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Sweeping of an instability : an alternative to self-organized criticality to get powerlaws without parameter tuning

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Abstract. — We show that a notable fraction of numerical and experimental works claiming the observation of self-organized criticality (SOC) rely in fact on a different physical mechanism, which involves the slow sweeping of a control parameter towards a global instability. This slow sweeping (which does not apparently involve a parameter tuning) has been the cause for the confusion with the characteristic SOC situation presenting truly no parameter tuning and functioning persistently in a marginal stability condition due to the operation of a feedback mechanism that ensures a steady state in which the system is marginally stable against a disturbance. The observation of power law distributions of events, often believed to be the hallmark of SOC, can be traced back to the cumulative measurements of fluctuations diverging on the approach of the critical instability. For non-critical instabilities such as first-order transitions, the power law distribution exists on a limiting size range up to a maximum value which is an increasing function of the range of interaction. We discuss the relevance of these ideas on the onset of spinodal decomposition, off-threshold multifractality, an exactly soluble model of rupture, the Burridge-Knopoff model of earthquakes, foreshocks and acoustic emissions, impact ionization breakdown in semiconductors, the Barkhausen effect, charge density waves, pinned flux lattices, elastic string in random potentials and real sandpiles.

1. Introduction.

It is often asserted that the hallmark of self-organized criticality is the *simultaneous* existence of two properties, observed without having to tune a control parameter : (1) power law distribution of events, and (2) spatial and temporal correlation functions decaying algebraically. Self-organized criticality (SOC) is a concept proposed [1] to describe the dynamics of a class of open non-linear spatio-temporal systems, which evolve spontaneously towards a critical state. To the best of our knowledge, most numerical simulations and experiments, which have aimed at identifying SOC in various physical systems, have reported results either on the first (1) or the second type (2) of property but not on both of them simultaneously. From our point of view, property (1) is more characteristic of the new class of phenomena described

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by the SOC concept, since it relies on the condition of a very slow driving of the system and the existence of fast burst-like responses of the system (see [2] for instance for a discussion of the different classes of SOC). This behavior is at the opposite to the standard slow response of a critical system at equilibrium characterized by « critical slowing down » under the fast sollicitation created by thermal noise. In the following, we will therefore not discuss the class of phenomena dubbed « generic scale invariance » [3] or critical phase transitions made self-organized [4] which present a SOC behavior in the sense of property (2). Apart from the absence of tuning and the property of self-organization, they are similar to standard equilibrium critical systems [5].

Is the observation of a power law distribution of events (1) sufficient to qualify a system as SOC ? If one regards power laws as a possible definition of SOC, the question makes no sense. Here however, we view SOC in a more restricted sense, i.e. SOC requires that, as a function of a tunable control parameter, one has a phase transition at some critical point, and that the dynamics of the system brings this parameter automatically to its critical point without external fine-tuning. Then, the purpose of this paper is to show that the answer is negative and that a notable fraction of numerical and experimental works claiming the observation of SOC in fact rely on a different physical mechanism. This mechanism, that we term « sweeping of an instability », involves the slow sweeping of a control parameter μ on the approach to and possibly beyond a global instability at μ_c. In other words, we claim that the existence of a power law does not necessarily mean SOC since a power law may also arise if a tunable parameter crosses over from one side of the critical point to the other side. In the next section (Sect. 2), we present our main idea and its mathematical formulation using as an example the Ising and percolation models and then discuss in section 3 its relevance to a variety of experimental systems.

2. The mechanism of « sweeping of an instability ».

Our basic idea can be understood by considering the Ising model and similarly an annealed version of the percolation model (i.e. allowing sites or bonds to appear and die randomly at constant average occupation probability μ). These models are among the simplest archetypes of critical transitions reached by finely tuning the temperature (Ising) or the concentration (percolation) to their critical values. Presenting criticality for some value of the control parameter, they are not self-organized. The thermal Ising problem and the geometrical bond percolation are closely related since they can be put in one-to-one correspondence, using Coniglio and Klein's recipe [6] . connectivity in the Ising model is defined in the sense of a physical « droplet », according to which two nearest neighbor spins are connected if they are both up and furthermore if the bond between them is active, a bond being active with a probability $p = 1 - e^{-2J/k_B T}$, where J is the exchange coupling constant between spins.

