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Abstract. — We study the local scaling properties of driven interfaces in disordered media modeled by the Edwards-Wilkinson equation with quenched noise. We find that, due to the super-rough character of the interface close to the depinning transition, the local width exhibits an anomalous temporal scaling. It does not saturate at a characteristic time $t_s(l) \sim l^z$ as expected, but suffers an anomalous temporal crossover to a different time regime t^{β_*}, where $\beta_* \simeq 0.21$. This is associated with the existence of a local roughness exponent $\alpha_{\rm loc} \simeq 1$ that differs from the global one $\alpha \simeq 1.2$. The relevance of the typical size of pinned sites regions near the critical point is investigated and the definition of the critical depinning is discussed.

1. Introduction

Rough surfaces and interfaces appear in many situations in nature. Deposition, erosion, fluid-fluid displacement in porous media or fire front motion are important examples [1,2] in which an interface kinetically becomes rough. A rough interface in dimension $d+1$ (d is the substrate dimension), which is described by its height $h(\mathbf{x},t)$ at position $\mathbf{x}$ and time t, has been treated as a self-affine fractal. In the case of interfaces driven by spatio-temporal noise the temporal evolution of $h(\mathbf{x},t)$ is given by the stochastic equations of Edwards-Wilkinson (EW) [3] and Kardar-Parisi-Zhang [4], which have been widely studied. The effect of quenched disorder (*i.e.* a noise frozen in time) on the interface dynamics is a more difficult problem and up to now there is not a complete and consistent picture. An important feature of the quenched disorder is that the interface can be dominated by pinning forces, which can slow down the motion of the interface in large regions. Pinning phenomena have a considerable importance in problems like the immiscible displacement of fluids in porous media or the motion of domain walls in magnetic systems.

During the last years much effort has been paid to this subject, but theory, simulations and experiments are not yet in good agreement (see [2] for recent reviews). In this paper we wish to show that the reason for this disagreement is certainly twofold. On the one hand, growth processes do not always generate truly self-affine interfaces [5,6] and different scaling

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behaviours may appear at short and large length scales. On the other hand, the quenched disorder introduces a new length scale, related to the size of pinned regions that becomes relevant and, as we will see later, changes the usual scaling laws.

The simplest stochastic differential equation that describes the evolution of the interface height in the presence of quenched randomness is the so-called quenched Edwards-Wilkinson (QEW) equation:

$$\frac{\partial h}{\partial t} = D\nabla^2 h + F + \eta(\mathbf{x}, h), \tag{1}$$

which is similar to the EW equation but with a quenched disorder term, $\eta(\mathbf{x}, h)$. The external driving force F controls completely the dynamics of the front. If F is larger than a critical force F_c, the interface moves with a finite velocity. However, the interface remains pinned by the disorder for $F < F_c$. The critical point $F = F_c$ is known as depinning transition. Above the depinning transition, $F \geq F_c$, a characteristic length, ξ, appears. ξ represents the typical size of the pinned regions of the interface and close to the transition scales as $\xi \sim (F - F_c)^{-\nu}$ As we will discuss later, ξ is very important to understand the scaling of the interface near the depinning transition.

The *global* interface width, which characterizes the scaling of the advancing front, is defined by

$$\sigma(L,t) = \left\{ \langle (h(\mathbf{x},t) - \langle h(\mathbf{x},t) \rangle)^2 \rangle^{1/2} \right\}, \tag{2}$$

where brackets are averages over the whole system and $\{...\}$ over realizations of the disorder. In the usual case, when the only relevant length is the system size, the width scales with time for the early times regime and saturates at long times:

$$\sigma(L,t) \sim \begin{cases} t^{\beta} & \text{if} \quad t \ll t_s \\ L^{\alpha} & \text{if} \quad t \gg t_s, \end{cases} \tag{3}$$

where α is the roughness exponent and β is the time exponent. This scaling picture is known as Family-Vicsek scaling ansatz [1, 2]. The saturation time t_s depends on the system size L because it is the time in which the horizontal correlation length $l_c(t) \sim t^{1/z}$ reaches the system size. The exponent z is called dynamic exponent and the scaling relation $\alpha = z\beta$ is fulfilled.

