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Interaction of Pancake Vortices with c-Axis Plasmon in Josephson-Coupled Layered Superconductors

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Abstract. — The power-law dependence of plasma resonance frequency on magnetic field applied along the c-axis is explained in the model of strong disorder in pancake positions along the c-axis. The line width of plasma resonance is explained by inhomogeneous broadening caused by random positions of pancake vortices in the vortex glass phase. This leads to a nearly temperature independent line width which is inversely proportional to magnetic field. The dynamic interaction of vortices with plasmon is calculated. It is ineffective in fields studied but may become stronger in crystals with weaker pinning or in higher magnetic fields.

1. Introduction

Highly anisotropic high-$T_c$ superconductors may be considered as a stack of superconducting CuO$_2$ layers coupled by Josephson interactions [1, 2]. The novel properties of these materials as compared with a single Josephson junction are associated with their multilayer structure and with the presence of pancake vortices when a magnetic field is applied along the c-axis. It was shown previously that pancake vortices induced by such a field strongly suppress the interlayer maximum superconducting current by inducing random phase differences between layers in the presence of disorder in pancake positions along the c-axis [3]. It was predicted [4] that this effect leads to a decrease of the c-axis Josephson plasmon frequency with magnetic field applied along the c-axis.

Recently, a sharp magnetoabsorption resonance was observed in the vortex state of the highly anisotropic layered superconductor Bi-2:2:1:2 by Tsui et al. [5] and Matsuda et al. [6] in the frequency range 30-90 GHz depending on magnetic field and temperature. The plasma resonance was observed as a peak in absorption in AC field and temperature when magnetic field is swept at fixed frequency, see Figure 1. The field behavior of this resonance (decrease of frequency with magnetic field applied along the c-axis) as well as its angular dependence (sharp decrease of the resonance frequency near orientations of the strong magnetic field parallel to layers) was found to be in agreement with predictions [4, 7] for the Josephson plasmon in layered superconductors. Matsuda et al. [6] and Tsui [5] confirmed that this resonance is a maximum when an AC electric field is oriented along the c-axis. This observation provides strong evidence that the observed resonance is indeed the c-axis Josephson plasmon. Thus, the plasma resonance found in Bi-2:2:1:2 superconductor is the extension of the Josephson plasmon

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Fig. 1. — Curve-fits (solid lines) to data obtained in reference [8] for Bi-2212 at $T = 4.2$ K assuming inhomogeneous broadening with $B \parallel c$ (see text). Experimental data of different frequencies are represented by different symbols. The vertical scale of individual curve has been normalized and shifted with an offset for the ease of comparison.

discovered in the single Josephson junction by Dahm et al. [8] to the multilayered system with Josephson interlayer coupling. In a single Josephson junction the plasma mode is the charge oscillation between two superconductors forming the junction, with the current between the superconductors being the Josephson tunneling current. In layered superconductors, the plasma mode is a charge oscillation between the top and bottom layers of the sample, and the corresponding currents flow between all layers forming the crystal.

The experimental studies [5,6] have established the following properties of the $c$-axis plasma resonance:

a) Below the irreversibility line in the $(B, T)$ plane the dependence of the plasma frequency $\Omega$ on the magnetic field $B \parallel c$ and the temperature $T$ has the form:

$$\Omega^2(B, T) = A_1 B^{-\mu} \exp(T/T_0),$$

where $A_1$ is a constant, $\mu \approx 0.7-0.8$ is temperature independent and $T_0 \approx 12.5$ K in fields 0.3-7 T and at temperatures 3-16 K [5]. Above the irreversibility line the power-law field dependence holds with $\mu \approx 0.9-1$ and here $\Omega$ drops with temperature at fixed magnetic field [6]. Thus, the temperature dependence of plasma frequency exhibits a cusp at the irreversibility line.

b) At low temperatures, when the magnetic field $B$ is tilted by the angle $\theta$ with respect to the $ab$ plane, at high angles $\theta > 10^\circ$ the only perpendicular component of the field $B_z = B \sin \theta$ is
effective, and equation (1) with \( B = B_z \) describes the data. In high magnetic fields above 2 T, at \( \theta \) smaller than 5° the plasma frequency decreases sharply as \( \theta \) approaches zero [5].

c) In a magnetic field \( B \parallel c \), and below the irreversibility line, the relative resonance line width, \( \Gamma/\Omega \), decreases weakly with magnetic field [5]. Here \( \Gamma \) is the line width. At \( T = 4.2 \) K the relative resonance line width, \( \Gamma/\Omega \), changes from 0.12 at \( \Omega/2\pi = 50 \) GHz to 0.087 at \( \Omega/2\pi = 30 \) GHz. The line width of resonance with respect to variable magnetic field at fixed frequency was found to be almost temperature independent in the vortex glass state [5]. The dependence of the relative line width on magnetic field below the irreversibility line may be described as \( \Gamma/\Omega = A_2 B^{-\nu} \), where \( A_2 \) is a constant and \( \nu \approx 0.25 \). Matsuda et al. found the relative line width \( \approx 0.15 - 0.2 \) below the irreversibility line and they observed its increase with temperature above the irreversibility line in the vortex liquid state up to 0.5 at \( T = 63 \) K. As for the resonance frequency, the relaxation rate has a cusp at the irreversibility line [5].

