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Short Communication

Study of the Response to Pulses and Possible Prediction of Catastrophes

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Abstract. — We show in a numerical study of the Burridge-Knopoff spring-block model of earthquake, that if one applies very weak pulses (fixed amount of momentum) to any particular block at regular intervals and studies the response on the same block (the stress developed on it), then accurate predictions of the timings of the imminent earthquakes are possible. This is shown by studying the (local) pulse susceptibility $\chi_p$, which grows exponentially with the inverse of the time interval from the next (global) earthquake and saturates to a (nonuniversal) value dependent on the model parameters (fixed for any particular series of earthquakes).

Considerable progress has recently been made in the statistical study of the (fracture, breakdown or earthquake) strength or magnitude distribution of disordered solids (porous media, random composites etc.), of granular packings, or of the dynamical models of earthquake, etc. [1]. Several simple models (at a semi-microscopic level) have been introduced for mechanical [2] and electrical [3] failure of such randomly disordered media, modelled by randomly disordered lattices with individual bonds breaking irreversibly. These theoretical results about the fracture or breakdown strength distribution etc. have also been checked in several experiments [4]. Failure processes playing intrinsic roles in many systems of industrial importance and in many natural catastrophic (disaster) phenomena, the above mentioned statistical studies establishing the nature of (non-self averaging) fracture-strength distribution, its (critical) fluctuations, consequent power laws for the average strength of such solids, etc. [1], are of extreme importance. Although a significant amount of literature has been developed for these studies of failure strength distribution, not much studies have been made regarding the dynamics of microscopic failures. Since the microscopic failures are irreversible and therefore require intermediate redistributions of the forces (potentials), the equations for the dynamics of failure are intrinsically nonlinear and dissipative. The formulation and studies of such equations are necessary for the search of any precursor effect of the macroscopic failure (if the macroscopic failure is at most a critical and not a fully chaotic phenomenon). In some recent experiments [5]

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on the dynamics of the cracks in thin glass plates, with thermal stresses, the dynamics seems
to undergo a sequence of (numerous but) reproducible instabilities (not sensitive to every
detail of the fluctuations in initial conditions). The dynamics of fracture is thus observed to be
mostly critical, on the verge of chaos (depending on the velocity of the crack tip), but not
quite chaotic [6]. Similar is the case for the dynamical (Burridge-Knopoff [7] type) models of
earthquake [8], where also one gets the Gutenberg-Richter type power law for the magnitude
(strength) variation of the density of quakes (failure distribution being critical).

There has been indications recently [9] that some linear properties (e.g., the ratio of shear
modulus and bulk modulus) show abnormal (but universal) properties near the breakdown
(macroscopic failure) point of disordered solids. The detailed study of such (linear) response
(before breakdown) would thus provide prior indication of the catastrophe. However, such
indications are found to be very selective and it has not been possible to study in cases of
important catastrophic failures like the earthquake etc. The study of the response of such
dynamical systems to short-duration pulses indicates that another kind of linear response,
namely the pulse-susceptibility [10], has indeed universal behaviour near the failure point.
It locates very accurately the magnetic transition point [10], and the self-organized critical
points [11] in various models [12]. We present here the results of our numerical study of pulse
response in the uniform Burridge-Knopoff model (Carlson-Langer [8]), indicating the possibility
of accurate prediction of “earthquakes” in the model.

It was seen [10], both in a Monte Carlo study and in the numerical solutions of the mean field
equation of motion of an Ising magnet, that if one applies an external magnetic field pulse of
height $h_p$ and width $\delta t$ on such a system in equilibrium, and if the response magnetization pulse
is characterized by its height $m_p$ and (half) width $\Delta t (\geq \delta t)$, then both the pulse susceptibility
$\chi_p = m_p/h_p$ and the width ratio $R_p = \Delta t/\delta t$ incorporates (senses) the equilibrium (critical)
fluctuations of the unperturbed system. Both $\chi_p$ and $R_p$ have sharp divergence (peak) at
the order-disorder transition point of the unperturbed ($h_p = 0$) system. The precision in the
location of the transition point can be made accurate with somewhat larger pulse amplitude
$h_p$ and extremely small width $\delta t$ (it may be noted that the pulse susceptibility $\chi_p$ reduces to
the ordinary susceptibility as $h_p \to 0$ and $\delta t \to \infty$). Similary, we found [12] in a computer
simulation study that a pulsed addition of “particles” at a central site for a short time $\delta t$
in the BTW model [11], after its avalanche dynamics has stopped, causes a successive series
of local avalanches persisting for a response time $\Delta t$ (after which the dynamics stops again).
The width ratio $R_p = \Delta t/\delta t$ was found to diverge with a power law as the average “slope”
approaches its self-organized critical value. In fact, one can predict quite accurately this critical
value of the slope by determining the extrapolated average slope value where the inverse width
ratio vanishes (with some appropriate power it gives linear fit) [12]. These successes of such
dynamic (pulse response) measurements to locate the (self-organized) critical points prompted
us to investigate similar response behaviour in the earthquake models [7,8].