An alternative way of determining the roughness exponent α is frequently used in simulations because of the long wait needed to get saturation in large systems, in which simulations are done. The width is calculated over a window of length $l \ll L$ and averaged over many pieces with the same length along the interface. In other words, we can study the scaling of the *local* width

$$\sigma(l,t) = \left\{ \langle (h(\mathbf{x},t) - \langle h(\mathbf{x},t) \rangle_l)^2 \rangle_l^{1/2} \right\}. \tag{4}$$

The local width scales in a similar manner as the global width and has a short time regime $\sigma(l,t) \sim t^{\beta}$. For small length scales (or equivalently at long times), $l \ll l_c(t)$, the local width is believed to be independent of time and to scale as

$$\sigma(l,t) \sim l^{\alpha_{\text{loc}}} \tag{5}$$

This method is very useful when one has no possibility of producing interfaces in different system sizes as occurs in real experiments. Most theoretical growth models generate *self-affine* interfaces and $\alpha = \alpha_{\text{loc}}$ [1, 2].

Much work has been carried out to obtain the critical exponents at the depinning transition. The roughness exponent has been the most controversial. Experiments performed in $d = 1$ [7], which are believed to be described by equation (1), gave roughness exponents around 0.75-0.9

in disagreement with a renormalization group prediction $\alpha = 1$ [8]. A scaling theory [9] gave an effective roughness exponent 0.8 in good agreement with the model reported in reference [10]. A number of numerical models for interface pinning by quenched disorder have been investigated. Roux and Hansen [11] found two different exponents: $\alpha_{\rm loc} = 0.86 \pm 0.03$ when the local width was used, and $\alpha \simeq 1.16 - 1.20$ looking at the scaling of the global width as a function of the system size. Results reported by Jensen [12], $\alpha_{\rm loc} \simeq 0.9$ and $\alpha \simeq 1.15$ are similar. Makse and Amaral [13] obtained $\alpha_{\rm loc} = 0.92 \pm 0.04$ or $\alpha \simeq 1.23 \pm 0.04$ depending on the method that they used. Also other numerical determinations of $\alpha_{\rm loc}$ gave values around 0.8 [14–18] and $\alpha \simeq 1.2$ [17,19].

It has been demonstrated that the reason for the existence of two different roughness exponents is the *super-rough* character of the interface close to the depinning transition. In reference [19] was already shown that the local width is bounded by $\sigma(l,t) \leq l$ and for that, it is just *technically* impossible to measure a roughness exponent larger than one using the local width. On the contrary the global width does not have this restriction so that it gives the *true* roughness exponent. However, we would like to note here that there also exist other effects [5,6] that may lead to the same anomalous scaling in surfaces even with $\alpha < 1$. Our aim in this paper is to show that the local width suffers a temporal crossover, instead of saturating, associated with a different interface scaling at short, $l \ll L$, and long, $l \sim L$, length scales. The scaling properties of the interface cannot be correctly described by the usual dynamic scaling hypothesis. We will see that a super-rough dynamic scaling gives the correct scaling of the front.