2. Field, Angular and Temperature Dependence of Plasma Frequency

The power-law dependence of the plasma frequency on \( B \) at \( B \parallel c \) was explained assuming that the vortex lattice is strongly disordered along the c-axis due to pinning in the vortex glass state or by thermal fluctuations in the liquid vortex state [9]. Deviations of pancake vortices from straight lines in equilibrium induce a nonzero phase difference \( \varphi_{n,n+1}(r) \) between neighboring layers \( n \) and \( n+1 \) at coordinate \( r = (x,y) \). This phase difference suppresses the average interlayer Josephson energy and maximum possible interlayer superconducting current, \( J_m = J_0 \cos \varphi_{n,n+1}(r) \). The parameter \( J_0(T) = e\Phi_0/8\pi^2 s\lambda_{ab}^2(T) \gamma \) characterizes the Josephson interlayer coupling at \( B = 0 \) and \( \langle . \rangle \) means averaging over space and disorder. Here \( \lambda_{ab}(T) \) is the London penetration length, \( s \) is the interlayer spacing and \( \gamma \) is the anisotropy ratio. The suppression of plasma frequency by pancake vortices was described assuming that plasma frequency is proportional to the maximum interlayer current [4,9]:

\[
\Omega^2 = \frac{8\pi^2 cs}{e_c \Phi_0} J_m = \frac{8\pi^2 cs}{e_c \Phi_0} J_0 \langle \cos \varphi_{n,n+1}(r) \rangle. \tag{2}
\]

Here \( e_c \) is the high frequency dielectric function for an electric field along the c-axis. Equation (2) corresponds to averaging over the Josephson interaction. In this mean field approach, pancake vortices are assumed to be fixed and effect of the inhomogeneity of Josephson interactions due to random positions of pancakes is not accounted for. It is the phase difference \( \varphi_{n,n+1}(r) \) which depends on the vortex structure and leads to the field dependence of \( \Omega \). The difference induced by pancake vortices is:

\[
\varphi_{n,n+1}(r) = \sum_{\nu} [f(r - r_{n,\nu}) - f(r - r_{n+1,\nu})], \tag{3}
\]

where \( f(r) = \arctan(x/y) \) and \( r_{n,\nu} = (x_{n,\nu}, y_{n,\nu}) \) are the positions of pancakes labeled by the index \( \nu \) in the \( n \)-th layer [3]. The expression in the sum is obtained by neglecting interlayer Josephson coupling, and it is correct at distances smaller than the Josephson length \( \lambda_J = \gamma s \) from neighboring pancakes at \( r_{n,\nu} \) and \( r_{n+1,\nu} \) in the \( ab \) plane. Equation (3) shows that many terms contribute effectively to the phase difference at given \( r \) since the phase difference for given close \( r_{n,\nu} \) and \( r_{n+1,\nu} \) is a slowly decreasing function of \( r \): it vanishes as \( 1/|r - r_{n,\nu}| \) far away from these vortices. As a result, the phase \( \varphi_{n,n+1}(r) \) obeys a Gaussian distribution with \( \langle \varphi_{n,n+1}(r) \rangle = 0 \). Therefore, plasma frequency may be expressed in terms of a Debye-Waller factor:

\[
\Omega^2 = \Omega^0_2 \exp \left(-\langle \varphi_{n,n+1}^2(r) \rangle/2 \right), \tag{4}
\]
where $\Omega_0 = c/\lambda_{ab} \sqrt{c/c}$ is the plasma frequency in the absence of magnetic field. Using equation (3) we see that $\langle \varphi_{n,n+1}^2(r) \rangle$ diverges logarithmically at large distances; the cut-off is determined by $\lambda_J$. As result, when $\lambda_J$ is larger than the intervortex distance $a = (\Phi_0/B)^{1/2}$ and the long range order of the vortex lattice in the $ab$ plane is absent, the power law holds:

$$\Omega(B)^2 \approx \Omega_0^2 (B_3 e^2/B)^\mu,$$

where $B_3 = \Phi_0/\gamma^2 s^2$. The exponent $\mu$ is determined by the correlation function $K(r) = \langle \rho_n(0) \rho_n(r) \rangle - \langle \rho_n(0) \rangle \langle \rho_n(r) \rangle$ of pancake density $\rho_n(r) = \sum_\nu \delta(r-r_{\nu n})$ by the relation:

$$\mu = (\pi/4) \int dr r^2 K(r).$$

For strong disorder along the $c$-axis we obtain $\langle \rho_n(0) \rho_{n+1}(r) \rangle \approx \langle \rho_n(r) \rangle^2$, and $\mu$ becomes field independent and of the order unity because $K(r)$ oscillates on the scale $a$ and decays at large $r$ in the $ab$ plane on the same scale or slowly. The parameter $\mu$ depends on the correlations of pancakes in the $ab$ plane and may be slightly different in the vortex glass phase and in the liquid phase. Thus, the field dependence of the plasma frequency characterizes the arrangement of pancakes along the $c$-axis: field independent $\mu$ of order unity means that vortices are disordered along the $c$-axis, while smaller $\mu$ decreasing with field indicates in the pancake ordering along lines at distances larger than the interlayer spacing. Note, that $\mu$ may depend on history as well.

