_a \quad (A6) \]

Note added in proof.

After the acceptance of this manuscript the author came across the paper by S Ciaponi and S Faetti entitled, «The effect of finite anchoring energy on the transient periodic structures in nematic liquid crystals» (Liq Cryst 8 (1990) 473) while perusing Current Contents [Physical, Chemical and Earth Sciences 30, number 45, 5th November 1990] Up to date the author has not been able to read the paper by Ciaponi and Faetti but the author believes that they may have independently anticipated some of the results on weak anchoring which have been qualitatively discussed in the present work.

References

[1] OSEEN C W., Trans Faraday Soc. 29 (1933) 883,
FRANK F C , Disc Faraday Soc 25 (1958) 19,
LESLEY F M, Adv Liquid Cryst, G H Brown, Ed (Academic Press) 1979, p 1,
CHANDRASEKHAR S, Liquid Crystals (Cambridge University Press) 1977,
DEULING H J, Solid State Phys Suppl 14 (1978) 77,
BLINOV L M, Electrooptical and Magnetooptical Properties of Liquid Crystals (John Wiley) 1983
BROCHARD F, Mol Cryst Liquid Cryst 23 (1973) 51
[5] GUYON E, MEYER R B and SALAN J, Mol Cryst Liquid Cryst 54 (1979) 261, see also references therein
McClymer J. P and Labes M M, Mol Cryst Liquid Cryst 144 (1987) 275,


REY A D, Liquid Cryst 7 (1990) 315


OLDANO C, Phys Rev. Lett 56 (1986) 1098,
ZIMMERMANN W. and KRAME R L, Phys Rev Lett 56 (1986) 2655,

KINI U D, Liquid Cryst 7 (1990) 185


[27] LESLIE F M, Mol Cryst Liquid Cryst 37 (1976) 335