For a given value of the control parameter μ, spontaneous fluctuations occur. These fluctuations in both models correspond to the clusters defined by the connectivity condition. These fluctuations can be visualized as cooperative spatial domains of all sizes between the microscopic scale up to the correlation length ξ in which the order parameter takes a non-zero value over a finite duration depending on the domain size. Their size distribution is given by [7]

$$P_{\mu}(s)\,\mathrm{d}s \sim s^{-a} f(s/s_0(\mu))\,\mathrm{d}s \tag{1}$$

with $a = 2 + 1/\delta$ ($= 2.05$ with $\delta = 91/5$ in 2D percolation [7]). Recall that the size s is the number of spins or sites belonging to a given cluster. $s_0(\mu)$ is a typical cluster size given by

$$s_0 \sim |\mu_c - \mu|^{1/\sigma} \tag{2}$$

with Fisher's notation $1/\sigma = \gamma + \beta$. $s_0(\mu)$ must be distinguished from the mean cluster size $\langle s \rangle\ (\mu)$, which scales as $\langle s \rangle\ (\mu) \sim |\mu_c - \mu|^{-\gamma}$. γ is the susceptibility exponent defined by the number of spins which are affected by the flip of single spin, which corresponds in the language of percolation to the mean cluster size. β is the exponent characterizing the way the order parameter goes to zero as $|\mu_c - \mu|^{\beta}$ as $\mu \to \mu_c$.

The scaling function $f(s/s_0)$ decays rapidly (exponentially or as an exponential of a power law) for $s > s_0$ [7]. Thus, for our purpose, equation (1) teaches us that the cluster or fluctuation size distribution is a power law $P_\mu(s) \sim s^{-a}$ for $s < s_0(\mu)$ and $P_\mu(s)$ is negligibly small for $s > s_0(\mu)$.

Now, suppose that one monitors the fluctuation amplitudes (i.e. cluster sizes) as the control parameter μ is swept across its critical value μ_c, say from the value $\mu = 0$ to some value $\mu_c + c$ (with $c > 0$) above μ_c. The total number of clusters of size s which are measured is then proportional to

$$N(s) = \int_0^{\mu_c + c} P_\mu(s)\, \mathrm{d}\mu \tag{3}$$

which can be written as

$$N(s) = \int_0^{\mu_c} P_\mu(s)\, \mathrm{d}\mu + \int_{\mu_c}^{\mu_c + c} P_\mu(s)\, \mathrm{d}\mu\ . \tag{4}$$

In writting expression (3), we have used the fact that the full distribution (1) of fluctuations exists for each value μ of the control parameter. The change of variable $\mu \to s_0(\mu)$ in the integral (3) gives

$$N(s) = s^{-a} \int_1^{+\infty} s^{-\sigma\left(\frac{1}{\sigma} + 1\right)} f\left(\frac{s}{s_0(\mu)}\right) \mathrm{d}s_0 \tag{5a}$$

$$N(s) \sim s^{-a} \int_s^{+\infty} s^{-(\sigma + 1)}\, \mathrm{d}s_0 \tag{5b}$$

using the fact that $f(s/s_0(\mu))$ is negligible for $s_0(\mu) < s$. Here, the symbol « $\sim$ » is taken as meaning the leading behavior in decreasing powers of s. Equation (5b) finally yields the power law

$$N(s) \sim s^{-(a + \sigma)}\ , \tag{6}$$

as its leading behavior. Note that we have not restricted μ to stop at its critical value μ_c but have allowed for a spanning of an arbitrary interval containing μ_c. The contribution to the cumulative fluctuation size distribution $N(s)$ for μ close to μ_c from above is in general similar to that for μ close to μ_c from below, and thus contributes to the numerical factor in front of the power law (6). In some cases, depending upon the physical variable which is measured, the fluctuations for $\mu > \mu_c$ may on the contrary be small and thus do not contribute to the leading power law behavior (6).

Expression (6) thus demonstrates our basis claim, i.e. that a continuous monitoring of events or fluctuations up to and above a critical point yields a power law even if a similar measurement for a fixed value of μ would only give a truncated power law (with a smaller exponent). Thus, by varying the temperature (Ising) or concentration (percolation), say, linearly in time from a value below the critical point to a value above the critical point and by integrating over all fluctuations observed during this whole time interval, one gets a power law in the distribution of fluctuations. Since only right at the critical point, large clusters exist, one gets a power law without cut-off for the time-integrated cluster number even if we do not stop at the critical point. Note the renormalized exponent $a + 1/(\gamma + \beta)$, stemming from the relatively smaller weight of large clusters which are found only in a narrow interval of the control parameter.