Experimental observation of a field independent $\mu \approx (0.7 - 0.8)$ confirms that strong disorder in pancake positions along the $c$-axis is present in the samples studied. This conclusion is indirectly supported by the fact that in high magnetic field (above 0.1 T) and at low temperatures the Bragg peaks have not been observed in neutron scattering experiments [10]. This can be also explained by the absence of pancake ordering along the $c$-axis.

Strong angular dependence of the plasma resonance in high magnetic fields $B \gg \Phi_0/s \lambda_J$ was predicted [7] as a result of interactions of pancake vortices, produced by the $B_z$-component of the field, with the dense lattice of Josephson vortices, induced by the field component parallel to the $ab$ plane. In the magnetic field parallel to the layers Josephson structure can move as a whole under the effect of electric field applied along the $c$-axis, and the only low laying resonance is at zero frequency. Pinning shifted this resonance to nonzero frequency. When component of magnetic field along the $c$-axis presents, pancake vortices provide such pinning. They form an almost hexagonal structure in the $ab$ plane and a zigzag structure along the $c$-axis, see Figure 2. The zigzag structure of pancakes minimizes the Josephson interlayer interaction and induces pinning for Josephson vortices. This results in a sharp enhancement of the plasma frequency when $\theta$ increases from zero and pancake vortices appear:

$$\Omega^2 = \Omega_0^2 \theta \lambda_{ab}^2 \left[ \ln \frac{\lambda_{ab}}{\sqrt{3} s^2} + \frac{1}{2} \lambda_{ab}^2 G^2 g^2 \left( \frac{h}{G} \right) \right]^{-1},$$

where $G = (8\pi^2 B \sin \theta/\Phi_0 \sqrt{3})^{1/2}$ is the reciprocal vector of pancake triangular lattice formed by the perpendicular component of magnetic field, $h = 2s B/\Phi_0$ is the reciprocal vector of the Josephson lattice formed by the parallel component of the magnetic field and $g(x)$ is the minimum value of $|m + x|$ with respect to integer $m$.

Next we discuss the temperature dependence of plasma frequency in the vortex glass phase. It includes the temperature dependence of $J_0$ and that due to phase fluctuations (thermally excited phase collective modes and pancake oscillations in pinning wells). $J_0(T)$ may be extracted from measurements of the plasma frequency at $B = 0$. The renormalized plasma frequency $\hat{\Omega}(T) = \omega_c(T) J_0(0)/J_0(T)$ characterizes the effect of phase fluctuations only.
Oscillations of pancakes around equilibrium positions in pinning wells renormalize the parameter $\mu$ according to equation (5). The maximum thermally induced mean square displacement of pancakes may be estimated by use of the classical expression, $\langle u^2_{n+1/2} \rangle_t = T/s\alpha_L$. Here $\alpha_L$ is the Labusch parameter estimated as $\alpha_L = J_{c,ab} F_0/c\xi_{ab}$ [11,12]. This gives $\alpha_L \approx 5 \times 10^5 \text{g/cm} \times \text{s}^2$ for $J_{c,ab} \approx 5 \times 10^6 \text{A/cm}^2$. Then $\langle u^2_{n+1/2} \rangle_t \approx 3 \times 10^{-14} \text{cm}^2$ and $\langle (u_{n+1/2} - u_{n+1/2})^2 \rangle_t$ is about the same. The pinning induced mean square relative displacements ($\approx a^2$ for strong disorder) are about two orders of magnitude larger. This means that thermal motion of pancakes may be neglected below the irreversibility line, and the parameter $\mu$ is practically temperature independent there in agreement with the results [5]. Thus, temperature dependence in equation (1) for plasma resonance cannot be explained by thermal depinning. As was argued [9], enhancement of plasma frequency with temperature may be caused by the effect low frequency thermal fluctuations of phase difference (thermally excited phase collective modes). These smooth out rapid changes of the phase difference produced by disordered pancake vortices and result in linear in $T$ increase of the average Josephson interaction and plasma frequency with temperature at low temperatures. Their effect on the plasma frequency may be compared with the effect of spin waves on the magnetic susceptibility of an antiferromagnet below the Néel temperature: magnetic susceptibility increases because spin waves make the antiferromagnet softer.

