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Metal-insulator transition in two-dimensional Ando model

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Abstract. — The transport properties of the Ando model on square lattice are studied numerically for box distribution of the diagonal disorder. The statistical properties of the Lyapunov exponents and of the conductance are discussed. It is shown that the statistics of the Lyapunov exponents agrees with the random matrix theory not only in the metallic regime, but even at the critical point. That confirms the applicability of the random matrix theory to the description of the metal-insulator transition. The limiting distribution of the conductance at the critical point of the metal-insulator transition is presented and its form is discussed on the base of the statistics of the Lyapunov exponents. Fluctuations of the conductance in the metallic and of the logarithm of the conductance in the insulating regimes indicate that the theory of the metal-insulator transition is one-parametric.

1. Introduction.

Recently, Evangelou and Ziman [1] and Ando [2] described the two-dimensional tight-binding models which, thanks to the spin-orbit coupling, exhibit the disorder induced metal-insulator transition (MIT). Besides their relevance to the real physical systems, these models are interesting for the theoretical studies of the MIT [3-6]: As representative of the symplectic symmetry class in terminology of the random-matrix theory (RMT) [7, 8] they offer new possibilities to test the applicability of the RMT to the transport problems. The lower dimension makes these models also attractive for numerical studies of the transport properties of the disordered systems close to the critical point of the MIT because much larger samples than that in three-dimensional models could be considered here.

The last feature motivated us to the present work. To understand the MIT, we have recently analyzed the statistics of the conductance of the three-dimensional Anderson model (3DAM) [9]. This model has been studied intensively on the base of the finite-size-scaling hypothesis [10-13]. To complete these studies we wanted to find the probability distribution $P(g)$ of the conductance $g$ at the critical point as well as in the metallic and the localized regimes. Our studies have been inspired by Shapiro [14] who proposed to generalize the scaling theory of
the MIT [15] and to formulate it in terms of the whole conductance distribution $P(g)$. Such formulation reflects the large fluctuations of the conductance, measured on mesoscopic samples in both metallic and insulating regimes [16] and found theoretically [17, 18]. Although we have succeeded to obtain the limiting (system-size independent) form of $P(g)$ at the critical point, we would like to repeat the numerical simulations also for the Ando model, where the lower dimension provides the possibility to analyze the systems up to $120 \times 120$. The second, not less important, motivation of the present work was the study of the statistics of all the Lyapunov exponents (LE) of the transfer matrix and the analysis of the possible differences between the orthogonal and symplectic ensembles. The knowledge of the statistical behaviour of LE could namely approve or disapprove the applicability of the random matrix theory to the description of the transport properties of disordered solids outside of the metallic region - close to the critical point of MIT [20] or even in the insulating regime [21].

In section 2 we define the Ando model and describe briefly the method of calculation of the conductance. Section 3 presents the function $\gamma_L(x)$, which characterizes the spectrum of Lyapunov exponents [22]. The statistics of the Lyapunov exponents of the transfer matrix and of the differences $\Delta$ of two successive LE is also presented. We show that $\Delta$'s are distributed according to the Wigner Surmise (WS) even at the critical point. Special attention is driven to the correlations between successive LE and their differences. The data are in very good agreement with RMT. In section 4, the distribution of the conductance in the metallic regime and at the critical point is presented. To determine the number of parameters, which determine the distribution $P(g)$, we tested the universality of the conductance fluctuation in the metallic regime and the relation between the mean and the variance of the logarithm of conductance in the localized regime.

2. Method and models.

The Hamiltonian of the Ando model has the form

$$\mathcal{H} = \sum_{n,s} \varepsilon_n \left| n \sigma > n \sigma \right| + \sum_{n,n'} V_{nn'} \left| n > n' \right|$$

(1)

where $n = (x,y)$, $x,y = 1,2,\ldots,L$ defines the sites on the square lattice. The energies $\varepsilon_n$ are considered to be randomly distributed. We will consider in this paper the box distribution

$$P(\varepsilon) = W^{-1} \text{ if } |\varepsilon| \leq W/2, \quad P(\varepsilon) = 0 \text{ otherwise.}$$

(2)

The $2 \times 2$ matrices $V_{nn'}$ represent the hopping between the nearest neighbour sites:

$$V_{xy;x+1y} = \begin{pmatrix} V_1 & -V_2 \\ V_2 & V_1 \end{pmatrix} = A_+, \quad \text{and} \quad V_{xy;xy+1} = \begin{pmatrix} V_1 & -iV_2 \\ -iV_2 & V_1 \end{pmatrix} = B_+.$$  

(3)

Evidently,

$$V_{xy;x-1y} = A_- = A^\dagger_+, \quad V_{xy;xy-1} = B_- = B^\dagger_+.$$  

(4)

In what follows we define the energy scale by relation $V_1^2 + V_2^2 = 1$ and denote the spin-orbit coupling by the parameter $S = V_1$.