2. Numerical Results

We begin by performing a numerical integration of the QEW equation in $d = 1$ to study the scaling behaviour of the local and global widths. To start we have to discretize the QEW equation in the horizontal direction

$$h_i(t + \Delta) = h_i(t) + \Delta(D(h_{i+1}(t) + h_{i-1}(t) - 2h_i(t)) + \Delta(F + \eta(i, [h_i(t)])), \qquad (6)$$

where the index $i = 1, \cdots, L$, being L the size of the system. $[h_i(t)]$ represents the integer part of $h_i(t)$. $h_0(t) = h_1(t)$ and $h_{L+1}(t) = h_L(t)$ are the boundary conditions. This is the usual discretization scheme used by many authors [12,15,18]. The random field $\eta(i,j)$ is Gaussian and its correlation $\langle \eta(i,j)\eta(i',j') \rangle = g\delta_{i,i'}\delta_{j,j'}$. Hence the noise is uncorrelated from a cell to another one and stands for the time of a run. The time step Δ (we used $\Delta = 0.01$ in all our simulations) and the size L have to be chosen in such a way that the QEW equation is obtained in the continuous limit ($\Delta \to 0$, $L \to \infty$). So, the following conditions must be verified: $\Delta D \ll 1$, $\Delta F \ll 1$, and $\Delta g \ll 1$. One has to check that changing these parameters does not affect the final numerical result. Following Kessler *et al.* [15] we distributed disorder sites randomly with a prefixed density p over the network. So, the disorder is nonzero with probability p and on these sites disorder is distributed according to a Gaussian distribution. The scaling exponents do not depend on the actual value of p but the crossover effects will be clearer for small values of p. As we were looking for crossover regimes, we used $p = 0.1$ in all our numerical simulations. We worked with networks of size $L = 1000$, and the results were averaged over 15-50 realizations of disorder. As expected we found a critical driving force F_c that separates the pinning and depinning phases. The value of F_c depends on the actual values of the parameters of simulation (p, D, L). For $p = 0.1$, $\Delta = 0.01$, $D = 5$, and $L = 1000$ we estimated $F_c \simeq 0.13$.

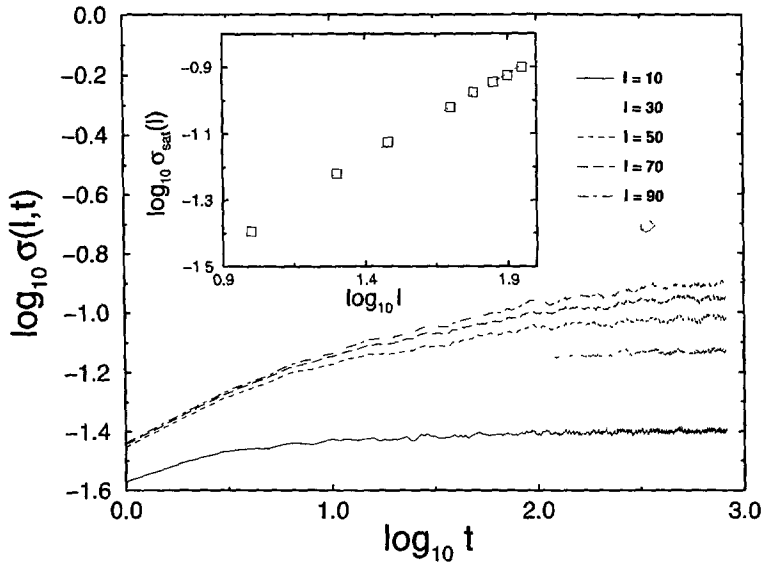

Fig. 1. — Time evolution of the local width on different length scales l (window size) for a driving force $F = 0.4$, which is far from the depinning transition. Local width saturates at longer times for larger length scales as corresponds to a standard scaling. The values of the width in saturation are taken and plotted *versus* scale l in a log-log plot (see inset). The slope of the line (inset) that fits the numerical data is 0.5 and gives a local roughness $\alpha_{\rm loc} \simeq 0.5$.

2.1. Scaling Far from the Depinning Transition. — Our numerical results indicate that the global width has two different time regimes. Close to the transition, $F \simeq F_c$, the width behaves as $\sigma(L,t) \sim t^{\beta}$, where $\beta \simeq 3/4$ is the time exponent near the critical point. Far from the pinning transition, $F \gg F_c$, $\sigma(L,t) \sim t^{\beta_{\rm EW}}$ and the time exponent $\beta_{\rm EW} \simeq 1/4$ corresponds to the time exponent of the EW equation. As we will see later, the values of the remaining exponents α and z indicate that the interface dynamics clearly belong to the EW universality class in the limit of large driving forces.

