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Absence of inflation symmetric commensurate states in inflation symmetric networks

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Abstract. — In this paper we present measurements of the normal-to superconducting phase boundary $T_c(H)$ for three different networks possessing inflation symmetry. Fluxoid quantization constraints induce the formation of a lattice of fluxoid quanta for any non-zero perpendicular magnetic field, and at particular fields, $T_c(H)$ exhibits cusp-like structure indicating that the lattice is commensurate with the underlying network geometry. For inflation symmetric networks studied in the past, all commensurate states have always been related to the inflation symmetry. In the three networks studied here, none of the commensurate phase boundary structure derives from the inflation symmetry. We propose that this structure can instead be understood by considering the Fourier transform of the network geometry and that the transform actually provides a more universal prescription for the identification of commensurate states. The relevance of the transform (as opposed to the inflation symmetry) in determining commensurate states in two dimensions is consistent with analytical work performed for one dimensional systems.

1. Introduction.

Fluxoid quantization in superconducting networks has proven to be a versatile and fruitful model system for studying commensurability effects in a substantial variety of network geometries. The behavior of this system is governed by the constraint that the magnetic fluxoid $(8\pi\lambda^2/c\Phi_0) \oint \mathbf{J} \cdot d\mathbf{l} + (1/\Phi_0) \oint \mathbf{A} \cdot d\mathbf{l}$ quantize to integer values for a path taken around any closed loop of superconducting material. The application of a magnetic field to a network made up of superconducting loops then leads to the formation of a lattice of fluxoid quanta. This quantization is accompanied by the generation of supercurrents around the loops which depresses the critical temperature $T_c$ of the networks below the zero field value. At specific values of the applied field, the fluxoid lattice registers with the underlying network geometry in energetically favorable configurations for which the induced supercurrents are reduced.

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These local energy minima translate to local maxima in the network $T_c$ and slight changes in applied field are accommodated by adding defects to these stable locked configurations of the quanta. Since the energy is proportional to the number of defects, linear cusps in the field-dependent normal-to-superconducting phase boundary $T_c(H)$ result, providing a signature for the existence of commensurate states and indicating the field values at which they occur.

Commensurate states have been studied in numerous wire array geometries including (but not limited to) periodic [1], quasiperiodic [2, 3], fractal [4], percolated [5, 6], and disordered [7-9] networks. One of the fundamental questions that arises from the results of these experiments is the following: what geometric property of a network dictates the nature of those states in which the magnetic fluxoid lattice is commensurate? In previous work, at least two possibilities have been considered. In studies of eight-fold [2, 9] and five-fold [3, 10] symmetrical quasiperiodic arrays, a type of self-similarity known as inflation symmetry has been central to an understanding of commensurate states in these geometries since it provides an excellent scheme for indexing the field values for which commensurate states are found. This symmetry implies the existence of an overlayer of supertiles, or inflated tiles, which covers the basic network, and the assumption of a systematic approach to filling these inflated tiles with fluxoid quanta is quite successful in explaining all the commensurability cusps found for these quasiperiodic networks. Commensurate states on periodic arrays such as the simple square net can be viewed similarly since commensurate states at rational fields $p/q$ are thought to be based on identical fillings of $q \times q$ supertiles with $pq$ quanta. (But see Ref. [11] in which $2q \times 2q$ supertiles are found to give lower energy states at some field values.)

However, the inflation symmetry is closely related to the structure of the network's diffraction spectrum (i.e., Fourier transform); in fact, the transform of the overlayer has a support (i.e., Bragg peak positions) identical to that of the basic network. This suggests another possible scheme for the determination of commensurate states: it may be that correspondences between the network transform and the fluxoid lattice transform are sufficient to determine the commensurate states [12, 9]. For instance, it has been proposed [12] that a fluxoid lattice configuration is commensurate with an underlying network if the support of its transform contains the support of the network's transform. Analytic results [13] have rigorously established that for one-dimensional (1D) networks, commensurate states are determined solely by the lattice's transform; specifically, each discrete wavevector $q$ in the 1D reciprocal lattice determines the fields $f = q/n$ (where $n$ is any integer) at which commensurate states will be found. However, the extension of such a prescription to two dimensions (2D) is still an unsolved problem.

In this paper we present measurements made on a number of inflation symmetric aperiodic networks for which the commensurate states do not exploit the inflation symmetry. Instead, these states can be related to a different symmetry of the lattice inherent in the lattice's Fourier transform. These results indicate that for 2D systems, as in 1D, it is the composition of a network's Fourier spectrum which determines at what fillings the states will be commensurate with the network geometry, and the presence of inflation symmetry is not directly relevant to commensurability. Furthermore, it appears that the strongest components in the transform determine those states which are most commensurate (e.g., as measured by the depth of their commensurability cusps on $T_c(H)$).