### 3. Equation for Phase Difference in Presence of Mobile Pancakes

The relatively large line width of the c-axis plasma resonance [5,6] is surprising. In a single Josephson junction the line width of the Josephson plasmon is associated with incoherent dissipative tunneling of quasiparticles. In a superconductor with gap $\Delta$ in the quasiparticle spectrum the concentration of quasiparticles is exponentially small at low temperatures $T \ll \Delta$ and vanishes linearly in $T$ for d-wave superconductors. Pancake vortices increase the number

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Fig. 2. — The zigzag structure of pancake vortices at high magnetic fields near $ab$ orientation at $h = G$. 

---

n+1
n
n-1

$B$

$B_x$

$y$

$x$

---

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of quasiparticles linearly in $B$ in s-wave superconductors and as $B^{1/2}$ in d-wave superconductor [13]. Thus we can imaging that a relatively large weakly temperature and field dependent line width can not be explained by an interlayer current of normal quasiparticles. Rather, it may be caused by the effect of pancake vortices on Josephson interlayer interaction; randomly positioned they induce inhomogeneity of the Josephson interaction which may lead to a significant broadening of the Josephson plasma resonance. In addition, plasma oscillations may mix with vortex oscillations and this effect also results in broadening of plasma resonance. To describe these effects quantitatively, we need time-dependent equations for the phase difference (like sine-Gordon equation for single Josephson junction) in the presence of mobile pancake vortices.

The equation for phase difference at equilibrium in the presence of pancakes is [14]:

$$-\sum_{m} L_{nm} \nabla^{2} \psi_{m,m+1}^{(r)} + \lambda_{J}^{-2} \sin[\varphi_{n,n+1}^{(v)} + \varphi_{n,n+1}^{(r)}] = 0.$$  \hspace{1cm} (8)

Here \( \varphi_{n,n+1}^{(r)} = \varphi_{n,n+1}^{(v)} + \varphi_{n,n+1}^{(r)} \) is the total phase difference, \( L_{nm} = (\lambda_{ab}/s)(1 - s/\lambda_{ab})^{n-m} \) is the mutual inductance of layers, and \( \varphi_{n,n+1}^{(v)}(r,r_{nv}) \) is the phase difference induced by pancakes at positions \( r_{nv} \) in the absence of Josephson interaction (at infinite \( \lambda_{J} \)). It is given by equation (3). The function \( \varphi_{n,n+1}^{(v)}(r) \) is singular at vortex positions \( r_{nv} \), while the function \( \varphi_{n,n+1}^{(r)}(r) \) is regular everywhere and describes the effect of three-dimensional screening caused by interlayer Josephson currents [14]. For mobile pancake vortices we insert in equation (3) time-dependent coordinates \( r_{nv}(t) \) and in equation (8) we need to add terms with time derivatives of the phase difference. These terms were obtained in the framework of time-dependent Ginzburg-Landau (TDGL) equation (see [15]) corresponding to the Lawrence-Doniach functional [16]. In the presence of the homogeneous external electric field \( \mathcal{E}_{2}(t) \) (for a sample placed in a cavity acting as a capacitor) the equation for \( \varphi_{n,n+1}^{(r)}(r,t) \) is:

$$\frac{1}{c_{0}^{2}} \hat{T}_{c} \frac{\partial}{\partial t} \left[ \varphi_{n,n+1}^{(r)} + \varphi_{n,n+1}^{(v)} \right] - \sum_{m} L_{nm} \nabla^{2} \psi_{m,m+1}^{(r)} + \frac{1}{\lambda_{J}^{2}} \sin \left[ \varphi_{n,n+1}^{(r)} + \varphi_{n,n+1}^{(v)} \right] = \frac{\hbar}{8\pi e\epsilon_{0}} \frac{\partial}{\partial t} \mathcal{D}_{2}(t), \quad \hat{T}_{c} = \frac{\partial}{\partial t} + i\Gamma_{c},$$  \hspace{1cm} (9)

where \( c_{0} = cs/\lambda_{ab}\sqrt{\sigma_{c}} \) plays the role of the Swihart velocity, \( \epsilon_{0} = \Phi_{0}^{2}s/16\pi^{3}\lambda_{ab}^{2} \) is the characteristic energy of pancake vortices, \( \omega_{ab} = c/\lambda_{ab} \) is the in-plane plasma frequency and \( \Gamma_{c} = 4\pi\sigma_{c}/\epsilon_{c} \), where \( \sigma_{c} \) is the c-axis conductivity due to quasiparticles. Here the function \( \varphi_{n,n+1}(r,t) \) is determined by solving TDGL equation for given vortex coordinates \( r_{nv}(t) \). This function coincides with \( \varphi_{n,n+1}^{(v)}(r,r(t)) \) when relaxation of the superconducting order parameter is neglected. The nonzero difference \( [\varphi_{n,n+1}(r,t) - \varphi_{n,n+1}(r,t)] \) is caused by retardation in the time variations of order parameter and superconducting currents when pancakes are moving. The effect of retardation becomes negligible far away from vortex center. Respectively, the difference \( [\varphi_{n,n+1}(r,t) - \varphi_{n,n+1}(r,t)] \) vanishes exponentially far away from vortices with the characteristic length of decay \( \Lambda_{ab} = (\hbar\sigma_{ab}/8e^{2}\gamma_{GL}|\Psi_{\infty}|^{2})^{1/2} \), where \( \gamma_{GL} \) characterizes the relaxation of the superconducting order parameter and may be estimated as \( \gamma_{GL} \geq \hbar\omega_{ab}^{2}/4e^{2}s^{2}|\Psi_{\infty}|^{2}\sigma_{ab}^{(n)} \).