A similar result holds for supercritical instabilities such as for instance the Rayleigh-Bénard instability, which corresponds to an out-of-equilibrium critical point [8]. In the Rayleigh-Bénard case, the order parameter is the average convection velocity and the fluctuations are associated to streaks or patches of non-zero velocity v occurring below the critical Rayleigh number at which global convection starts off. Note that the divergence of the spatial diffusion coefficient $D(\xi)$ of the fluctuations which have been found in this problem [9] can be understood as a consequence of the existence of a power law similar to (1) describing the distribution of velocity fluctuations prior to the instability. We can write the diffusion coefficient as the product of an average square velocity times a characteristic time scale :

$$D(\xi) = \alpha(\xi)\langle v^2 \rangle \; \tau(\xi) . \tag{7}$$

$\tau(\xi) \sim \xi^z$ is the typical duration of a fluctuation of spatial extension ξ (here z is the dynamical critical exponent). Using $\langle v^2 \rangle \sim |\mu_c - \mu|^{-\gamma}$, we thus get $D(\mu) \sim \alpha(\xi)(\mu_c - \mu)^{-(\gamma + \nu z)}$ Within mean field which applies for the Rayleigh-Bénard supercritical instability, $\gamma = 1$, $\nu = 1/2$ and $z = 2$ which gives $\gamma + \nu z = 2$. For an hydrodynamic instability, one must in addition take into account the hydrodynamic drag which results in the correction $\alpha(\xi) \sim \xi^{-1}$, yielding $D(\mu) \sim (\mu_c - \mu)^{-3/2}$, which recovers previous results [9]. This shows that the enhancement of the spontaneous fluctuations near the instability produces a measurable singular diffusion coefficient. The occurrence of a singular diffusion coefficient has been suggested as the origin of SOC [10] , here, it is clear that such a singularity only reflects the existence of very large fluctuations. It is probably more correct to interpret the singular behavior of the diffusion coefficient as the *consequence* and not the cause of the critical behavior.

The mechanism of « sweeping of an instability » for generating power law distributions of events should be contrasted with the SOC phenomenon, best exemplified by the « sandpile » cellular automaton introduced initially by Bak, Tang and Wiesenfield [1]. The main difference is that the sandpile automaton is functioning persistently in a marginal stability condition due to the operation of a feedback mechanism (decrease of the local slope upon reaching a threshold and loading of neighbors) that ensures a global steady state in which the system is marginally stable against a disturbance. The fundamental origin of the power law distribution of avalanches is basically not understood since one is unable to state *a priori* if a given model or modified version will exhibit power law behavior. The absence of a deep undertanding of SOC is probably also at the origin of the confusion in the interpretation of experiments and numerical simulations which have often misattributed the observation of power laws to SOC, as we discuss below in specific examples.

3. Physical illustrations of the mechanism of sweeping of an instability.

3.1 FLUCTUATIONS BEFORE THE ONSET OF SPINODAL DECOMPOSITION WITHIN MEAN FIELD THEORY. — We would like first to point out that the above scenario is present in the standard thermal equilibrium problem of spinodal decomposition [11] Spinodal decomposition is indeed a critical instability within mean field theory occurring on the so-called spinodal line (defined by $\partial^2 F / \partial A^2 = 0$, where F is the free energy and A the order parameter) inside the domain of coexistence in a liquid-gas or binary mixture demixing first-order transition. Outside the spinodal domain, spontaneous density or concentration fluctuations occur which are distributed in size according to the law (1) with the exponent $a = 2.5$ within meanfield percolation and spinodals [12]. Linked with this distribution, the fluctuations in space present a fractal and even multifractal structure [13]. Note that a similar multifractal structure appears in standard second-order phase transition and has been used to rationalize the distribution of particles numbers in high-energy nuclear collisions [14] interpreted using phase transition concepts. The physical interpretation of the distribution (1) is particularly clear in the spinodal case, due to the mapping between spinodal decomposition onto a percolation problem, or, more precisely a cluster growth problem [13]. In this case, the distribution of fluctuation sizes (1) takes a genuine geometrical sense. This is a general fact that the fluctuations and droplets appearing as precursors of a spinodal decomposition are distributed in size according to a power law. A similar results holds for a first-order phase transition but then the power law holds up to a maximum size which is an increasing function of the range of the interaction [11-13]. Then, measuring the total number of clusters when spanning the control parameter (for instance the temperature) up to the spinodal instability will yield the power law distribution (6).