2. Experimental details.

The three experimental aperiodic geometries presented here can be found in Tilings and Patterns by Grunbaum and Shephard [14]. We refer to the first as the "L-similarity" tiling; an electron beam micrograph of a small portion of a typical sample is shown in figure 1a. It is constructed by repeated inflations of the basic L-shaped tile into four similarly shaped tiles as
Fig. 1. — Electron micrographs of inflation symmetric experimental Al networks with lattice constants of \(~2 \mu m\). Below, inflation schemes are illustrated. (a) L-similarity tiling. (b) Quasiperiodic Ammann Golden tiling. (c) Equal-area non-Golden tiling.

shown at the bottom of figure 1a. Each of the smallest tiles can be inflated to obtain the next level of construction, and so the \(n\)th inflation will contain \(4^n\) tiles. Our experimental samples consisted of a square portion of a ninth level inflation and contained roughly 87,400 basic tiles.

We refer to the second geometry, illustrated by the micrograph in figure 1b, as the Ammann Golden tiling. It contains two basic tiles whose area and number ratios are both given by the golden mean \(\tau \equiv (\sqrt{5} + 1)/2\). The lattice is inflation symmetric, and the inflation of the two tiles is shown at the bottom of figure 1b. The ratio of the inflated tile areas to those of the basic tiles is also given by \(\tau\). Additionally, it can be shown that the lattice possesses quasiperiodic long-range translational order by an appropriate decoration of it using Ammann bars; details can be found in reference [14]. It is interesting to note that the area, number, and inflation ratios are also given by \(\tau\) in the celebrated five-fold Penrose tiling [15, 16], so that one might expect similarities in the phase boundaries for the Ammann Golden and Penrose tilings. Our experimental patterns were generated using 12 levels of inflation and contained about 57,000 tiles.

The third tiling studied, shown in figure 1c, is derived from an approximately inflation symmetric variant of the Ammann Golden tiling which has two tiles with area ratio 3:2 instead of \(\tau\):1. (We call it “approximately inflation symmetric” since successive inflations rapidly converge to the geometry of the Golden tiling.) Those two “non-Golden” tiles are then decorated with two additional tiles as shown at the bottom of figure 1c; what is interesting about the two final tiles is that while they are different shapes, they have identical areas. This hasimportant consequences with respect to fluxoid quantization effects.

The actual samples were fabricated at the National Nanofabrication Facility using electron beam lithography with standard metallization and liftoff techniques. They consist of 500 Å thick, 2500 Å wide aluminum wires with lattice constants on the order of 2 \(\mu m\). Network \(T_e\)'s were about 1.25 K with transition widths (10%-90%) of 3-5 mK, and four-probe resistances measured in a van der Pauw geometry were typically a few ohms.

\(T_e(H)\) phase boundaries were measured by holding the network resistance at a fixed fraction of its normal value while sweeping the magnetic field \(H\) and measuring the temperature. Since the application of a magnetic field shifts the entire resistive transition to a different temperature, maintaining a fixed network resistance and monitoring temperature gives the transition
temperature $T_c$ as a function of the field. Locking the resistance lower in the transition tends to yield enhanced commensurability structure, so most measurements were made by locking in to a resistance of between 1% and 5% of the normal resistance. A parabolic background due to expulsion of flux from the bulk of the wires is subtracted to isolate the fluxoid quantization effects.

3. Experimental results.

3.1 L-similarity tiling. — The $T_c(H)$ phase boundary for the L-similarity tiling is shown in figure 2a. It has a periodicity of $H_0 = 7.0$ G since at this characteristic field exactly one fluxoid quantum is applied to each network tile without the presence of any supercurrents. There is also additional fine structure apparent at fractional values of the unit field $H_0$; these structures at 3/16, 3/8, 5/8, and 13/16 are labeled by the arrows in figure 2a. This structure is not related to a direct inflation of the lattice, but can instead be understood with the help of figure 2b. Let us assume a length scale such that each basic L-tile consists of three unit squares which form the L-shape. Then we can break up the network into square supertiles which consist of $4 \times 4 = 16$ unit squares and contain five of the basic L-tiles with an extra unit square in one of the corners. (This extra square belongs to an L-tile which is shared by three of the square supertiles.) Notice that this supertile, indicated by hatching in figure 2b, occurs with four different orientations in an aperiodic fashion. The 3/16 state then corresponds to putting a quantum (indicated by dots in Fig. 2b) in one of the L-tiles (area = 3) in each square supertile (area = 16). Most likely, it is the L-tile in the center of each supertile which is filled, although it is possible that this state could have up to five-fold degeneracy: instead of filling every center tile, the other possibly degenerate states would have a different tile filled (upper right, upper left, lower right, or lower left). In the same way, the 3/8 filling is a state with two of the L-tiles filled on each supertile, and so forth for the 5/8 and 13/16 states.

