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Reply to Mikheev’s Comment on the Black-Scholes Pricing Problem

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Mikheev’s stimulating comment [1] gives us the opportunity to clarify important points which were probably not sufficiently detailed in our paper [2]. Our general method for pricing options is criticized on two aspects: first, it is claimed that in the case of a non-zero average return of the underlying stock our method does not reproduce the Black-Scholes result (fundamentally based on a Gaussian, continuous time model) which gives in fact a price independent of this putative average return. Second, and more importantly, the very procedure of fixing the price by an average ‘fair game’ argument is said to be limited to an ‘imaginary risk neutral world’ [1] rather than the real, risk averse, world. Here, we refute both points by expanding our previous theory [3], and insist on the operational value of our approach to real world, as opposed to an ‘imaginary’ Gaussian, time continuous world.

Let us briefly recall our approach. We take the point of view of the seller of the option, who receives the price of the option $C$ at time $t = 0$, has to pay back $\theta = \min(x(T) - x_c, 0)$ at time $T$ ($x(t)$ is the value of the stock at time $t$) and in the meanwhile trades and earns

$$\int_0^T dt \, \phi(x,t) \frac{dx}{dt},$$

where $\phi$ is the amount of stock in the seller’s portfolio at time $t$. The ‘fair price’ $C$ is obtained by demanding that, on average, the sum of these three terms is zero. If the average return (or more precisely the excess average return over the risk free interest rate) is zero, then the last term is identically zero and one gets [2] $C \equiv < \theta >_m$, where $< \ldots >$ denotes averaging over the stochastic process governing the stock, which ever it might be. If on the other hand $<dx/dt> \neq 0$, then the third term corrects $C$ – but one has to know $\phi$. Our procedure to determine $\phi$ is to ask for minimal uncertainty on the outcome for the seller [2]. The result of this is that [3] for a Gaussian (or log-normal) continuous process, the uncertainty can be made to vanish by choosing a certain ‘optimal’ $\phi^*$ which turns out to be independent of $m$ and equal to the one determined by Black-Scholes. Furthermore, the correction to $C$ given by $-m \int_0^T dt \, \phi^*(x,t)$ precisely compensates the difference between $< \theta >_m$ and $< \theta >_{m=0}$, i.e., $C$ is given by $< \theta >_{m=0}$ for all $m$, in full agreement indeed with the original result of Black and Scholes. This is not surprising since in the Gaussian case, our balance is nothing but the time-integrated version of the differential equation of Black and Scholes [3].
Now, Mikheev argues that this procedure can only be valid in a risk neutral world where operators can be satisfied with average results and not be sensitive to risk. This is certainly true and emphasizes the interest of our approach. Indeed, the model of Black and Scholes leads to zero residual risk which is well known to be very far from reality – while in our point of view, the residual risk is only minimized, and can be accurately estimated for arbitrary stochastic processes. As already mentioned in [2], the seller is now free to correct the fair game price \( C \) by a suitable fraction of this residual risk, obtained as a trade off between – say – minimizing the probability of losing and the fact that an option is itself a commercial product and its price must remain low because of competition. Hence the importance of minimizing the residual risk as a criterion to fix an optimal \( \phi^* \).

In summary, in the real world where zero risk is sham, we argue that the risk must be minimized precisely because of the extra cost incurred in a risk averse world. We end by noting that when applied to real data [4], our formulae show very clearly that the Black-Scholes price based on the historical volatility severely underestimates the 'true' price – a fact well appreciated by professionals who, correspondingly, use an overestimated effective volatility (volatility 'smile')

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**References**