3.2 OFF-THRESHOLD MULTIFRACTALITY. — Roux and Hansen [15] have shown that, if a physical quantity Q exhibits a multifractal behavior at the percolation threshold and more generally at a critical point μ_c, the histogram of the quantity Q recorded during an evolution of the control parameter μ (fraction of bonds in the percolation problem, temperature, . .), away from μ_c up to μ_c, will show a power law behavior. This result is very similar to our previous reasoning going from equation (1) to equation (6), and in fact constitutes a generalization of it. These authors have illustrated their result for the current distribution in diluted lattices but have failed to discuss its broader application to other phenomena discussed here.

3.3 AN EXACTLY SOLUBLE MODEL OF RUPTURE. — Let us consider the « democratic fiber bundle model » (DFBM) [16-23] of rupture phenomena, which has been introduced initially to describe long flexible cables or low-twist yarns. It is one of the few exactly soluble models of rupture which will allow us to test our ideas in this context. Indeed, rupture phenomena seem interesting candidates for applying the idea of « sweeping of an instability ». Indeed, one often measures bursts of acoustic emission or jumps of elastic constants associated to sudden internal damage or cracking as the stress is increased up to total rupture. The point of global breakdown is similar to an instability ; in fact the analogy with nucleation is particularly clear with the Griffith criterion for the unstable growth of a single crack. The analogy has been studied recently [24].

The DFBM is made of N independent parallel vertical fibers with identical spring constant k_p and identically distributed independent random failure thresholds X_n, $n = 1, . ., N$, distributed according to some cumulative probability distribution $P(X_n < x) = P(x)$. A total force F is applied to the system and is shared democratically among the N fibers. As F increases, more and more fibers break down until the final complete rupture. F is thus the analog of the control parameter μ and the force threshold F_c corresponding to complete collapse is the analog of μ_c.

An exact derivation of the distribution of rupture sizes (i.e. number of fibers Δ which break simultaneously in a single « avalanche » process) has been given in references [23, 25]. Here, Δ is the analog of s. In particular, reference [25] has shown the coexistence of 1) a differential distribution of burst of size Δ given by equation (1) with $a = 3/2$, with a cut-off exponent $\sigma = 1$, and 2) a total number of bursts of size Δ up to the run away scaling according to equation (6) with an exponent $a + \sigma = 5/2$, in agreement with the above derivation. The exponent 5/2 reflects the occurrence of larger and larger events when approaching the total breakdown instability. Note again that this global power law distribution is not associated with criticality but to fluctuations accompanying the onset of global rupture. This onset of global rupture is in fact a first-order transition since a finite fraction of fibers break simultaneously at threshold F_c. The existence of « critical » fluctuations of arbitrary sizes up to the threshold results from the infinite range of the interaction between the fibers, due to the democratic reloading on all surviving fibers.

3.4 THE BURRIDGE-KNOPOFF MODEL OF EARTHQUAKES. — Soon after the introduction of the SOC concept, it has been suggested [26-30] that earthquake dynamics and notably their power law distribution in sizes constitute a vivid example of SOC. We are still of this opinion for genuine earthquakes. However, we would like here to underline that some of the mathematical models that have been introduced exhibit power laws in earthquake size distribution, not because the models are SOC but because they somehow produce the mechanism of « sweeping of an instability ». Furthermore, foreshocks (i.e. small earthquakes which are precursors of large earthquakes) are also an example of this phenomenon.

This idea is most simply illustrated using the correspondence between a mean field version of the Burridge-Knopoff block-spring stick-slip model of earthquake faults and a cycled generalization of the democratic fiber bundle model (see previous Sect. 3.3) [25]. The exactly soluble democratic fiber bundle model suggests that the Gutenberg-Richter power law of earthquake size distribution is not associated, in the Burridge-Knopoff model, to stationary criticality but to fluctuations accompanying the nucleation of a large earthquake run away. This view point is also defended in reference [31] where an analogy between failure dynamics in a class of Burridge-Knopoff models and a mean field spinodal line has been proposed.