What is quite curious about this result is that a commensurate state with the structure of a periodic square lattice has formed on a network which is non-periodic! To try to see how this can be the case, it is helpful to consider the Fourier spectrum of this lattice. By using the sample geometry to diffract a He-Ne laser, we have found that the optical transform of the L-similarity tiling consists of the superposition of transforms for square lattices of different sizes; i.e., Bragg peaks are given by $|g(k)|^2$ where $g(k)$ has the form

$$g(k) = \sum_i \sum_{m_i, n_i} \delta \left( k_x - \frac{2\pi m_i}{a_i} \right) \delta \left( k_y - \frac{2\pi n_i}{a_i} \right) A_{m_i, n_i} .$$

In this expression, $a_i = 2^i a_0$, $a_0$ is the length of the smallest line segment in the real space lattice, and $A_{m_i, n_i}$ is the peak amplitude at $k_i = \frac{2\pi}{a_i}(m_i, n_i)$. We have verified this form by taking the numerical transform of a lattice made up of points placed at the inner vertices of all the tiles. This numerical transform shows that the peaks corresponding to a real space periodicity of $a_i$ have an intensity roughly proportional to $1/(4i)$, i.e., $A_{m_i, n_i} \propto 1/(4i)$. (Note that this network is aperiodic since its transform contains an infinite number of frequencies given by the $a_i$ for all integers $i$; but it is not quasiperiodic since these frequencies are not irrationally related).

It then seems that the commensurate states measured on this network (which are based on a superlattice with a square geometry) are related to a subset of the lattice's transform: specifically, one frequency component of the multiple frequency reciprocal lattice. This frequency is given by the $i = 1$ component. It is likely there are commensurate states corresponding to
frequency components given by \( i > 1 \), but we assume that because these components are substantially weaker in amplitude, their corresponding commensurate states were experimentally inaccessible in our measurements.

We should point out that these measured states can be related to a second-order inflation of the lattice where each super tile contains 16 of the basic tiles. From this point of view, a state at \( f = n/16 \) would result from the decoration of each 16-tile super tile with \( n \) flux quanta. (For instance, the decoration presented in Fig. 2b could be interpreted as one such case with \( n = 3 \).) However, we find this interpretation to be unsatisfactory. The beauty of the inflation scheme as applied in previous studies was its generality: the first inflation always yielded the strongest (non-trivial) commensurate states, and higher order inflations gave additional, but weaker states. However, in the present case, there are no commensurate states corresponding to the first inflation of the L-similarity tiling, and therefore, an explanation invoking the second-order inflation seems inappropriate.

3.2 Ammann Quasiperiodic Golden Tiling. — We now consider the results for the Ammann Golden tiling in figure 1b; a section of its measured \( T_c(H) \) phase boundary is presented in figure 3a. As is ubiquitous in fluxoid quantization experiments on networks, we expect to find commensurate states at fields which automatically apply a nearly integral amount of flux (in units of \( \Phi_0 \)) to each tile. For networks with two tiles, we can label these fields with the notation \( (N_l, N_s) \) where \( N_l(N_s) \) is the number of flux quanta in each large (small) tile. Then the most commensurate fields occur when \( N_l/N_s \) gives the best approximation to the tiles' area
Fig. 3. — (a) Experimental phase boundary $T_c(H)$ for the Ammann Golden tiling. The labels $(N_l, N_s)$ indicate states with $N_l$ ($N_s$) quanta in each large (small) tile. Some of these states can also be indexed by $\tau^n H_0$ where $H_0 \equiv (1, 1)$. There are additional fine structure states corresponding to fractional values of the $\tau^n H_0$ states. (b) Possible configuration for the commensurate state at $H_0/2$; filled tiles indicate placement of quanta.

ratio. We have previously referred to these as “basic lattice” states [9] since they are completely determined by the number and area ratios of the tiles in the basic lattice; the local ordering of the lattice plays no role in determining these states. If we define a unit field $H_0 \equiv (1, 1)$, then many of these basic lattice states can be indexed by $\tau^s H_0$; for instance, $(1, 0) = \tau^{-1} H_0$, $(1, 1) = \tau^0 H_0$, and $(2, 1) = \tau^1 H_0$. Note that the five-fold Penrose tiling has the same exact basic lattice states as this network because of the geometric similarities mentioned in section 2; see reference [3] for $T_c(H)$ measurements on the Penrose tiling.

