Effect of coupling to the leads on the conductance fluctuations in one-dimensional disordered, mesoscopic systems
Asok Sen, S. Gangopadhyay

To cite this version:
Asok Sen, S. Gangopadhyay. Effect of coupling to the leads on the conductance fluctuations in one-dimensional disordered, mesoscopic systems. Journal de Physique I, EDP Sciences, 1994, 4 (10), pp.1373-1378. <10.1051/jp1:1994193>. <jpa-00246998>

HAL Id: jpa-00246998
https://hal.archives-ouvertes.fr/jpa-00246998
Submitted on 1 Jan 1994

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Short Communication

Effect of coupling to the leads on the conductance fluctuations in one-dimensional disordered, mesoscopic systems

Asok K. Sen and S. Gangopadhyay(*)

Low Temperature Physics Section, Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Calcutta 700 064, India

(Received 22 November 1993, revised 21 June 1994, accepted 28 July 1994)

Abstract. — We study the electronic transport in a one-dimensional disordered chain with both site-diagonal and off-diagonal (hopping) disorder, the latter being perfectly correlated to the diagonal disorder at the nearest neighbor sites in a linear fashion. Since we allow the hopping in the perfect leads to be different from the average hopping in the sample, the average two-probe conductance \( \langle g \rangle \) and the conductivity \( \sigma \) decay non-monotonically with length. Because the peak regions of conductivity represent nearly constant \( \sigma \)'s, these domains are quasi-Ohmic with their "effective" mean free paths equal to the stationary values of \( \sigma \) in these domains. Indeed, when \( \langle g \rangle \) passes through these quasi-Ohmic regions, the standard deviation of \( g \) latches on to almost constant values of about \( 0.3e^2/h \), appropriate for 1D (as observed by us in a recent paper). The evolution of the probability distribution \( P(g) \) with length demonstrates that it is unusually broad (nearly uniform) around the first quasi-diffusive regime and that for larger lengths, \( P(g) \) narrows down in general, but becomes non-monotonically broader whenever \( \sigma \) peaks up again, i.e., around the other (than the first) quasi-diffusive regimes.

Conductance fluctuations in disordered, mesoscopic, quantum systems continues to be a topic of great interest. In a recent paper [1] on disordered one-dimensional systems, hereafter referred to as I, we have reported the existence of a quasi-Ohmic regime in a range between \( 2.5\ell - 4\ell \) \( (\ell \) being the elastic mean free path of the electrons) and a concomitant "universal" conductance fluctuation (UCF) of the two-probe conductance in the regime \( 2.5\ell - 6\ell \) in 1D disordered quantum systems in the absence of any inelastic scattering, i.e., at zero or a small but finite temperature such that the inelastic scattering length \( \ell_i \) is much larger than the localization length \( \xi \approx 5.5\ell \) (as obtained from small length scale behavior). The fluctuation is universal in the sense that its magnitude is about 0.3 (in units of \( e^2/h \)) irrespective of the average value within the specified lengths, disorder, Fermi energy, and finally the Hamiltonian involved. The last point was checked by using both a tight binding Hamiltonian with nearest neighbor hopping and a Schrodinger Hamiltonian.

(*) Permanent address: Binodim Girls' School, 2 Bank Plot, Dhakuria, Calcutta 700031, India.
One important assumption used in the tight binding model in I is that the hopping term in the disordered sample is identical to that in the semi-infinite perfect leads. In a recent paper, Iida et al. [2] introduce off-diagonal disorder as well in a quasi-one-dimensional wire (i.e., a system having many channels but with its transverse dimensions being much smaller than \( \ell \)). It appears that Iida et al. have allowed for a statistical correlation between the diagonal and the off-diagonal disorder within a slice of length \( \ell \). They have also accounted for a realistic coupling between the many channel sample at its interfaces with the perfect leads. Then they find using statistical scattering theory that the coupling to the leads strongly affects the mean and the standard deviation (SD) of the conductance for sample sizes smaller than a length of the order of several tens of the elastic mean free path \( (L_0 \sim 10\ell) \), but becomes unimportant for length scales sufficiently larger than that. Thus a realistic coupling to the leads gives rise to another energy scale in the problem (different from the Thouless energy) and its effect is felt only up to a length scale of \( L_0 \). The SD of the conductance increases monotonically and does not achieve its UCF value in quasi-1D for \( L < L_0 \), but does so for lengths much larger than that as long as it is within mesoscopic regime. So, at least in a short length scale the one-parameter scaling theory [3] seems to be violated. Inspired by this work, we have investigated the effect of a model of correlated disorder along with a concomitant "noisy" coupling to the leads in an exactly on-dimensional system and thus generalize our work in I. It may be noted here that Iida et al. consider the scattering events in different slices (of length \( \ell \)) to be independent. But it is well-known that phase-coherent backscatterings play a very important role in disordered systems. Thus, even though qualitatively alright for behavior at \( L \gg \ell \), the independence of the scatterings may become suspect for \( L \sim \ell \), and in our work using transfer matrices there is no need to make any such assumption. The results, as we see later, are qualitatively different from those of Iida et al. and indeed whether one can uniquely define the elastic mean free path, \( \ell \), become an important issue now.