This idea is further confirmed by a recent work [32] on a 1D dynamical version of the Burridge-Knopoff model for earthquakes with a velocity weakening friction law, in fact exactly the version studied in reference [28] constituting a rediscovery of the initial Burridge-Knopoff model [33]. Depending on the system size, two types of solution have been found which are in general present simultaneously : chaotic motion and solitary wave propagation. The solitary wave propagation, which can be seen as the existence of propagative localized macro-dislocations, is always present. For certain values of the system size, there is a kind of resonance such that the chaotic motion disappears and only the completely coherent solitary propagation is observed. This corresponds to a macroscopic run away which covers the system endlessly as a domino line falling over and over along the system in order to accommodate the slow imposed tectonic plate velocity. For other system sizes which are out of resonance, the velocity of the solitary macro-dislocation is not matched to the size of the system and the slow imposed tectonic velocity, involving a kind of frustration. A macro-dislocation therefore possesses a finite lifetime. Its appearance is in general preceeded by the chaotic phase, characterized by a power law distribution of small slidings up to a maximum size. It is thus tempting to view this chaotic phase producing the power law distribution as the phase preceeding the nucleation of the macro-dislocation. Due to frustration, the macro-dislocation is not stable and eventually decays away. One thus observes recurrent random nucleation of macro-dislocations in between chaotic phases. This is in close analogy to the behavior described above as well as the behavior observed in real sandpile avalanches (see below Sect. 3.8).

3.5 FORESHOCKS AND ACOUSTIC EMISSIONS. — Foreshocks are precursors of large earthquakes which have often been observed to cluster in time and increase in size on the approach of the onset of the large rupture (see Ref. [34] and references therein). In other words, seismicity prior to a great earthquake often shows a marked increase of activity in a way similar to the increase of fluctuations prior to the onset of a critical instability. A typical observation, well described by Omori's hyperbolic law [35], is that the rate of energy release $\mathrm{d}E/\mathrm{d}t$ by small earthquakes is found to increase on average as a powerlaw $(t_r - t)^{-\alpha}$ as the time t approaches t_r with an exponent α close to one. In addition to this average behavior, the existence of larger fluctuations in the rate of energy release dE/dt from systems to systems and its sensitivity to the initial inhomogeneity configuration has been demonstrated in model earthquakes [34b]. It is also characteristic of field observations. Furthermore, the distribution of the foreshock sizes is also governed by the Gutenberg-Richter power law.

Very similar observations are ubiquitous in the acoustic emission literature [36]. Acoustic emission is now a standard technique to monitor and measure the progressive deformation, damage and cracking within materials submitted to increasing stresses. An acoustic wave burst is generated each time an acceleration appears within the material as a result of the creation or motion of a dislocation, the growth of a crack, etc. Standard measurements quantify the total energy of each acoustic burst, their duration, etc. Omori's law for the increase of activity prior to global rupture and the Gutenberg-Richter size distribution law are also very often observed. This even provides practical tools to predict the incipient global rupture [37].

Our purpose here is to argue that this power law distribution is not due in general to some SOC phenomenon but again results from the characteristic nature of the fluctuations on the approach towards a global instability with long-range elastic interactions, here the onset of global rupture. This is contrast to a recent claim [38] that SOC is the origin of the power law properties of acoustic emission. We also argue that dislocation glides and abrupt deformation associated with twinning processes [39] in metals are also the result of the approach of a global instability.

The existence of the Omori's law and of the Gutenberg-Richter power law for foreshocks can be rationalized using the tools presented in references [40]. Essentially, the idea is to assume an arbitrary distribution of initial flaws reflecting a microscopic disorder always present in an experiment or in nature. Then, in the presence of a growth law expressing the subcritical crack growth velocity as a power of the crack length [40], the Omori's law and the Gutenberg-Richter power law are easily derived. The exact solution of the DFBM of rupture discussed above also provides an alternative way to view these phenomena.

Concerning the claim of reference [38], we would like to stress that it is subtantiated solely on the basis of the observation of a power law distribution. However, we note that the acoustic emissions occur during the α-β phase transformation during the hydrogen doping of Nb samples upon a constant cooling rate. This hydrogen doping and the associated acoustic emissions cannot be sustained for ever since the sollicitation stops when the temperature is too low. This problem can not be a SOC phenomenon, since it cannot operate persistently. It is much better described by the sweeping of a control parameter, here the amount of hydrogen content within the metal, which makes the system span its phase diagram from the single α phase to the mixed β phase, in a way very similar to the spanning of a coexistence diagram in a first order phase transition. We note that the authors themselves stress that the hybride precipitation is a first order transformation. We thus interpret the power law distribution as stemming from the fluctuations associated to the contral parameter spanning the phase diagram.