We have studied the stick-slip model of earthquake [7]. In this model, a linear array of $N$
blocks, each coupled to its nearest neighbours by elastic springs and each connected to a rigid
support (at the top) by elastic springs, is put on a uniformly moving rough platform. The
equations of motion (after linear scale transformation) for any of the blocks, in the uniform
spring constant Burridge-Knopoff model (Carlson-Langer [8]), can be written as:

$$
d^2U_j/d\tau^2 = l^2(U_{j+1} - 2U_j + U_{j-1}) - U_j - \phi(2\alpha v + 2\alpha(dU_j/d\tau)),
$$

(1)

where, $U_j$ denotes the displacement of the $j$-th block from its equilibrium position, $l^2$ is the
ratio of the spring constants, $\alpha$ is a constant (linear transformation factor), and $v$ is the uniform
pulling velocity of the rough (frictional) platform. The nonlinear friction force is given by the
function $\phi(y) = \text{sign}(y)/(1 + |y|)$. 

Fig. 1. — (a) The time variation of the total elastic energy $E_T$ of the entire system. (b) shows the time variation of the elastic stress (force) $\Delta U_k$ developed on the block on which pulse has been applied ($k = 50$ here). (c) Time variation of inverse of the logarithm of the pulse susceptibility ($\chi_p$). The linearly extrapolated saturation ($\sim 0.25$) point (time) of this quantity gives the possible location (time) of the imminent earthquake (compare with the major energy release points in (a)).
In such dynamical models, the elastic stresses (energy) developed compete with the (velocity dependent) frictional force as long as the blocks stick to the moving platform. As it fails, there occurs local failures (slippage for a finite number of blocks) or global failures (simultaneous slippage of almost all the blocks). Initially the (elastic) energy increases and then suddenly falls (causing an avalanche or quake) releasing the excess elastic energy. As the released energy is often more than the “excess”, the system again becomes “sub-critical”. The energy starts

Fig. 2. — A close view of the pulse susceptibility variation near some major events or earthquakes (enlarged from a portion of Fig. 1). The time variation of the total elastic energy is also compared.
to build up again and after some time it falls; and the process continues. The strain energy released in any such failure is identified as the magnitude (strength) of the earthquake in this model system. We have solved the above set of equations using fourth-order Runge-Kutta method taking 100 blocks \((N = 100)\). We have checked the distributions etc. of earthquakes (the Guttenberge-Richter law) for the model and reproduced the previous results \([8]\) for various initial randomness of the block velocities and positions. We then apply very weak pulses to an arbitrarily chosen (central; \(k\)-th) block (giving fixed arbitrary \(\delta \bar{U}_k \sim 0.01\) for a duration \(\delta t \sim 0.01; k = 50\) here) at a regular time interval of \(T = 1000\). We then calculate again the total elastic energy \(E_T(= (\sum_j l^2(U_{j+1} - 2U_j + U_{j-1})^2 + U_j^2)/2)\) at each time step and plot them against time (Fig. 1a). In all the figures we show the typical results for \(N = 100, l^2 = 10, \alpha = 2.5, v = 0.01\), although the general features we discuss are observed for any combinations of these parameter values. We also show in Figure 1, the time variation of the elastic stress \(\Delta \bar{U}_k = l^2(U_{k+1} - 2U_k + U_{k-1} - \bar{U}_k)\) developed on the \(k\)-th block, on which the pulses are being applied (Fig. 1b). We also show there (in Fig. 1c), the time variation of the pulse susceptibility \(\chi_p\), defined as \(\chi_p = \Delta \bar{U}_m^p/\bar{U}_m^p\) with \(\Delta \bar{U}_m^p\) denoting the maximum of \(\Delta \bar{U}_k\) within the pulse period and \(\bar{U}_m^p = \int_0^{\delta t} \delta \bar{U}_k dt\). We find \(\chi_p \sim \exp[A(t_{c_n})/(t_{c_n} - t)]\) for \(t < t_{c_n}\), where \(A(t_{c_n})\) is a constant and \(t_{c_n}\) denote the onset time of the \(n\)-th earthquake or catastrophe. It may be noted (from Figs. 1c and 2) that \(\log(\chi_p)^{-1}\) finally saturates to a value around 0.25 before the earthquakes. Indeed the linear extrapolation (with time) to this saturation value of \(\log(\chi_p)^{-1}\) gives quite accurate prior indication about the location (time) of the imminent earthquake (compare with the large elastic energy releases for the entire system, shown in

\[\text{Fig. 3. — Earthquake energy distribution, giving the variation of the number } N(E) \text{ of quakes releasing elastic energy } E: (\diamond) \text{ For unperturbed system. (+) For the same system perturbed with the application of pulses.}\]
Figs. 1 and 2). This is quite similar to the observation of Sahimi and Arbabi [9] regarding the convergence of the elastic constant ratio to a universal value before fracture. However, the saturation value (for \( \log x_p^{-1} \)) in this case is found to be nonuniversal and depends on the parameters of the dynamics (\( I, \alpha \) and \( v \); the saturation value decreases as \( v \) increases). Still, the saturation value being fixed for a particular series of earthquakes, the above procedure for the prediction of the earthquakes should be useful. In Figure 3, we just show the distribution \( N(E) \) of earthquakes releasing energy \( E \). The distribution indeed follows the Gutenberg-Richter type power law decay (\( N(E) \sim E^{-\beta}, \beta \sim 2.0 \) [7]), both in the unperturbed system (without pulse) and the system perturbed by weak pulses (on the \( k \)-th block) at regular intervals. It may be noted however, although the nature of the distribution remains unchanged, because of the sensitivity of the initial conditions, the exact magnitude and the timings of the quakes are somewhat different in the two cases.

In summary, we find that by applying very weak (mechanical) pulses on any particular block, and by studying the local response (studying the pulse susceptibility \( x_p \) at the same block), one can locate very accurately almost all (more than 75 per cent) of the major (global) earthquakes (for the entire system) in the Burridge-Knopoff type model.

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