The $4L \times 4L$ transfer matrix in the site representation reads

$$M = \begin{pmatrix} A_- & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} E - \mathcal{H} & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & A_- \end{pmatrix}.$$  

(5)

where $E$ is the energy.
The method of calculation of the conductance is based on the "classical" relation for the conductance \( g \) (in units \( e^2/h \)) [23]

\[
g = \sum_{i=1}^{N} \frac{4}{1 + \cosh z_i}
\]

(6)

where \( N = L \) is the number of channels, \( z_i = 2\gamma_i L \) are the "extensive" LE, which determine the eigenvalues \( e^{\varepsilon_i} \) of the matrix

\[
A = T^\dagger T.
\]

(7)

In (7), \( T \) is the transfer matrix in the channel representation [23, 8]: \( T = Q^{-1}M^Q \), where the (nonunitary) matrix \( Q \), \( (Q^{-1}) \) consists of right (left) eigenvectors of \( M \) in the absence of the disorder, respectively.

For the periodical boundary conditions in the \( x \)-direction, the right eigenvectors \( \Psi_R \) could be found from Ansatz

\[
\Psi_R^q(x) = \begin{pmatrix} a_R e^{ikx} \\ b_R e^{ikx} \end{pmatrix}
\]

(8)

\( k = 2\pi n_k/L_x, n_k = 0, 1, \ldots, L_x - 1 \). The equation \( M\Psi_R^q(x) = \Lambda \Psi_R^q(x) \), where \( \Lambda = e^{iq} \) is the corresponding eigenvalue, reduces then to relations for two-components spinors \( a_R, b_R \)

\[
A_- \{ E - (B_+ e^{ik} + B_- e^{-ik}) \} a_R - A_+^2 b_R = \Lambda a_R
\]

(9)

\[
a_R = \Lambda b_R.
\]

(10)

Their combination gives

\[
[A_+ e^{iq} + A_- e^{-iq} + B_+ e^{ik} + B_- e^{-ik} - E] a_R = 0
\]

(11)

which has a nontrivial solution \( a_R \neq 0 \) only when

\[
\cos q = \frac{1}{2} \left\{ (E - 2V_1 \cos k) \pm \sqrt{4(1 + \sin^2 k) - (E - 2V_1 \cos k)^2} \right\}.
\]

(12)

There are four different values of \( q \) for each \( k \); two of them represent the waves propagating to the left, the other two the waves to the right. This degeneracy is absent only for \( k = 0 \) and \( k = \pi \) (the last case appears only for \( L \) even), where \( k = -k \) (mod 2\pi). Nevertheless, for nonzero disorder, the Lyapunov exponents of the matrix \( A \) appears always in pairs, in agreement with the theoretical predictions [3]. We consider therefore only \( N = L \) different LE, each of them characterizes one channel. The contribution of each channel to the conductance is due to the two possible spin orientations, less than or equal to 2.

The corresponding (right and left) eigenvectors are easy to obtain. The only complication consists in the necessity to orthogonalize the eigenvectors appropriate to the same eigenvalue.

After transformation of the transfer matrix (5), we diagonalize matrix \( A \) and use the relation (6) to calculate the conductance. The detailed description of the method of the numerical calculation of eigenvalues of \( A \) are given elsewhere [24].