Here \( \sigma_{ab} \) is the conductivity due to quasiparticles, \( \sigma_{ab}^{(n)} \) is the conductivity in the normal state and \( |\Psi_{n}| \) is the amplitude of the superconducting order parameter. For typical parameters the inequality \( \Lambda_{ab} \ll a \) holds, and the difference \( [\varphi_{n,n+1}(r,t) - \varphi_{n,n+1}(r,t)] \) is of the order unity in the region of the radius \( \Delta_{ab} \) around each pancake and rapidly vanishes outside. It
is this difference which determines the dynamic interaction of plasmon with pancakes. The equation of motion for pancake coordinates \( r_{n\nu}(t) \) should be added to equation (9) to obtain the complete set of equations for coupled phase difference and pancake coordinates.

4. Inhomogeneous Broadening of Plasma Resonance

We consider now the solution for \( \varphi_{n,n+1}^{(r)}(r,t) \) and vortex coordinates \( r_{n\nu}(t) \) in the presence of DC magnetic field \( B \parallel c \) and an oscillatory weak external field \( D_z(t) \). Then the phase collective modes are excited and in addition vortices oscillate near equilibrium positions \( r_{n\nu}^{(0)} \) due to phase variations. Then we obtain equations which describe small variations of the phase difference and small amplitude vortex oscillations. We denote pancake deviations by \( u_{n\nu}(t) = r_{n\nu}(t) - r_{n\nu}^{(0)} \) and expand \( \varphi_{n,n+1}^{(v)}(r,t) \) in \( u_{n\nu}(t) \):

\[
\varphi_{n,n+1}^{(v)}(r,t) = \sum_{\nu} [f(r - r_{n\nu}^{(0)}) - f(r - r_{n+1,\nu}^{(0)})] + \varphi_{n,n+1}^{(v)}(r,t),
\]

(10)

\[
\tilde{\varphi}_{n,n+1}^{(v)}(r,t) = \sum_{\nu} D(r - r_{n\nu}^{(0)}) \cdot u_{n\nu}(t) - D(r - r_{n+1,\nu}^{(0)}) \cdot u_{n+1,\nu}(t),
\]

(11)

where \( D(r) = (-y/r^2, x/r^2) \) for \( a < r < \lambda_3 \) and \( D(r) \) drops much faster with \( r \) for \( r > \lambda_3 \). We also expand \( \varphi_{n,n+1}(r,t) \) in \( u_{n\nu}(t) \). We present the phase difference as

\[
\varphi_{n,n+1}(r,r_{n\nu},t) = \varphi_{n,n+1}^{(0)}(r) + \varphi_{n,n+1}^{(v)}(r,t) + \varphi_{n,n+1}^{(r)}(r,t),
\]

(12)

where \( \varphi_{n,n+1}^{(0)} \) is the phase difference at equilibrium when \( D_z = 0 \). It is determined by equation (8) at \( r_{n\nu} = r_{n\nu}^{(0)} \). The contribution \( \varphi_{n,n+1}^{(v)} \) accounts directly for vortex motion and is given by equations (10, 11), while \( \varphi_{n,n+1}^{(r)} \) accounts for the rest of the phase variations. Both, \( \varphi_{n,n+1}^{(v)} \) and \( \varphi_{n,n+1}^{(r)} \) are small in a weak external electric field. Thus we can expand in equation (9) and obtain a linear equation for \( \varphi_{n,n+1}^{(r)} \):

\[
\frac{1}{c_0^2} \frac{\hbar}{8\pi\epsilon_0\epsilon_c} \frac{\partial}{\partial t} \varphi_{n,n+1}^{(r)} + \sum_m L_{nm} \nabla^2 \varphi_{m,m+1}^{(r)} + \frac{1}{\lambda_3} \cos \varphi_{n,n+1}^{(0)}(r) \varphi_{n,n+1}^{(r)} = \frac{\hbar}{8\pi\epsilon_0\epsilon_c} \frac{\partial}{\partial t} D_z(t) - \frac{1}{\lambda_3} \cos \varphi_{n,n+1}^{(0)}(r) \varphi_{n,n+1}^{(r)} - \frac{1}{c_0^2} \frac{\hbar}{8\pi\epsilon_0\epsilon_c} \frac{\partial}{\partial t} \varphi_{n,n+1}^{(r)}. \]