3.6 CHARGE DENSITY WAVES, PINNED FLUX LATTICES, ELASTIC STRING IN RANDOM POTENTIALS. — Models of charge-density waves (CDW's) usually consist in an elastic array of particles submitted to a driving field E and interacting with impurities at random positions [41]. For fields E below a threshold E_c, the CDW is pinned whereas for $E > E_c$ the systems has a nonzero average velocity. Recently, Middleton and Fisher [42] have found in 2D simulations that the polarization diverges as $P(E) \sim (E_c - E)^{-\gamma+1}$, with $\gamma = 1.58 \pm 0.12$, as $E \to E_c$ (from below). From the size dependence of threshold fields and polarizations, they also find that the largest correlation length ξ diverges as $\xi \sim (E_c - E)^{-\nu}$, with $\nu = 1.05 \pm 0.04$. In a related work using a similar model inspired from the physics of pinned flux lattices, Pla and Nori [43] have found that the distribution $D(d)$ of sliding bursts of size d, measured in narrow intervals of driving fields E at a finite distance below threshold E_c, scales as $D(d) \sim d^{-\beta}$, with $\beta \approx 1.3$. They infer that this reflects a SOC state. Very interesting experimental results have also been reported on the electric behavior of the quasi-one-dimensional CDW compound $K_{0.3}MoO_3$ at liquid-helium temperatures [44]. In this system, there exists a threshold voltage at which an abrupt increase in the conductivity takes place. The authors [44] report that the onset of this transition in conductivity is marked by a hysteresis, pointing to a first-order type instability. Note that this is of no consequence for our proposed scenario in which the fluctuations can appear in both cases to produce a power law distribution of burst sizes as long as the interaction is long range. Indeed, they report, in the low-conducting branch of this hysteresis (i.e. prior to the global threshold), an intermittently spiking current in the time domain characterized in particular by a power law scaling of the firing and of the waiting times. The authors conclude that these features are characteristic of a SOC phenomenon.

It is now becoming clear from the above discussion that the onset of global sliding of a CDW or of a pinned flux lattice is similar to a spinodal point or a critical point. Again, any of these systems cannot function persistently in a SOC. It is only due to the spanning of the electric field up to the critical value E_c that the distribution of bursts is found a power law, in agreement with the mechanism proposed in this paper. In reference [45], we have given a mean field theory based on an extension of the DFBM made critical which allows to rationalize the numerical results quoted above. Similar behavior is observed for an elastic string in a random potential [46].

3.7 IMPACT IONIZATION BRAKDOWN IN SEMICONDUCTORS AND BARKHAUSEN EFFECT. — Very exciting experimental results have been reported a few years ago [47] on the low-temperature impact ionization breakdown of p-Ge. The authors claimed that this was the first experimental verification of the SOC idea. Let us briefly recall the nature of the problem and the main results. The system consists in a slightly doped semiconductor cooled down to a low temperature such that it constitutes an almost ideal insulator, because most of the extrinsic carriers are frozen out at the impurity atoms. If an applied electric field exceeds a critical value, the few remaining carriers can gain enough energy to release the bound carriers by impact ionization. This autocatalytic process ends up in an avalanche breakdown of the resistivity of the sample. There are three main regimes exhibited by the systems : a) at low and intermediate bias voltage, short current pulses occur with a statistical temporal and size distribution ; b) for a larger bias voltage such that the system functions in the non-linear S-part of the macroscopic I-V characteristics of the sample, ordered current spikes occur in a quasi-regular pattern characteristic of oscillations of relaxation ; c) at higher values of the bias voltage, the ordered oscillation mode becomes qualitatively different both in amplitude and frequency. The statistical analysis of reference [47] concerns the first regime. In the spirit of our previous discussions, it is tempting to view the voltage limiting the non-linear S-region of the I-V characteristics as an instability threshold for the onset of current spikes oscillations of relaxation. The ordered large current spikes observed in regime b) correspond to the bifurcation to an ordered phase with a non-zero order parameter. We interpret regime a) as the fluctuation regime prior to the global instability. Then, it is not surprising that the distribution of time intervals between current spikes, closely related to that of the current spike amplitude distribution (see Ref. [26] for a discussion of the relation between amplitude distribution and time interval distribution in the case of earthquakes), is given by a power law. The exponent is found equal to -1.33, close to the simulations of Pla and Nori [43] and to our predictions [45] based on a mean field model. This system is similar to those discussed in the previous section 3.6, notably in the feature that the system is not functioning persistently in a fixed dynamical state, since a macroscopic control parameter is spanned slowly. We thus conclude that the claim that the experimental results are the signature of SOC is not substantiated.