Due to the randomness of the energies \( \varepsilon_i \), all the eigenvalues of \( A \) and, consequently, the conductance \( g \) are statistical quantities. To find the probability distribution of conductance and of Lyapunov exponents, we construct the statistical ensemble consisting of \( N_{\text{stat}} \) samples, which differ only in the realization of the disorder. In contrast to the Anderson model, \( N_{\text{stat}} \) could be rather large (\( \sim 10^3 \)), which makes the statistics better and the result more accurate.
Thanks to the detailed numerical work of Fastenrath and colleagues [5], many critical points in the three-dimensional space of the external parameters $E, W, S$ are known for the Ando model. In this paper we concentrate only on one of them,

$$E = 0.1, \quad \text{box distribution} \quad W = 5.75 \quad S = 0.5$$

The analysis of the conductance distribution at other critical points and for other forms of distribution of the disorder is given elsewhere [25].

3. Lyapunov exponents.

Pichard and Andre [22] characterized the spectrum of transfer matrix by the function $\gamma_L(x)$, defined for $0 \leq x \leq 1$ as

$$\gamma_L(x) = \frac{1}{2L} \langle z_i \rangle, \quad x = i/2L.$$  \hspace{1cm} (14)

[22] (Fig.1). In the metallic regime, $\gamma_L(x)$ is linear:

$$\gamma_L(x) = \xi x + \nu_L$$  \hspace{1cm} (15)

(see Fig. 1a for $W = 2$). It is what one assumes to obtain on the basis of RMT [8]. Both $\xi_L$ and $\nu_L$ seem to converge to the limiting, $L$-independent values for large $L$. For $\xi$, we demonstrate this convergence in the inset of figure 1a. It would mean that not only $\gamma_L$, but also the number of open channels [26] and so the mean conductance $\langle g \rangle$ converge to constants in the limit $L \to \infty$. This contradicts the log $L$-dependence of $\langle g \rangle$, predicted by field theory [27]. To solve this paradox, one can assume that $\xi \sim \log^{-1} L$ for $L \to \infty$. However, no log $L$-corrections have been found in $\xi_L$ for $L < 120$.

At the critical point, the smallest LE are size-independent; the limiting function $\gamma(x)$ exists. As is shown in figure 1b, it is close to linear for small values of $x$.

The random matrix theory predicts [8] that the normalized differences

$$\delta_i = \Delta_i / \langle \Delta_i \rangle, \quad \Delta_{i+1} = z_{i+1} - z_i$$  \hspace{1cm} (16)

of the Lyapunov exponents should be distributed according to the Wigner surmises

$$W_4(x) = \frac{2^{18}}{3^6 \pi^3} x^4 \exp \left\{ -\frac{64}{9 \pi} x^2 \right\}.$$  \hspace{1cm} (17)

(index 4 identifies the parameter $\beta$ in the classification of the random-matrix theory [19]). The distributions of $\delta_2$ for the metallic regime and for critical point (13) are present in figures 1a, b. The agreement with (17) is very good. To characterize the distribution of the higher $\Delta$'s, we present in the inset of figure 2 the $i$-dependence of the variable $y(\delta_i)$, defined as

$$y(x) = \frac{\sqrt{\text{var}(x)}}{\langle x \rangle}, \quad \text{var}(x) = \langle x^2 \rangle - \langle x \rangle^2.$$  \hspace{1cm} (18)

The distribution of $x$ has the form of (17) if $y(x) = y_{WS4} = \sqrt{45 \pi / 128 - 1} \approx 0.3232$. Figure 2 shows that $y(\delta_i) \approx y_{WS4}$ for $i \leq L/2$ and so $\delta$'s are distributed according to the distribution (17) in both the metallic and the critical states.

In this picture, the first LE $z_1$ plays a special role. $P(z_1)$ differs considerably from Wigner surmises (17). In the metallic regime $P(z_1)$ is rather similar to (but not identical with) Wigner surmise