In general, the temporal behaviour of the global width is well understood near and far from the depinning transition and it has been analyzed by many authors [10,13,18]. On the contrary, we focus here on the scaling behaviour of the local width. Firstly, as far as time evolution is concerned, in the strong pushing regime the local width scales with time for early times and saturates at time $t_s(l)$ as corresponds to the Family-Vicsek scaling picture (see Fig. 1). The value of the width at saturation can be used to determine the roughness exponent. We found that the saturated local width, $\sigma_{\rm sat}(l)$, scales as $\sigma_{\rm sat}(l) \sim l^{0.5}$ (see Fig. 1 inset). Thus, for large driving forces the exponents ($\beta \simeq 1/4$ and $\alpha_{\rm loc} \simeq 1/2$) of the EW universality class are obtained, as expected [9,10,13].

2.2. Local Scaling at the Depinning Transition. — At the critical value, $F = F_c$, the complete scaling description of the local width is much less trivial than in the strong pushing limit discussed above. In Figure 2 we display the time evolution of the local width $\sigma(l,t)$ for different window sizes l. After an early time regime, the local width crosses over to t^{β_*}, where the asymptotic time exponent is $\beta_* \simeq 0.25$. This scaling region becomes shorter as the length scale is larger, $l \to L$. Moreover, there is no saturation of local widths at times $t_s(l) \sim l^z$, contrary to what is generally thought saturation of the local width occurs only when the whole

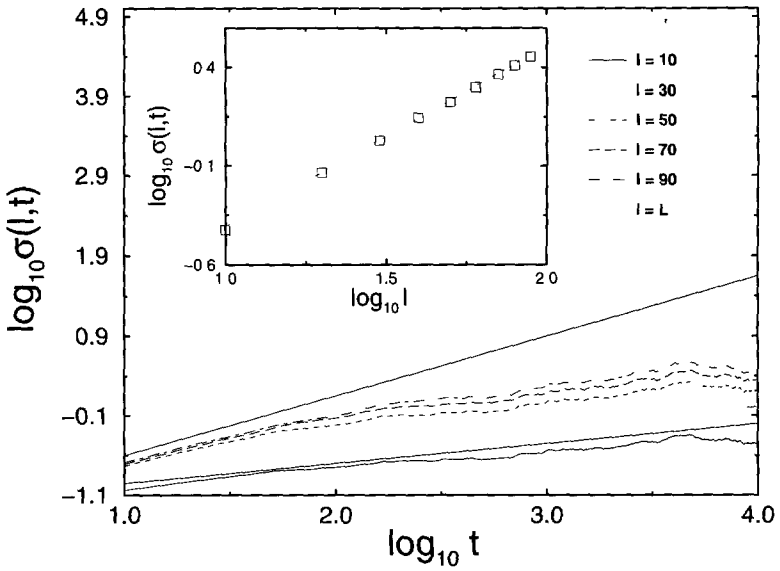

Fig. 2. — Time evolution of the local width at the depinning transition, $F = F_c \simeq 0.13$. The global width, $l = L$, is also displayed. The continuous straight lines have slopes 3/4 and 1/4 and are shown to guide the eye. A simple comparison with Figure 1 reveals the anomalous scaling behaviour for this case. The local width does not saturate, but crosses over to an anomalous time regime, t^{β_*}, where $\beta_* \simeq 0.25$ and saturation occurs at the same time for the whole system. Inset shows the width in true saturation (flat part in the plots) *versus* the scale l in a log-log plot. The line (inset) fits the data and the slope corresponds to $\alpha_{\rm loc} \simeq 0.92$.