(13)

The last two terms in the right hand side describe the excitation of the phase collective mode due to vortex oscillations \( u_{n\nu}(t) \). The equation of motion for vortex deviations, \( u_{n\nu}(t) \), which couple with phase oscillations, will be presented later. After solving these two coupled equations for \( \varphi_{n,n+1}^{(r)} \) and \( u_{n\nu}(t) \), we obtain the electric field between layers \( n \) and \( n+1 \):

\[
E_{z:n,n+1}(r,t) = \frac{\hbar}{2\epsilon_0} \frac{\partial}{\partial t} \left[ \varphi_{n,n+1}^{(r)}(r,t) + \varphi_{n,n+1}^{(v)}(r,t) \right],
\]

(14)

and then we calculate the inverse dielectric function \( 1/\epsilon(\omega) = (E_z(r,\omega))/D_z(\omega) \), which describes the plasma resonance.

Now we solve equation (13) with fixed vortices, when \( u_{n\nu}(t) = 0 \). Then the two last terms in the right hand side of this equation are absent. We assume that the inhomogeneous part of \( \varphi_{n,n+1}^{(r)} \), which is \( \Theta_n(r,t) = \varphi_{n,n+1}^{(r)}(r,t) - \bar{\Theta}(t) \), is much smaller than the homogeneous
part \( \theta(t) = \langle \cos \varphi_{n,n+1}(r,t) \rangle \). We use here the perturbation theory in the continuous spectrum, see [17]. Averaging over space in equation (13) we obtain:

\[
\frac{1}{c_0^2} \frac{\partial \theta(t)}{\partial t} + \frac{b}{\lambda_3^2} \theta(t) + \frac{1}{\lambda_3^2} \left[ \cos \varphi_{n,n+1}(r) - b \right] \Theta_n(r,t) = \frac{\hbar}{8\pi e e_0} \frac{\partial}{\partial t} D_z(t),
\]

where \( b = \langle \cos \varphi_{n,n+1}(r) \rangle \). The inhomogeneous part is determined by

\[
\frac{1}{c_0^2} \frac{\partial \Theta_n(r,t)}{\partial t} + \frac{b}{\lambda_3^2} \Theta_n(r,t) - \sum_m L_{nm} \nabla^2 \Theta_m(r,t) = -\frac{1}{\lambda_3^2} \left[ \cos \varphi_{n,n+1}(r) - b \right] \theta(t).
\]

The solution of this equation in the Fourier representation \( \omega, k, q \) with respect to \( t, r, n \) is:

\[
\Theta(\omega, k, q) = \frac{\Omega_0^2 \left[ \cos \varphi_{n,n+1}(r) - b \right] k q}{\omega_0^2 - \omega^2 - i \Gamma_c(\omega)} = \frac{\omega}{\Omega_0^2 \Omega_0^2 - \omega^2 - i \Gamma_c(\omega)}.
\]

\[
\Omega_0^2 = \frac{\omega_0^2}{\Omega_0^2 - \omega^2 - i \Gamma_c(\omega)}.
\]

Inserting this expression into equation (15) we obtain the solution for \( \theta(\omega) \):

\[
\theta(\omega) = -\frac{\hbar \Omega_0^2}{8\pi e e_0} \frac{\omega}{\omega_0^2 - \omega - i \Gamma_c(\omega)} \theta(\omega) = \int \frac{dk dq}{(2\pi)^3} \frac{\Omega_0^2 F(k, q)}{\omega_0^2 - \omega - i \Gamma_c(\omega)}.
\]

The dispersion relation for the phase collective mode, equation (18), is valid for frequencies \( \hbar \Omega(k, q) \ll \Delta \), and thus integration over \( k \) in equation (20) is restricted by \( k < \Delta/c_0 \).

In calculation of the correlation function \( F(r, n) \) we follow the approach [9] used above to calculate \( b \). We consider zero temperature limit. In the Fourier representation for \( r \) and \( n \) we obtain \( F(k, q) \approx a^2 [\pi \mu/(2\mu - 1)] \) at \( ka \leq 1 \) and \( F(k, q) \approx (2\pi)^{1/2} a^{1/2} e^{-3/2} \cos(ka - 3\pi/4) \) at \( ka \gg 1 \). Integration over \( k \) and \( q \) in equation (20) yields:

\[
\Im w(\omega) = \frac{\Gamma_{\text{inh}}^2}{\Omega_0^2} \approx \frac{\pi(4\mu - 1)}{2(2\mu - 1)} \frac{a^2}{\lambda_3^2} \approx \frac{\pi \mu}{2(2\mu - 1)} \frac{B_j}{\Delta_0^2}.
\]