$$W_1(x) = \frac{\pi}{2} x \exp \left\{ -\frac{\pi}{4} x^2 \right\}$$  \hspace{1cm} (19)
Fig. 1. — Function $\gamma(x)$ vs. $x$ close to the origin. a) Metallic regime, $W = 2$: ($L = 20$ (□), $L = 40$ (Δ), $L = 80$ (Δ), $L = 12$ (○). Solid lines are fits (15). Inset: the convergence of the the slope $\xi_L$ vs. $1/L$: solid line is linear fit $\xi_L = 0.185 + 1.27/L$. The second inset: the whole function $\gamma(x)$. b) Critical point, $W = 5.75$. Symbols are the same as in figure 2b. The linear fit through the data for $L = 80$ (A) reads $\gamma(x) = 1.126x - 0.000359$. Inset: the whole function $\gamma(x)$. The second inset: $i$-dependence of the variations of the Lyapunov exponents $\text{var}(\langle x_i \rangle) = \langle x_i^2 \rangle - \langle x_i \rangle^2$ at the critical point.
Fig. 2. — a) Distribution of the normalized difference $\delta_2$ for $W = 2, E = 0.1, L = 20(\square), L = 30(\triangle), L = 40(\Diamond)$. b) Distribution of the $\Delta_2$ at the critical point for $L = 10(\circ), L = 20(\square), L = 30(\triangle), L = 40(\Diamond), L = 50$ (stars), $L = 60$ (crosses). Solid lines are Wigner surmises (17,19). Insets: the variations of the normalized differences $\delta$. Note that $y \approx y_{WS4} = 0.3232$ for $x \leq 0.5$. The large values ($\sim 0.56$) close to the origin correspond to the first Lyapunov exponent.
Fig. 3. — a) Distribution of the normalized first LE in metallic regime. b) Distribution of the \( z_1 \) at the critical point. The solid lines are Wigner surmises (17, 19). The meaning of the symbols is the same as in figure 2. Inset: the same distribution in the logarithmic scale.
Fig. 4. — Correlation coefficients for Lyapunov exponents (solid curves) and for the Δ's in the metallic regime $W = 2, L = 30(×)$, at the critical point $W = 5.75, L = 40(△), L = 80(□)$, and in the localized regime $W = 17, L = 30(△)$.

(19) (Fig. 3a); $y(z_1) \approx 0.56$ ($y_{WS1} = \sqrt{4/\pi - 1} = 0.5227$). It could be explained on the basis of the random-matrix theory: the repulsion of the first LE from its "negative image" is namely $\beta$-times weaker than that between two successive LE [8]. It is also the reason why $\langle z_1 \rangle$ does not lie on a linear curve (15). The proper analysis shows [28] that $\langle z_2 \rangle / \langle z_1 \rangle \approx 4.05$ instead of 2.

At the critical point, $y(z_1) \approx 0.56 - 0.58 > y_{WS1}$. The difference between $P(z_1)$ and Wigner surmise (19) is visible in figure 3b; $P(z_1)$ decreases evidently more slowly than (19) for large values of $z_1$. As disorder grows, $y(z_1)$ decreases, in accordance with the Gaussian shape of $P(z_1)$ in the insulating regime.

In the insulating regime ($W = 15, L = 20$) we have found all $y(Δ)$ close to 1.0, which corresponds to the Poisson distribution of $Δ$'s [24].

We also calculated the correlations of the two successive LE and differences $Δ_t, Δ_{t+1}$. The correlation coefficients $C(z) = C(z_1, z_{i+1})$ and $C(Δ) = C(Δ_t, Δ_{t+1})$, where

$$C(x, y) = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\sqrt{\text{var}(x) \text{var}(y)}}$$

(20)

in all three regimes are plotted in figure 4. $C_z \approx 0.8$ indicates that the correlation of Lyapunov exponents are weak. The correlations between Δ's are even weaker: independently of the regime the system is in, the correlation $C_Δ$ lies in the interval $-0.4 \leq C_Δ \leq -0.2$ (Fig. 4). It is in very good agreement with the predictions of RMT [7], which predicts $C_Δ = -0.34$ for symplectic ensemble.

From the "practical" point of view, one can consider all Δ's to be almost statistically independent; the only correlation consists in the tendency of the spectrum to rigidity: if any Δ is much smaller than its mean value, the neighboring ones tend to exceed their mean value. This corresponds to the negative sign of $C_Δ$. 

\[ C(x, y) = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\sqrt{\text{var}(x) \text{var}(y)}} \]
Fig. 5. — The distribution of the 2nd LE \( z_2 \) and the 8th LE \( z_8 \) at the critical point and their comparison with the Gaussian distributions with the same mean and variance.

A similar effect (almost uncorrelated \( \Delta \)'s) has been found in orthogonal ensembles by numerical studies of 3D Anderson model (for orthogonal ensembles, RMT predicts \( C_\Delta \sim -0.27 \)). However, due to the smaller statistics and smaller system size, the data are not so conclusive.