system saturates (*i.e.* $t_s(l) \sim L^z$ for any l). As we did in Figure 1 for the strong pushing limit, we can use the saturated values of the local width in Figure 2 to determine the local roughness exponent (see Fig. 2 inset), $\sigma_{\rm sat}(l) \sim l^{0.92}$ From Figure 2 it is clear that the dynamic behaviour of the interface near the depinning transition cannot be completely described with a Family-Vicsek scaling ansatz. According to this scaling picture the local width $\sigma(l,t)$ should saturate when the horizontal correlation length reaches the size of the sample, $l_c(t_s) \sim l$, as occurs in the strong pushing limit (see Fig. 1). However, the local width displayed in Figure 2 scales with time for length scales $l \ll l_c(t)$ and a new time exponent β_* has to be introduced.

This anomalous time regime of $\sigma(l,t)$ has been observed in other growth processes and termed the problem of *anomalous* kinetic roughening in the literature [5,6]. In reference [5] we have shown that for a broad class of growth processes the scaling for $l \ll l_c(t)$ is actually given by

$$\sigma(l,t) \sim t^{\beta - \alpha_{\rm loc}/z}\, l^{\alpha_{\rm loc}}, \tag{7}$$

when a local exponent $\alpha_{\rm loc} \neq \alpha$ exists, *i.e.* when anomalous roughening occurs. This gives rise to an anomalous temporal regime with $\beta_* = \beta - \alpha_{\rm loc}/z$. Only when the interface is self-affine one has $\beta_* = 0$ and the Family-Vicsek scaling for the local width is recovered. As demonstrated in reference [5] this may occur for growth with α below or above one and thus it is not necessarily associated with super-roughening. More precisely, super-roughening corresponds to an anomalous scaling like (7), but $\alpha_{\rm loc} = 1$ and so $\beta_* = \alpha - 1/z$ in this particular case. At true saturation, when $l_c(t) \sim L$, the local width scales as

$$\sigma(l, t \gg L^z) \sim l^{\alpha_{\rm loc}} L^{z\beta_*} \sim l^{\alpha_{\rm loc}} L^{\alpha - \alpha_{\rm loc}},$$

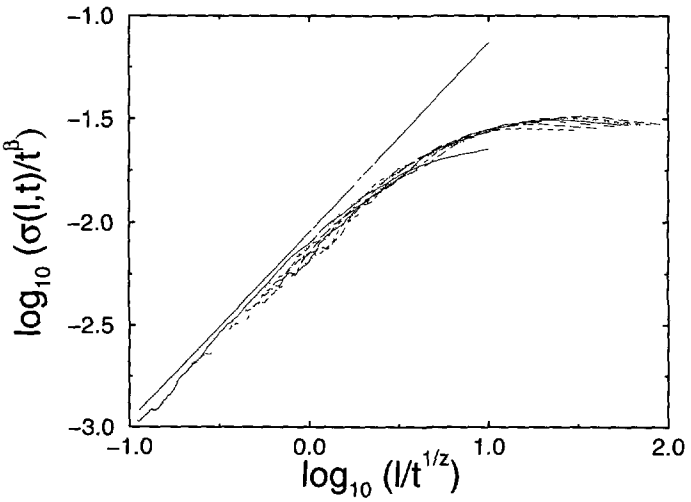

Fig. 3. — Data collapse, according to (9), of the results displayed in Figure 2. The exponents $\beta = 0.81$ and $z = 1.53$ were used. The slope of the straight line gives the local roughness exponent $\alpha_{\rm loc} = 0.92$.

that corresponds to the anomalous dependence of the local width on the system size already observed in simulation [13].