We consider a tight binding Hamiltonian for the disordered chain with site energies \( \varepsilon_n \) obtained from a uniform distribution \([-W/2, W/2]\) and hopping terms \( V_{n,n+1} = V_0 + 0.5\delta(\varepsilon_n + \varepsilon_{n+1}) \). One notes that this is a very simple-minded model of an off-diagonal disorder which is perfectly correlated to the diagonal disorders at the nearest neighbor sites in a linear fashion. The sample is connected to semi-infinite perfect leads with zero site-energies and a hopping energy \( V_{\text{lead}} = 1 \) to set the energy scale. The underlying lattice constant for the chain is chosen equal to one to set the microscopic length scale. We apply the transfer matrix formalism as discussed in I to obtain the two-probe conductances (in units of \( e^2/h \)) for chains of various lengths keeping disorder and other parameters constant.

In figure 1 we have shown the "average" conductance, its standard deviation and the "average" conductivity \( \sigma \) as a function of length for chains with a site disorder \( W = 0.6 \), \( \langle V_{\text{sample}} \rangle = V_0 = 7.0 \) and \( \delta = 0.5 \). It may be mentioned at the outset that one obtains qualitatively similar graphs for any other \( V_0 \neq V_{\text{lead}} \) and \( \delta \). If \( V_0 \) is reduced keeping \( \delta \) fixed, then our results indicate that the oscillations in \( \sigma \) becomes less pronounced and a smaller number of peaks encountered (e.g., for \( V_0 = 5.0 \), we can find only two peaks as opposed to three in Fig. 1). On the other hand, if \( \delta \) is varied keeping \( V_0 \) constant, then the peak positions and magnitudes changes a little bit but their numbers remain the same. Particularly the graph for the case of \( \delta = 0 \) (i.e., no disorder in the hopping term), not shown here because of its repetitive nature, is quite similar to that for figure 1. The main point is that the disorder in hopping does not seem to control the physics which is here guided by the disorder in the site energy and the inequality of the hopping term in the lead from that in the sample. The average conductance \( \langle g \rangle \) at some specified length intervals of the chain are calculated as described in I (by first calculating the \( \langle \ln g \rangle \) for an ensemble of different realizations and then taking its antilog). Using this \( \langle g \rangle \), the \( \sigma \) and SD \( g \) are calculated at those specified
Fig. 1. — The “average” conductance, \( <g> \), SD \( g \), and the average conductivity \( \sigma \) (scaled down by a factor of 1000) for chains with a site diagonal disorder \( W = 0.6 \), and a perfectly correlated off-diagonal disorder such that \( <V_{\text{sample}}> = V_0 = 7.0 \) and \( \delta = 0.5 \) (see text). The incident, reflected and transmitted electrons at Fermi energy \( E = 0.01 \) are supported by perfectly ordered chains with site energy \( \epsilon = 0 \), and hopping \( V_{\text{lead}} = 1.0 \).

lengths. The number of realizations taken for the above averaging process is at least 5000 and is about twice the number of sites even for quite large chains. This means for example that for \( L = 6000 \), the “mean” is over at least 12000 configurations. By increasing the number of configurations significantly at some arbitrary fixed lengths, say from 10000 to 15000 (using sometimes up to 25000 configurations), we have ensured that the values of \( <g> \) and SD \( g \) do not change before the third or fourth significant digit. Electrons in the form of plane waves and at a Fermi energy of \( E = 0.01 \) (chosen close to the band center for this communication) enters one end of the sample and leaves out of the other in the form of plane waves with the same energy but reduced amplitude. The effect of changing the Fermi energy is similar to what has been described in I, in the sense that the localization length away from \( E = 0 \) becomes smaller. But the period of oscillation (in a pure sample) also becomes smaller. Hence for \( E \) farther away from the origin, one sees qualitatively very similar graphs except for a different number of discernible peaks within \( L = \xi \). Again we have not shown such graphs to avoid repetition.