We propose similar considerations to another experimental report [48] on the Barkhausen effect, consisting in the measurements of irreversible domain-wall jumps in a ferromagnetic metallic glass by a pickup coil sensitive to variations of magnetic fields. Here again, there exists three main regimes : a) for small applied field, one observes small reversible domain-wall changes and the system remains magnetically elastic ; b) as the applied field increases and approaches the magnitude of the coercive force, the magnetization increases very rapidly and the response is characterized by large and irreversible domain-wall jumps along with rotations within domains ; c) at saturation, the entire specimen is magnetized in the direction of the applied field and no more rearrangements occur. We note that a typical experiment is carried on by applying a constant slow magnetic field rate (1 Oe/s) and that the coercive force of the ferromagnetic samples is small (≈ 0.1 Oe). Since about 3 000 pulses separated in time from 50 μs to 1 000 μs were detected in a measurement, the total duration of such a recording is about 0.3 s or more, taking as an order of magnitude an average time interval between consecutive pulses of 100 μs. This means that the experiments reported in reference [48] correspond to a sweeping of the control parameter, the applied magnetic field, up to complete magnetization, since the field spanned during the time of an experiment is of the order of or larger than the coercive force and hysteresis loops are rectangular [48]. Viewing the state of saturated magnetization as the ordered state resulting from a nucleation process (here the nucleation of the favored magnetization at the expanse of other directions), the Barkhausen noise can thus be interpreted as resulting from the fluctuations announcing the cooperative ordering of the domains. We believe that the observed power law results from the combination of the long range magnetic interaction and the nucleation process. We note again that the SOC interpretation given in [48] cannot hold since the system cannot operate persistently in the same macroscopic state but rather evolves progressively to its completely ordered state. This remark is common to all systems previously discussed.

3.8 REAL SANDPILES. — Since the introduction of the SOC concept [1] based on cellular automata model sandpiles, it is only natural that experiments has been carried on real sandpiles in order to test the application of these ideas. The results of the first experiments [49] devised to observe a power law distribution of avalanches have been discouraging : the avalanches occur in a quasi-regular fashion with a well-defined mean size, lifetime and average period. These averages are decorated by fluctuations which however are not larger than about 10 % of the means. The physics of avalanches seems thus well-described as oscillations of relaxation. The slope is steadily and slowly increased by addition of grains or by a slow tilting or rotation of the sandpile. When the maximum angle of repose θ_m is reached, an avalanche occurs which corresponds to a flow of grains not only at the surface but also within the bulk of the sandpile. The avalanche stops and defines a new angle which is smaller that θ_m by about 2 degrees. This induces an hysteresis and the sandpile is stable until addition of grains allows the slope to reach again its instability threshold θ_m. We thus again have the existence of an instability, with the control parameter being the slope θ, leading to oscillations of relaxation. These oscillations of relaxation are due to the fact that the instability triggers an avalanche which then takes away some of the grains and thus relaxes the control parameter below its threshold value θ_m. Addition of new sand grains brings back θ to its threshold θ_m. The observation [49] of fluctuations decorating this average behavior is similar to their ubiquitous observation close to instabilities.

According to our scenario, these fluctuations should be characterized by a power law distribution if the interaction between grains is sufficiently long range. This should be the case since the interaction between two distant grains is mediated by the transport of one grain close to the other in successive avalanches. However, the power law distribution of avalanche sizes seemed absent in the previous works [49]. The first experiment [50] which found these power law fluctuations has followed a slightly indirect course. The main difference of the work of reference [50] with respect to others was the study of *small* sandpiles such that the angle $(180/\pi)\, R/L$ with which one sees a grain of size R at a distance equal to the sandpile size L is less or equal to about 2 degrees. In these small sandpiles, the quasi-regular oscillations of relaxation were completely absent and replaced by a power law distribution of small avalanches, which are completely confined to the surface or first grain layer. The observation of a power law behavior for small systems but its disappearance for large systems is at odds with the standard finite size effects expected in critical systems and cast a strong doubt on the SOC interpretation of the results of reference [50]. Our interpretation is rather that the observed power law distribution of small avalanches is the signature of the fluctuations which are the precursors of the instability at θ_m. For small systems, the global instability is essentially suppressed due precisely to finite size effects. Furthermore, the grain-grain dilatancy effect and inertia effects are much less efficient the smaller the system is and essentially disappear for a system size L such that $(180/\pi)\, R/L < 2$ degrees. This then allows the observation of the fluctuations, which are always present but which were concealed by the large oscillations of relaxation in the larger sandpiles. Our interpretation is confirmed by a recent analysis [51] of avalanches in the same type of sandpiles but with *large* ones. Use of digital image analysis of the avalanches occurring at the surface of large sandpiles show clearly the coexistence of 1) the large oscillations of relaxation avalanches which are quasi-regularly spaced in time in agreement with [49] and 2) a power law distribution of sizes for smaller avalanches occurring between the large sliding events. These small avalanches were not visible in the previous experiments [49] because they do not reach the bottom rim and thus remain within the sandpile. They also produce very little acoustic emission and are thus difficults to detect by other means than direct visualization. It is also interesting to note that the number of small avalanches increases in number as $(\theta_m - \theta)^{-1}$, in remarkable similarity to Omori's law of foreshocks preceeding a large earthquake (see Sect. 3.5 above). In fact, the mechanism is analogous, the large avalanches playing the role of the large earthquakes, and Omori's law results from the amplification of a small initial disorder in grain packing on the approach of the instability.