Not surprisingly, there is also fine structure present on the Ammann Golden tiling phase boundary; however, none of it seems to be related to the inflation symmetry. Instead, these structures occur at fractional values of the $\tau^s H_0$ basic lattice states, i.e., at fields $p \tau^s H_0/q$, where $p$ and $q$ are integers. Some of these states have been labeled in figure 3a, such as $H_0/2$, $\tau H_0/2$, and $\tau^2 H_0/2$. This result is radically different from what has been found for other quasiperiodic networks [2, 3, 10, 9], where all the fine structure was related to the inflation symmetry and there were no fractional states.

Closer inspection of the Ammann Golden network makes the fractional states quite plausible. In figure 3b, we demonstrate a possible commensurate state in which there are alternating filled and empty tiles in the horizontal and vertical directions without any inconsistencies. This obviously creates a fractional state analogous to the 1/2-filled (checkerboard) state on a simple square lattice. Although on occasion sections of some links are shared by two filled (or empty) tiles, these occur relatively infrequently and it is clear that this configuration of fluxoid...
quanta will certainly reduce the currents generated in the network just as it does in the square lattice. It is also clear that we can create other fractional states in the same way; e.g., the phase boundary in figure 3b appears to have a commensurate state at $H_0/4$.

It is not obvious how to visualize the extension of this fractional filling procedure to the other states $p\tau^s H_0/q$ when $s \geq 1$. Nevertheless, this form is entirely consistent with that found for commensurate states in 1D in the analytical work of Griffiths and Floria [13]. They have shown that in 1D quasiperiodic lattices, commensurate states will be found at fields $f = (l + m\tau)/n$ where $l$, $m$, $n$ are integers and $\tau$ is the irrational number which characterizes the quasiperiodic sequence. (This follows directly from their general solution $f = q/n$ mentioned above since wavevectors $q$ of a 1D quasiperiodic sequence are of the form $q \propto (l + m\tau)$.)

One final issue regarding this network is the composition of its Fourier spectrum. Although this has not been calculated directly, the network's quasiperiodic structure (evidenced by its decoration using Ammann bars [14]) suggests that it should have a Fourier spectrum consisting of peaks at reciprocal lattice vectors $q \propto 2\pi(m/a + n/b)$ where $a$ and $b$ are incommensurate lengths in real space. The structure in the optical transforms for this lattice is consistent with this assumption.

3.3 Equal-area non-Golden tilings. — At this point we present results for our final aperiodic tiling which is based on a quasiperiodic non-Golden version of the Ammann Golden tiling just discussed. As explained in section 2, this non-Golden variant has approximate inflation symmetry since its successive inflations rapidly converge to the Golden geometry. One source of interest in this particular non-golden Ammann tiling is the fact that the tiles can be decorated in one last step by two different tiles which both have the same area; refer to the bottom of figure 2c. Like the phase boundary for the L-similarity tiling, the $T_c(H)$ phase boundary of this “equal-area non-golden” network will be periodic since all the network tiles have the same area. However, one might expect that the approximate inflation symmetry could influence the nature of higher order commensurate states.

The measured phase boundary for this network is illustrated in figure 4a, where commensurate states have been labeled by arrows. Just as in the Ammann Golden tiling, these commensurate states occur at fractional values of the unit periodic field (here, $H_0 = 4.2$ G), and there is no evidence that the approximate inflation symmetry of the network plays any role in the formation of commensurate states. Although higher order states are fairly weak, the cusp at 1/2 is quite deep and is comparable to the 1/2 state on a square lattice. The rectilinear nature of this network’s geometry allows us to employ the same scheme used for the Ammann Golden network to explain the presence of the fractional states; see figure 4b for a possible configuration of the 1/2 state.