In figure 5 we present the distribution of the higher LE \( z_2 \) and \( z_8 \) at the critical point. Already \( z_2 \) is almost Gaussian distributed, as well as for orthogonal ensembles \([19, 9]\).


Figure 6 shows the distribution of the conductance in the metallic regime \((E = 0.1, W = 2\) and \(40 \leq L \leq 100\)). The inset shows the \( 1/L \)-dependence of the mean value \( \langle g(L) \rangle \) and \( 2/\xi_L \). The data confirm the relation

\[
\langle g \rangle = \frac{2}{\xi},
\]

although the agreement is worse for large \( L \). Large discrepancies (more than 30 \%) have been found between \( \langle g \rangle \) and the quantity \( 1/(\xi_1) \). As we show elsewhere [28], this discrepancy does not mean the failure of the finite-size-scaling theory. As could be seen from linear and logarithmic fits for the mean conductance, it is difficult to distinguish the finite-size corrections (proportional to \( 1/L \)) from the physical, \( \log L \)-dependence, predicted by Wegner [27].

We have also been interested in the fluctuations of the conductance, measured by the variance of the conductance in the metallic, and by the variance of the logarithm \( g \) in insulating regimes:

In the metallic regime, \( \text{var}(g) \) fluctuates between 0.15-0.20 (in units \((e^2/h)^2\); see inset of Fig. 6), which is four times smaller than that for the universal variance \( \text{var}(g) \) in the two-dimensional orthogonal ensembles \([19, 29]\), and agrees with the theoretical prediction of Lee, Stone and Fukuyama [18].

In the insulating regime, we studied the systems with box, binary, Gaussian and Cauchy distribution of the random energies and for different parameters \( E, W, L \) and found the linear
Fig. 6. — The distribution $P(g)$ of the conductance in the metallic regime, $W = 2$ for $L = 20$, $L = 40$, $L = 80$, $L = 100$, (from left to right). Inset: the $L$-dependence of $\langle g \rangle$ and $2/\xi$. Dashed lines are the linear fits $\langle g \rangle = 11.13 - 60.7/L$ and $2/\xi = 10.6 - 52.5/L$, solid line is the logarithmic fit $\langle g \rangle \approx 9.266 - 50.28/L + 0.4 \times \log L$. Second inset: variation of the conductance $v.s.$ its mean value for different systems in the metallic regime.

The most important question is that about the existence of large fluctuations of the conductance at the critical point. As is seen from figure 8, no large fluctuations have been found in our calculations. The maximal conductance, obtained in all ensembles we studied, was approximately the same, independent of whether $L = 10$ or $L = 80$ and of the number of samples ($500 \leq N_{\text{stat}} \leq 10000$). It is given by the the large value of $\langle z_1 \rangle$, which remains $\sim O(1)$ at the critical point and by the form of their distribution which are narrow Gaussian (Fig. 5). These two circumstances prevent our finding the sample in which a large number of channels are opened. The situation here differs completely from that in $2 + \varepsilon$-dimensional orthogonal system ($\varepsilon << 1$) [17, 31], where the critical disorder $W_c$ and all differences between LE are $\propto \varepsilon$.

Following the argumentation of reference [9] we conclude that the extremely large values of

\begin{equation}
\text{var}(\log g) = \alpha(\log g) + \beta, \tag{22}
\end{equation}

which favours the one-parametric scaling hypotheses (Fig. 7). We could not, however, exclude that the deviations from the one-parametric relation (22) arise in the larger systems.
Fig. 7. — Variation of the log $g$ vs. \( \langle \log g \rangle \) for different systems in insulating regime. The solid line is the linear fit (22) with $\alpha = 1.26$ and $\beta = 0.74$. The random energies are distributed according to box (○), binary (□), Gauss (△) and Cauchy (ϕ) distribution. Inset: function $\gamma(x)$ for $L = 20$, $W = 15$. The second inset: distribution of log $g$ for $W = 15$, $L = 16$ (○) and $L = 20$ (□) and the Gaussian fits through the data.

The conductance appear only with the probability

$$P(g) \propto \exp\{-c g^2\}. \tag{23}$$

The last equation follows from the linearity of the function $\gamma(x)$ and from an assumption of the Gaussian distribution of the higher LE (Fig. 5). The probability to have the sample with very large conductance decreases faster than it was in 3DAM.