The first thing to note in figure 1 is that \( <g> \) does not decay exponentially as in I, but rather decays non-monotonically with several peaks. As a consequence, the mean conductivity \( \sigma = <g> L \) also has several peaks. That this oscillatory nature of \( \sigma \) or \( <g> \), albeit its overall decaying behaviour over large lengths, is due to the increase in the average hopping integral inside the sample, may be seen as follows. For example, the change in \( <g> \) in going from its first trough to the second peak is in the second digit whereas error due to the statistics of a finite number of samples is beyond the second or third digit as discussed above. This rules out the possibility of statistical errors as the origin of these discernible peaks. Further, we looked
at a few specific samples of arbitrarily chosen configurations and find that their conductances follow similar, broad resonance-like structures around the lengths corresponding to the peaks of the “average” sample. Thus these peaks seem to be persisting, albeit with reduced amplitude, even in the average sample. Indeed, instead of looking at a few specific realizations, one may look at the full probability distribution of \(g\) in search of an explanation, and we have done that in the sequel.

Each of the local peaks in \(\sigma\) corresponds to a nearly stationary value within a finite length domain around the peak and thus, according to our arguments in I, corresponds to an almost Ohmic (or an almost diffusive) regime with its own “effective” mean free path equal to the nearly constant, dimensionless conductivity in that regime. In the present case, we see from figure 1 that there are three such quasi-Ohmic regions: (i) around \(L = 550\) and \(< g > = 0.28\) where the mean free path \(\ell_1 = 150\), (ii) around \(L = 3600\) and \(< g > = 0.14\) where the mean free path \(\ell_2 = 500\), and (iii) around \(L = 7600\) and \(< g > = 0.06\) where the mean free path \(\ell_3 = 480\). The almost constant values of the conductance fluctuations in these three regimes simply follow, as expected, from the almost diffusive nature of transport in these regimes. What is more interesting for the purpose of this communication is that the fluctuation in each case is quite close to the UCF value of 0.3 (except for the last case which is also not too far off) as obtained in I. Actually, the SD \(g\) is 0.30-0.31 for 250 < \(L\) < 650, 0.27-0.28 for 3000 < \(L\) < 4200 and 0.24-0.25 for 6200 < \(L\) < 7400. In passing, we note that \(< g >\) goes through a value of \(\exp(-2) \approx 0.14\) first at about \(L = 1000\) and a local minimum value of 0.089 at a length between 1700-1800. Applying our arguments in I naively, one would then like to think that \(\xi\) is about 1000. But that certainly cannot be the case as \(< g >\) rises again to the value of 0.14 at a length of about 3600. On the other hand, we have the Thouless formula [4] for \(\xi\) (obtained using second-order perturbation theory and the Herbert and Jones [5] result relating spectral properties to the localization length):

\[
\xi = 24 \left( 4V_{\text{sample}}^2 - E^2 \right) / W^2. \tag{1}
\]