Next, we are going to show that for the QEW model (1) the anomalous scaling behaviour of the local width at the depinning transition is also given by equation (7) with $\alpha_{\rm loc} = 1$ in this case. In Figure 3 we plot a collapse of the data of Figure 2. An inspection of Figure 3 reveals that the dynamic scaling form of the local width is consistent with

$$\sigma(l,t) \sim t^{\beta} f(l/t^{1/z}). \tag{8}$$

The scaling function is given by

$$f(u) \sim \begin{cases} \text{const} & \text{if} \quad u \gg 1 \\ u^{\alpha_{\rm loc}} & \text{if} \quad u \ll 1, \end{cases} \tag{9}$$

where $\alpha_{\rm loc} \simeq 0.92$, $\beta \simeq 0.81$ and $z \simeq 1.53$ are the exponents giving the best data collapse. This scaling behavior leads to a local width $\sigma(l,t) \sim t^{\beta - \alpha_{\rm loc}/z}\, l^{\alpha_{\rm loc}}$ in the intermediate regime $l \ll t^{1/z} \ll L$ that corresponds to the scaling proposed in (7) with $\beta_* = \beta - \alpha_{\rm loc}/z \simeq 0.21$ (to be compared with our previous determination $\beta_* \simeq 0.25$). In agreement with other authors [11, 13, 20], we thus found $\beta \simeq 0.81$, $z \simeq 1.53$ and a global roughness exponent $\alpha = z\beta \simeq 1.2$ from the data collapse shown in Figure 3.

Similar results were obtained when the structure factor was used. The structure factor, which is related to the Fourier transform of the height-height correlation function can be used to obtain the critical exponents

$$S(k,t) = \langle \widehat{h}(k,t)\widehat{h}(-k,t)\rangle \sim k^{-(2\alpha+1)} s(k^z t), \tag{10}$$

where $\widehat{h}(k,t) = L^{-1/2}\sum_x [h(x,t) - \overline{h}(t)] \exp(ikx)$ and the scaling function is

$$s(u) \sim \begin{cases} \text{const} & \text{if} \quad u \gg 1 \\ u^{2\alpha+1} & \text{if} \quad u \ll 1, \end{cases} \tag{11}$$

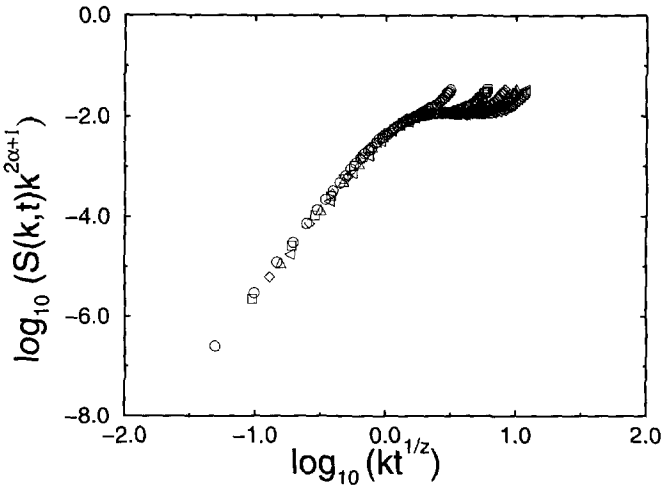

Fig. 4. — Structure factor data collapse for the QEW equation at the depinning transition in a system of size $L = 128$. Data shown a good collapse for $z = 1.67$ and $\alpha = 1.25$. The deviation from the scaling at large abscissa values is due to discrete lattice effects.

whenever the interface satisfies a standard Family-Vicsek scaling. Also in the case of anomalous scaling due to super-roughening, equation (11) gives the correct scaling of the structure factor (see [5b] for details).

We have determined the structure factor at the depinning transition by performing simulations of equation (1) in a system of total size $L = 128$ and a density of pinning sites $p = 1$. For this system size, the critical force was found to be $F_c \simeq 0.067$. Figure 4 shows the data collapse for the exponents $\alpha = 1.25$ and $z = 1.67$ (and then $\beta = \alpha/z \simeq 0.75$) in agreement with the exponents that we found in Figure 3.