4. Discussion.

Insofar as they support the relevance of the network spectrum in determining commensurate states, these experiments establish a greater consistency between results concerning commensurability in 1D and 2D. In 1D, Griffiths and Floria [13] have shown analytically that the field values at which commensurate states are found are directly related to wavevectors in the network transform and that the absence of inflation symmetry does not compromise the existence of commensurability. For a 1D quasiperiodic network in which $\tau$ characterizes the quasiperiodicity, they showed that commensurate states exist for all fields of the form $H \propto (l + m\tau)/n$ for integers $l$, $m$, and $n$. On the other hand, for eight- and five-fold quasiperiodic networks in 2D, the nature of commensurability appeared to be fundamentally different. All states could be indexed using a scheme based on inflation symmetry, and no “fractional” states (which
are given by $H = pr^s/q$ with $q > 1$ and are unrelated to the inflation symmetry) were ever found. However, the 2D quasiperiodic Ammann networks presented here do exhibit fractional states, and even though these networks are inflation symmetric, non-trivial inflation states (at $H = pr^s/q$ for $s < 0$) are absent. Note that the solution of Griffiths and Floria for 1D allows for both of these kinds of states since their form for commensurate fields $H \propto (l + m\tau)/n$ can be made equivalent to $pr^s/q$ for any value of $s$. (This follows from the fact that $\tau^s$ can always be written as $(l + m\tau)$ with appropriate choices of $l$ and $m$; the inverse is true if we are allowed to use multiple exponents $s$, i.e., $(l + m\tau)/n \rightarrow \sum p\tau^s/q$ where $p$ and $q$ are generally different for each value of $s$.)

There is additional evidence that the network spectrum will provide the most valuable information for identifying commensurate field values. The analytical establishment of this point of view in 1D has already been discussed. The commensurate states on the L-similarity tiling (for which the spectrum is most accessible of the tilings presented in this paper) help to substantiate the use of the strongest spectrum components in determining the existence of commensurate states; even though this tiling has a precise and simple inflation symmetry, the commensurate states found experimentally were not related to it. Furthermore, our results from previous studies [9] of phason-disordered quasiperiodic networks can also be interpreted from this perspective. The introduction of phason disorder, which amounts to local rearrangements of tiles, was found to destroy the inflation states which are present when a quasiperiodic network is ordered. Our calculations of the spectrum for a phason-disordered 1D quasiperiodic geometry showed that the loss of commensurate states was mirrored by a qualitatively similar decrease in
intensity of the Fourier peaks corresponding to these states. (The effect of the phason disorder is a shift of spectral weight from the discrete components to a diffuse background.) Although we did not extend our calculations to the 2D phason-disordered networks, we believe the decay of 2D inflation states with increasing phason disorder can also be related to the fading of specific Fourier components in the 2D spectrum.

One issue requiring further attention concerns exactly how the spectrum should be used to identify commensurate states. In 1D, the analytic solution demonstrates that a single Bragg peak at wavevector $q$ will result in commensurate states at fields $f = q/n$ for all integers $n$. It is not yet clear whether a “single peak” prescription of this sort is applicable in 2D; instead, it may be necessary to consider a greater amount of spectral information such as the degree of overlap between the supports of the transform of a potentially commensurate configuration and the underlying lattice. The precise overlap of the supports of the inflation and the underlying network in the eight- and five-fold quasiperiodic tilings suggests that the favored nature of the inflation states in these geometries can be understood just as convincingly by considering the network spectrum.

There is also no precise description of what role the peak intensities will play in 1D, but we suspect that peak intensities can be directly related to commensurability strength for the fields $f = q/n$ where $n = 1$ with an $n$-dependent decrease in strength for larger $n$. (We find that this type of relationship is at least qualitatively correct in 1D calculations.) Likewise, if a “support overlap” point of view is more appropriate in 2D, we believe that the degree of overlap is likely to determine the strength of commensurate states.

5. Conclusions.

Since commensurability effects are so prevalent in numerous physical systems, the nature of their origin is a compelling problem. For the particular case of fluxoid quantization experiments in which a superconducting network supports a lattice of magnetic fluxoid quanta, it is physically intuitive that the most uniform distribution of quanta will provide the most energetically favorable states. To find such a distribution, it is obvious that one should attempt to exploit long-range order inherent in the network. The simple translational long-range order of periodic networks makes this process simple; for aperiodic geometries, determining these states is not so straightforward. The use of inflation symmetry in explaining the nature of commensurate states in quasiperiodic geometries proved to be helpful in past studies of eight- and five-fold geometries, but in this paper we have established that there also exist inflation symmetric networks in which the inflation symmetry is of no use in explaining the resulting commensurate states. Therefore, we need a different unifying criterion with which to explain commensurability effects.

It has already been shown that for 1D systems, the spectrum allows a simple determination of those fillings for which the states are commensurate. Above, we have argued that a similar approach provides a consistent framework for our various results from 2D experimental systems and that inflation symmetry is relevant to the existence of commensurate states only to the extent that it is manifest in the network transform.

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