It may be mentioned that exponential localization is implicit in the derivation of equation (1). When one puts \(E = 0.01\), \(W = 0.6\) and \(V_{\text{sample}} = 1\) (as in one of the cases in I), \(\xi \approx 270\) (large length scale property), which compares reasonably well with the numerical value of \(\xi = 290\) as extracted from small length scale behavior in I. It is not quite expected that this formula should hold that well when disorder in both \(\epsilon\) and \(V\) are present. But since the disorder in \(V_{\text{sample}}\) follows that in \(\epsilon\) deterministically (correlated disorder), we would still like to apply it here to get an estimate of \(\xi\). When we put \(V_{\text{sample}} = 7.0\), as in the present case, we find that \(\xi \approx 13000\). Now, if we look at our figure 1, we find that for \(L = 10000\), \(< g > = 0.042\) and for \(L = 13200\), \(< g > = 0.026\). If we assume that \(< g(L) > = < g(L_0) > e^{-2(L-L_0)/\xi}\), we find from our numerical results that \(\xi \approx 13300\). Similarly for \(L = 12400, < g > = 0.029\) and for \(L = 13600, < g > = 0.024\). This again gives a \(\xi \approx 12700\). These results demonstrate exponential localization for large \(L\) with the expected \(\xi\). It may also be mentioned here that, as noted in a previous paragraph, even for \(\delta = 0\) (no disorder in hopping), the peak structure in \(< g >\) and/or \(\sigma\) seem to be only a little bit rearranged (without any qualitative change at all) and the decay property of \(< g >\) for large lengths because of only site disorder is found to be very similar to that for \(\delta = 0.5\). This implies that the use of the Thouless formula to obtain \(\xi\) is probably reasonable. In short then this large \(\xi\) of 13000 lattice units for the case studied here indicates that, as expected, all the interesting quasi-Ohmic behaviour (along with quasi-UCF) occur within the mesoscopic regime.

To understand the probabilistic origin of this intriguing behavior, we have also looked at the full probability distribution (not just the first two moments) of the two-probe conductance
Coupling to the Leads and Conductance Fluctuations

1377

Preparing to couple TO THE LEADS AND CONDUCTANCE FLUCTUATIONS

The probability distribution $P(g)$ for the same parameters as in figure 1 and for four different lengths to show its non-monotonic evolution through four different regimes, (a) for $L = 400$, (b) for $L = 1200$, (c) for $L = 1800$, (d) for $L = 3600$.

at some specific lengths. It is usually believed that the distribution in the UCF region is broader compared to the regions outside of it. For the simpler case described in I, it was found that [6] the distribution $P(g)$ evolves continuously from a strongly peaked function near $g = 1$ (nearly ballistic regime) towards another strongly peaked function near $g = 0$ (strongly localized regime) with a nearly uniform distribution in the UCF (weakly localized) regime. Further the peak shifts towards $g = 0^+$ while increasing and narrowing down monotonically as localization effects grow stronger. That does not quite happen in the present case and we find that the distribution itself evolves non-monotonically in the regions between the peaks. From figure 2a we find that $P(g)$ for $L = 400$ is very broad (nearly uniform) indicating the existence of a quasi-Ohmic region as argued above. Next from figure 2b at $L = 1200$, we find that the distribution is becoming narrower and the peak height has increased from 2 to about 7.7. Next we pick an $L = 1800$ where the first trough of average $\sigma$ occurs, and we find that the peak in $P(g)$ has increased to a value of about 9.3 and has grown still narrower while the peak itself moves towards $g = 0$ progressively. But as we approach the second peak in $\sigma$, we find that $P(g)$ becomes less strongly peaked, even though nearer to $g = 0$ than before. Indeed, for $L = 3600$ (very close to the second maximum in $\sigma$) peak value has reduced to 5.5 and is indeed much broader (with a significant tail contribution) indicating the existence of another quasi-Ohmic region. Thus we do indeed find that the distribution becomes broader for length scales near the peaks of the average conductivity, and since there are more than one peak,
the distribution itself must evolve non-monotonically as the length scale is increased within the localization length. In this context, it is probably useful to remind the reader that all the above histograms have been normalized to an area value (below the curve) of unity.

Finally we comment on the independent scaling parameters in disordered, mesoscopic systems. Because of the non-uniqueness of the effective mean free path (at least three of them, as far as we can resolve in the present case) in the system, a single formula for the localization length $\xi$ in terms of $\ell$ (e.g. $\xi = 4\ell$ as obtained from asymptotic behaviour [7] or $\xi \approx 5.5\ell$ as obtained in the case of I from small length scale behavior) does not exist. Thus $\xi$ and $\ell$ are not dependent on each other in open mesoscopic systems with lead characteristics (i.e., hopping) different from that of the sample, and this fact points towards the existence of two independent length scales in such systems.

Acknowledgements.

AKS would like to acknowledge discussions with R. Mills, J. Wilkins and R. Smith at the Ohio State University while he was a visitor there. The warm hospitality at OSU where part of this work was done is also gratefully acknowledged. AKS also had some useful discussions with B. Altshuler.

References