2.3. Dynamic Exponent at Depinning. — There is another point that merits some discussion related to the complete scaling of the QEW model. From Figures 2 and 3 we have obtained a value for the dynamic exponent $z \approx 1.5$–1.6 in agreement with several previous simulations [13, 20] of the QEW equation (and related models) in which an exponent $z \simeq 1.5$ was reported. However, in these numerical works z was always calculated from the scaling relation $z = \alpha/\beta$, as we have done in this paper. In principle, in order to correctly determine the dynamic exponent z one should be able to use its definition in $l_c(t) \sim t^{1/z}$, where the correlation length $l_c(t)$ can be obtained from an appropriate correlation function. Let us consider the height correlation function $\Gamma(l,t) = \{\langle (h(x+l,t) - \langle h \rangle)(h(x,t) - \langle h \rangle)\rangle\}$, which relates the heights at two positions x and $x + l$. This correlation function becomes zero at distances l larger than the correlation length, thus $l_c(t)$ can be determined from

$$l_c(t) = \frac{\int l\Gamma(l,t)\mathrm{d}l}{\int \Gamma(l,t)\mathrm{d}l} \sim t^{1/z} \tag{12}$$

Note that equation (12) should be the correct way of obtaining z while we are interested in checking the validity of the dynamic scaling ansatz. Direct determination of the dynamic exponent from equation (12) was reported in references [10, 21], where $z' \simeq 2$ for a wide

range of driving forces even at the transition (here the prime denotes a direct measure using equation (12)). Surprisingly, the value of the exponent $z' \simeq 2$ obtained in this way differs from the one that we have measured by an *indirect* method $z \simeq 1.5$ (and consequently the corresponding global roughness exponents $\alpha \simeq 1.2$ and $\alpha' \simeq 1.5$ are different as well). A value of the dynamic exponent $z' \simeq 2$ and the time exponent $\beta \simeq 0.75$ leads to $\beta_* = \beta - 1/z' \simeq 0.25$, in agreement with our previous estimation for the anomalous time exponent.

In order to try to explain this puzzling situation we have considered the effect of the characteristic size of the pinned regions on the interface local scaling. Besides the super-rough character of interfaces generated by the QEW equation and the corresponding anomalous scaling of the local width just discussed above, there is also another important effect that conspires to complicate even more the QEW problem. As we have already mentioned, the quenched disorder in equation (1) can slow down the motion of the interface in large regions of length ξ in which the interface temporarily remains pinned. In an infinite system, the size of the pinned regions, ξ, diverges as $\xi \sim (F - F_c)^{-\nu}$ when approaching the depinning threshold F_c from above and the exponent $\nu \simeq 1.35$ [13] (or $\nu \simeq 4/3$ in Ref. [14]) can be measured in simulation from a direct observation of these pinned regions in the interface. For a system of finite size L the depinning transition occurs at a value of the driving force $F = F_d(L)$ for which $(F_d - F_c)^{-\nu} \sim L$. As a consequence, in a finite system the depinning force $F_d(L)$ depends on the system size, and close to the depinning transition both lengths ξ and L become relevant in the dynamics. Thus, in any proper numerical determination of the global roughness exponent at the depinning threshold one must have in mind two important facts. First, to obtain the global exponent α' (where $\alpha' = z'\beta \approx 1.5$) one has to perform simulations in systems with different sizes L and, after saturation for $t \gg L^z$, one can fit the power law $\sigma(L, t \gg L^z) \sim L^{\alpha'}$ Secondly, since the critical force $F_d(L)$ is different for a different size of the system, in order to keep the system at the depinning transition one must vary the driving force F in such a way that $\xi \sim L$. These requirements lead to a dependence of the saturated global width on the relevant lengths ξ and L at the depinning transition as