Another problem is the existence of the extremely small values of $g$, or large negative fluctuations of log $g$. As explained in [9], the probability to have $|\log g| \gg 1$, $\log g < 0$ is determined by the form of $P(z_1)$ for large $z_1$. As was shown in the previous section, we do not know the form $P(z_1)$ completely. Supposing $P(z_1) \sim \exp -a z_1^n$, one deduces

$$P(g) \propto g \exp\{-\log^a g / 2(z_1)\}. \tag{24}$$

Weak statistical correlations between $\Delta$'s inspired us to rewrite relation (6) into the form

$$g = \sum_{n=1}^{N} 4 / [1 + \cosh \sum_{i=1}^{n} \Delta_i] \quad (\Delta_1 \equiv z_1) \tag{25}$$
Fig. 8. — Probability distribution of the conductance at the critical point $E = 0.1, W = 5.75, S = 0.5$ for 6 different system sizes $L = 10 - 80$. a) Logarithmic scale The solid line is $P^{(4)}(g)$ (25). b) Linear scale.
and to approximate

\[ P(\Delta_1, \Delta_2, \ldots) \approx P(\Delta_1)P(\Delta_2) \ldots \]

Supposing that each \( P(\Delta_i) \) is determined only by one parameter (the mean value \( \langle \Delta_i \rangle \)), \( P(g) \) could be determined completely only by the universal function \( \Delta(x) \)

\[ \Delta(x = i/2L) = \frac{(z_i - z_{i+1})}{2L}, \]

or, equivalently, by \( \gamma(x) \) (14). The approximate form of \( P(g) \), \( P^{(m)}(g) \), could be calculated numerically as

\[ P^{(m)}(g) = \int \prod_{i=1}^{m} (d\Delta_i P_i(\Delta_i)) \delta[g - \sum_{n=1}^{m} \frac{4}{1 + \cosh \sum_{i=1}^{n} \Delta_i}]. \]

The result of numerical integration for \( m = 4 \) is given in figure 8. We used our numerical data for \( \langle \Delta \rangle \)'s, and assumed the Wigner surmise (19) for \( z_1 \), and (17) for higher \( \Delta \)'s. For not too small values of \( g \), the agreement is quite good, while large differences are visible for \( g \ll 1 \), probably due to the differences between \( P(z_1) \) and the Wigner surmise (19).

To end this section, let us mention, that, due to the statistical character of the conductance, the minimal metallic conductivity losses its meaning in 2D systems. Namely one obtains at the critical point \( g \neq 0 \) with the probability 1 for any actual sample one studies. This actual value of the conductance is, however, unpredictable. It could be infinitesimally small as well as surprisingly large. One can only consider the mean conductivity at the critical point. Our data provide

\[ \langle \sigma(W = W_c) \rangle \approx 1.49. \]

(29)

(Let us note, that in 3D systems, where \( \langle g \rangle \sim O(1) \), \( \sigma \sim O(1/L) \) at the critical point and so \( \langle \sigma(W = W_c) \rangle = 0 \).)

5. Conclusion.

We obtained the size-independent conductance distribution at the critical point of the metal-insulator transition. The analysis of the large systems confirms our conclusions from the studies of the three-dimensional Anderson model, namely that large conductance fluctuations are absent at the critical point. It is caused by the discrete spectrum of the "external" Lyapunov exponents.

The universal distribution of the differences of Lyapunov exponent together with the weak statistical correlations of \( \Delta \)'s enables us to determine the conductance distribution \( P(g) \) only in terms of the spectrum of transfer matrix.

In the localized regime, the universal relation between variance and the mean value of \( \log g \) was found.

In the metallic regime, LE and the conductance possess the \( 1/L \) finite-size dependence. The \( \log L \)-corrections, predicted by Wegner [27] are visible only for larger system sizes, not accessible in our calculations.

In the metallic regime the distribution of the conductance is Gaussian. The \( L \)-independent width of the distribution confirms the universality of the conductance fluctuations. The relation between the mean conductance and the slope of the \( \gamma \) function has been proved.

The studies of the statistics of LE, especially the correlations between differences of LE indicate that the random matrix theory could be successfully applied to the studies of the 2D samples in all three regimes.
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