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To cite this version:

P. Sutton, D. Hunter, N. Jan. The ground state energy of the ±J spin glass from the genetic algorithm. Journal de Physique I, EDP Sciences, 1994, 4 (9), pp.1281-1285. <10.1051/jp1:1994112>. <jpa-00246991>

HAL Id: jpa-00246991
https://hal.archives-ouvertes.fr/jpa-00246991
Submitted on 1 Jan 1994

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Classification
Physics Abstracts
02.50 — 05.50

Short Communication

The ground state energy of the $\pm J$ spin glass from the genetic algorithm

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(Received 27 June 1994, accepted 4 July 1994)

Abstract. — The ground state energy of the $\pm J$ 2-dimensional square and 3-dimensional cubic spin glasses are determined by the Genetic Algorithm. System sizes of linear dimensions 6 to 20 are considered in 2-dimensions and from 4 to 9 in 3-dimensions. We estimate that the ground state energy of the infinite system is $-1.400 \pm 0.005$ (2d) and $-1.765 \pm 0.01$ (3d) in units of $J$.

The Genetic Algorithm [1-5, 15, 16] is the term used to describe a computing strategy which mimics evolutionary principles in the search for an optimal solution. A population of programs, or as in our case, configurations of the spin glass, are selected with a certain probability for contributing off-springs to the subsequent generation. The selection process is weighted by the individual's 'fitness' as compared to the 'fitness' of the other members of the population. Mutation and 'sex' are included in the reproduction process and the new members' fitnesses are weighted in the selection for the subsequent generation. We will describe below the actual implementation used for the spin glass problem but we refer the reader to references [1-5] for details of the algorithm and its application to a vast array of distinct, novel and challenging problems.

The determination of the ground state energy of the 2-dimensional and to a lesser extent the 3-dimensional $\pm J$ spin glass has attracted attention over an extended period and has been a testing ground for several strategies [6-14]. It is generally believed that the ground state energy of the square lattice is $-1.403 \pm 0.005$ in units of $J$ [10] and $-1.76$ or $-1.80$ [14] for the 3-dimensional simple cubic lattice. We describe below our implementation of the Genetic Algorithm for this NP problem.

A random arrangement of ferromagnetic and antiferromagnetic bonds are placed on the lattice and a random population (300 to 3000) of spins configurations also created at the start. The energies of these configurations are measured for the given bond arrangement. The next generation of the population is constructed in the following manner: potential members are selected from the present generation with a probability proportional to their energy difference.
Ground State Energy of the 2-d Spin Glass

Fig. 1. — The ground state energy of the 2-dimensional $\pm J$ spin glass as a function of inverse system size, $1/L$. We extrapolate the ground state energy of the $\infty$ system to $-1.400 \pm 0.005$ in units of $J$.

with the maximum energy; selected pairs have a probability of $0.5$ of interchanging segments of spin states - this mimics the genetic mixing of material; ten percent of the spins are given random mutations - flipped without consideration to increase or decrease in the energy of the system. Next ALL mutations leading to a decrease of the energy are allowed [11, 12]; we visit every site of the lattice in a type-writer fashion and the spin is flipped if this leads to a decrease of the energy of the system otherwise the state of the spin is left unchanged. A mutation is simply a flip of the spin from its present state to the opposite state. We have now constructed the population of the first generation. This process is continued for a finite set of generations which we estimate is necessary to find the lowest energy state. We make a generous estimate of this number - about ten times the value indicated in figure 2. We store the lowest value of the energy observed during the evolution of the configurations and this is the estimate of the ground state energy for the configuration of bonds.

We checked that the algorithm is able to successfully accomplish its task: for the smaller system sizes ($L = 5$) we enumerated all the states of the 25 spins (2-dimensions) and checked the ground state energy for a few particular bond arrangements; for larger systems we repeated the search several times with the same bond configuration but starting with different initial random populations and a different sequence of random numbers. We always arrived at the same lowest energy in all our tests. We also increased the number of generations and compared the final average ground state energy with the previous estimate. There was always a measure of agreement within the expected statistical fluctuations; the results were less than a standard
Fig. 2. — The average number of generations required to find the ground state energy for a fixed population of 300 as a function of system size, \( L \) for the 2-dimensional spin glass.

deviation apart.

Figure 1 shows the data for the average ground state energy for system sizes varying from \( L = 4 \) to \( L = 20 \) in 2-dimensions plotted vs. \( 1/L \). We extrapolate the ground state energy of the infinite system to \(-1.400 \pm 0.005\) in units of \( J \). Figure 2 shows the average number of generations required to determine the ground state energy for a population of 300. The number of generations appear to increase exponentially as the system size for this fixed population. Another measure of the optimal property of this search method is to compare the total number of possible configurations with a rough estimate of low energy states. The number of configurations for \( L = 10 \) is \( 2^{100} \) and if we approximate that there are \( 10^3 \) low energy states then one would expect to generate of the order \( 10^{26} \) configurations in a random search for one of these low lying states. In fact the genetic algorithm finds the solution for \( L = 10 \) with an average of 3500 configurations. Figure 3 shows the ground state energy of the 3-dimensional system as a function of \( 1/L \). We estimate that the ground state energy of the infinite system is \(-1.765 \pm 0.01\) in agreement with some of the earlier results.

In summary, we have used the genetic algorithm to determine the ground state energy of the 2-dimensional and 3-dimensional \( \pm J \) spin glasses on the square and cubic lattices. The result agrees with earlier results reported in the literature [10, 14]. An advantage of our approach is that with little effort the method may be extended to higher dimensional systems. This luxury is not afforded by some of the other methods. We have not attempted to calculate the ground state energy of higher dimensional systems because of our rather limited computer
resources - three workstations took about six weeks to complete this project. There are certain features of our implementation that require further investigation: optimal criterion for selecting members for the subsequent generation (our condition depended linearly on the energy of the configuration) and whether it is more efficient to optimise random sections of a large system rather that attempting, as we did, to optimise the complete system. We expect that the Genetic Algorithm will play a more important role in the solution of spin glasses and related problems in Statistical Physics.

This research is supported in part by NSERC.

References