The above picture can be completed in analogy with first order transition lines and their terminal critical point by mentionning the suggestion [52] based on the physical description of sand in terms of plasticity theory and the Granta-Gravel model that a sandpile should become truly critical at all scales for a special value of the packing density. We also would like to point out the relation with the dynamical system theory [53] of post-bifurcation localization in non-cohesive brittle media (i.e. sand). It provides a framework for understanding the development of complex deformation patterns from the mechanics of localization and rupture and for connecting the simple localizing behavior at small times (the small deformation regime) to the complex faulting at large times (corresponding to the large deformation regime). Fluctuations are also often observed [54] in this quasi-static deformation experiments which should be described in a similar vain.

4. Conclusions.

We have shown that a notable fraction of numerical and experimental works claiming the observation of self-organized criticality (SOC) rely in fact on a different physical mechanism, which involves the slow sweeping of a control parameter towards a global instability. The experimental examples which have been discussed are foreshocks and acoustic emissions, impact ionization breakdown in semiconductors, the Barkhausen effect, charge density waves, pinned flux lattices, elastic string in random potentials and real sandpiles. We have also proposed analogies between the mechanism of a sweeping of an instability and the fluctuations accompanying the onset of spinodal decomposition, off-threshold multifractality, rupture (through an exactly soluble model) and earthquakes (through the Burridge-Knopoff model).

We have noted that the existence of a slow sweeping of the control parameter, which does not apparently involve a parameter tuning, has been the cause for the confusion with the characteristic SOC situation presenting truly no parameter tuning and functioning persistently in a marginal stability condition. In the physical situations that have been examined, the systems cannot operate persistently since the control parameter is swept and does not remain constant as in the model sandpile for which the average slope is attracted to its marginal stability limit. For critical instabilities (CI), the observation of power law distributions of events is due to the cumulative measurements of fluctuations diverging at the instability. For non critical instabilities such as first-order transitions (FOT), the power law distribution exists on a limiting size range up to a maximum value which is an increasing function of the range of interaction. Their observations reflect the long range nature of the interaction in the various systems which have been studied : foreshocks and acoustic emissions (FOT with long range elastic force), impact ionization breakdown in semiconductors (FOT with long range electric interaction), the Barkhausen effect (FOT with long range magnetic interactions), charge density waves (CI), pinned flux lattices (CI), elastic string in random potentials (CI) and real sandpiles (FOT with long range grain-grain interaction by diffusion and convection).

In contrast to standard critical hydrodynamic instabilities [8, 55] where the effect of diverging fluctuations has been predicted some time ago [56] but detected only recently due to its smallness [57], the fluctuations, precursors of the instabilities discussed in this paper, are characterized by their large amplitude and relatively easy observation. It is at this quantitative level that the analogy proposed here breaks down. We attribute this quantitative difference mainly to the threshold nature of their dynamics which amplifies fluctuations and also maybe to the general long range nature of the interactions. Indeed, in each system, specific physical mechanisms operate to enhance the role of fluctuations. Let us cite one example, the case of sand. It is clear in this case that fluctuations are important due to the conjunction of rotation and dilatancy effects which create a kind of frustration, similarly to what occurs in spin glasses. In presence of frustration and disorder, fluctuations are known in general to be important, both from sample to sample and also in the evolution of a given sample.

What is finally the difference between the class of systems discussed here and those obeying SOC ? In the present language, a self-organized critical system is one which functions persistently at or near a global instability. The only difference therefore is the existence of some mechanism which attracts the dynamics to the instability. We recover here the ideas proposed more specifically to make self-organized the standard critical phase transitions [4].

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