The term \(-i(\Im w)\Omega_0^2 \) in the denominator in equation (19) for \( \theta \) describes broadening of the plasma mode due to random space variations of the Josephson interaction in the presence of pancakes. It may be described also as a decay of the homogeneous phase collective mode into inhomogeneous phase collective modes. The rate of this decay is \( \Gamma_{\text{inh}}^2/2\Omega \ll \Omega \) at \( a \ll \lambda_3 \). Thus, neglecting pancake oscillations. we obtain the dielectric function:

\[
\frac{1}{\epsilon(\omega)} = \frac{1}{\epsilon_c} \frac{\omega^2}{\omega(\omega - i\Gamma_c) - \Omega^2 - i\Gamma_{\text{inh}}^2},
\]

\[
\Omega^2 = b\Omega_0^2.
\]

We obtained \( \Gamma_{\text{inh}} \) at \( T = 0 \). Below the irreversibility line \( \Gamma_{\text{inh}}^2 \) is anticipated to be almost temperature independent because it is determined mainly by short range correlations of \( \cos \varphi_{n,n+1}(r) \) which depend weakly on thermal phase fluctuations \( [F(r, 0) \approx 1/2 \) at small \( r \).
The value $\Gamma_{\text{inh}}$ determines the line width of plasma resonance $\Gamma = \Gamma_{\text{inh}}^2 / 2\Omega$ due to inhomogeneous Josephson interaction in the presence of pancake vortices. For this mechanism, the line shape of resonance with respect to $B$ at fixed $\omega$ is determined by the function:

$$T_m = \frac{1}{\epsilon(\omega, B)} = \frac{A_1^{-1} C_1 B^{-1} B_0^{-\mu}}{(B_0^{-\mu} - B^{-\mu})^2 + A_1^{-2} C_1^2 B^{-2}}, \quad C_1 \approx \frac{\pi \mu}{2(2\mu - 1)} \Omega^2 B_0,$$

where $\mu = 0.8$ and $\omega^2 = A_1 B_0^{-\mu}$ (note that $\Omega^2 = A_1 B^{-\mu}$). This expression describes very well the experimental data [5,16] for line width at $\gamma = 300$, see Figure 1.

5. Dissipation of Plasma into Vortex Oscillations

Next we present the equation for pancake oscillations coupled with phase variations. Then we solve the equation for $\Theta(t)$ taking into account vortex motion to obtain the decay of plasma modes into vortex oscillations. In writing down the equation of motion for pancakes we will consider the case of strong pinning centers for simplicity and focus mainly of terms which describe the interaction of vortices with phase collective modes. The equation for pancake deviations is:

$$\eta \dot{u}_{n\nu} + \alpha_M (n \times \dot{u}_{n\nu}) + \alpha_L u_{n\nu} = F_{n\nu}.$$

$$F_{n\nu}(t) = -\frac{\epsilon_0}{s} \sum_m \int dx dy \left\{ \frac{1}{\lambda^2} \varphi_{m,m+1}^{(r)}(r,t) \frac{\partial \varphi_{m,m+1}^{(v)}(r,t)}{\partial r_{n\nu}} - \left[ \frac{1}{c_0^2} \frac{\partial}{\partial t} \varphi_{m,m+1}^{(r)}(r,t) + \frac{1}{c_0^2} \frac{\partial}{\partial t} \varphi_{m,m+1}^{(v)}(r,t) - \frac{\hbar}{8\pi\epsilon_0} \frac{\partial D_z}{\partial t} \varphi_{m,m+1}^{(v)}(r,t) \right] \right\}.$$ (26)

Here $\alpha_M$ is the Magnus force coefficient and we replace all elastic moduli and effects of pinning by the Labusch parameter $\alpha_L$. Such an approach is correct for strong identical pinning centers and it gives only an order of magnitude estimate for typical Bi-2:2:1:2 single crystals. The estimate for $\eta$ is the Bardeen-Stephen expression $\eta = \Phi_0 \sigma_{ab}^{(n)} / 2\pi \epsilon_0 c^2$, where $\sigma_{ab}^{(n)}$ is the normal state conductivity. For $\alpha_M$ the estimate is the hydrodynamic expression $\alpha_M = \frac{\hbar}{\pi n_s}$, where $n_s$ is the density of superconducting electrons [11,18].

Our focus is the forces $F_{n\nu}$ acting on pancake $n\nu$ when the phase difference deviates from that at equilibrium. Using equation (13) we obtain:

$$F_{n\nu}(t) = \frac{\epsilon_0}{s} \sum_m \int dx dy \left\{ \sum_k L_{mk} \nabla^2 \varphi_{k,k+1}^{(r)}(r,t) \frac{\partial \varphi_{m,m+1}^{(v)}(r,t)}{\partial r_{n\nu}} + \frac{1}{\lambda^2} \cos[\varphi_{m,m+1}^{(0)}(r,t)] \varphi_{m,m+1}^{(r)}(r,t) \frac{\partial}{\partial r_{n\nu}} \left[ \varphi_{m,m+1}^{(v)}(r,t) - \varphi_{m,m+1}^{(v)}(r,t) \right] \right\}.$$ (27)