$$\sigma(L, t \gg L^z) \sim L^{\alpha'} \left(\frac{\xi}{L}\right)^{\theta} \sim L^{\alpha'-\theta} (F_d(L) - F_c)^{-\nu\theta}, \tag{13}$$

where θ is an exponent that characterizes the dependence of the global width on the correlation length ξ. It is worth remarking here that only when $\xi \sim L$ the system is certainly being maintained at the depinning transition and the exponent α' can be measured. On the contrary, numerical simulations done in systems with different sizes, but in which the driving force F is not adjusted to the corresponding $F_d(L)$, always yield a different roughness exponent $\alpha = \alpha' - \theta$. The exponent α would appear for any F somewhat larger than F_d and gives the global roughness in the *moving* phase.

In reference [13] an anomalous dependence of the prefactor of the width on the driving force as $(F - F_c)^{-\phi}$ was already found, with $\phi \simeq 0.44$. We believe that the dependence of the saturated width on the driving force close to the depinning transition indicates that the system is not just at the transition. Taking into account the measured values $\nu \simeq 1.35$ [13, 16] and $\phi \simeq 0.44$ [13] we have $\theta = \phi/\nu \simeq 0.33$. So, we obtain a global roughness exponent $\alpha = \alpha' - \theta \simeq 1.17$ in agreement with our numerical determination ($\alpha \simeq 1.2$, see Figs. 3 and 4) as well as with previous simulations [11–13, 17, 19].

3. Discussion

In summary, we have found that the complete scaling behaviour of the QEW equation is not trivial. At the depinning transition the interface is super-rough and this leads to the violation

of the usual scaling ansatz. In particular, the local width exhibits a new time regime with an exponent $\beta_* \simeq 0.21$. We have shown that this is associated with the existence of two different roughness exponents describing the scaling of the local and global fluctuations. A new dynamic scaling, which was already used in other growth processes, has been successfully applied to understand the scaling behaviour of the local width.

Accordingly to this scaling, equation (7), the measured value of the local roughness, close to unity $\alpha_{\rm loc} \simeq 0.92$, suggests that the anomalous scaling of the local width is due to super-roughening, in such a way that a global roughness exponent larger than one must exist. A direct determination of the time exponent gives $\beta \simeq 0.75 - 0.81$ in agreement with most of the previous works. On the contrary, a direct determination of the global roughness exponent from the scaling of the saturated global width with the system size can be controversial since variation of the total size L leads to a change in the depinning threshold F_c. The effect of the characteristic length of the pinned sites regions is relevant close to the depinning transition. It seems then that it is possible to obtain two different values for the global roughness exponent depending on the procedure used. The most usual reported value of the global roughness exponent $\alpha \simeq 1.2$ (and $z \simeq 1.5$) is obtained by us from collapses of the local width as well as from the behaviour of the structure factor. However, a direct measure of the correlation length reflects that there is a dynamic exponent $z' \simeq 2$ [10, 21] (and consequently $\alpha' \simeq 1.5$). Moreover, it remains unclear for us which of the mentioned methods (and the corresponding set of critical exponents) is the proper to describe completely the scaling behaviour of the interface at depinning. We believe that it is a very unsatisfactory situation the fact that, on the one hand, the depinning threshold changes with the system size and, on the other hand, one of the exponents of interest, the global roughness exponent, has to be measured by changing the system size. It seems difficult to define a correct roughness exponent at critical depinning as the exponent α in the power law $\sigma(L, t \to \infty) \sim L^{\alpha}$ while a variation of L yields a shift in the critical point.

From an experimental point of view the local roughness exponent is very important since it is the exponent accessible in experiments, in which the size of the sample remains fixed. We suggest that the anomalous scaling (7) might be useful for clarifying whether the QEW equation certainly describes the motion of interfaces in disordered media, since in many experiments it may be possible to study the time evolution of the local width and to determine the existence of an anomalous exponent β_*.

Acknowledgments

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