Next we find pancake displacements caused by the force $F_{n\nu}(t)$ and insert the solution for pancake displacements into the right hand side of equation (13) and account for the homogeneous collective mode only. The first term in the right hand side of equation (27) vanishes for homogeneous part of plasma mode and thus may be omitted. The second term leads to dynamic vortex-plasmon interaction which originates from regions near vortices only because the function $\varphi_{m,m+1}^{(v)}(r,t) - \varphi_{m,m+1}^{(v)}(r,t)$ vanishes exponentially far away from vortex centers. This results in the additional contribution to the left hand side of equation (15):

$$\overline{\Theta}(\omega) = \frac{2 \epsilon_0}{\lambda^2 s a^2} \int dF(r,0) \delta(r) = \frac{\mu \epsilon_0}{(2\mu - 1) \lambda^4 s} \frac{\omega \eta + \alpha_L}{S(\omega)},$$ (28)
where $S(\omega) = (i\omega\eta + \alpha_L)^2 - \omega^2\alpha_M^2$ and $C$ depends on the difference $\vartheta_{n,n+1} - \varphi_{n,n+1}^{(v)}$ and is of the order unity. This additional term in equation (13) has an imaginary part due to vortex viscosity and it determines the dissipation rate $\Gamma_v$ of plasma modes into vortex oscillations:

$$\frac{\Gamma_v}{\Omega} = \frac{\mu C \epsilon_0 \Omega_0}{2(2\mu - 1)\lambda^2} \frac{\eta}{\alpha_L^2} \left( \frac{B}{B_{c2}^2} \right)^{\mu/2}, \quad (29)$$

Here we assume that pinning is strong, $\alpha_L/\Omega \gg \eta, \alpha_M$. $\Gamma_v$ depends weakly on $B$, and $\Gamma_v/\Omega$ increases with $B$ in contrast with inhomogeneous broadening. There is also shift of plasma frequency due to dynamic plasmon-pancake interaction.

For the contribution $\Gamma_v$ due to dissipation of a plasmon into vortex oscillations, we anticipate a significant increase with magnetic field, $\Gamma/\Omega \propto B^{\mu/2}$; see equation (29). We anticipate also significant temperature dependence due to that of the viscosity coefficient $\eta$. Up to now we cannot estimate this contribution to the line width because $\eta$ is unknown. However, predicted field and temperature dependence is in obvious disagreement with experimental data [5, 6].

We conclude that this mechanism is ineffective in magnetic fields used to study the c-axis plasmon so far; $B \leq 7$ T. We note that in higher magnetic fields or weaker pinning coupling of plasmon with vortex oscillations may become more effective. If so, plasma frequency shift will change a power law dependence of $\Omega(B)$ and relative line width will increase with magnetic field. Thus experiments in high magnetic fields may uncover a vortex mechanism of plasma dissipation and provide information on vortex dynamics.

To conclude discussion of dynamic plasmon-pancake coupling we consider the possibility of exciting a plasma mode via vortex oscillations by applying an AC magnetic field. Let us assume that vortices oscillate homogeneously, $u_{n\nu}(t) = u(t)$, around their equilibrium positions under the effect of such a field. Then by use of equation (11) we obtain on the right hand side of equation (13) for $\varphi_{n,n+1}^{(v)}$, the term

$$- \frac{1}{\lambda^2} [\cos \varphi_{n,n+1}^{(0)}(r)] \varphi_{n,n+1}^{(v)} = - \frac{1}{\lambda^2} [\cos \varphi_{n,n+1}^{(0)}(r)] \sum_\nu [D(r - r_{n\nu}^{(0)}) - D(r - r_{n+1,\nu}^{(0)})] \cdot u(t), \quad (30)$$

and a similar contribution originates from the term with $\vartheta_{n,n+1}$. Both terms vanish after averaging over space. Thus homogeneous plasma mode cannot be excited by homogeneous coherent vortex oscillations; only inhomogeneous phase collective modes may be excited by an AC external magnetic field. Similarly, a plasma mode is coupled with inhomogeneous vortex oscillations only, and an AC electric field excites such vortex oscillations but not homogeneous ones.

6. Conclusions

We derived equations which describe oscillations of the phase difference coupled with pancake oscillations in the vortex glass phase. Using these equations we proved that plasma frequency is determined by equation (1) when magnetic field is applied along the c-axis.

We derived also line width of the plasma resonance due to interlayer tunneling of quasiparticles, arising from mixing of the homogeneous plasma mode with inhomogeneous phase collective modes in the presence of randomly positioned pancakes (inhomogeneous broadening), and due to the decay of the plasma mode into vortex oscillations.

We attributed the observed line width of the c-axis plasma resonance in magnetic fields below 7 T to the inhomogeneous broadening. The positional disorder of pancake vortices induced by pinning leads to the possibility of exciting many phase collective modes by an
homogeneous external AC electric field. This mechanism results in the practically temperature independent line broadening which decreases as a power-law with magnetic field in agreement with experimental data [5].

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