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Observation of diffraction radiation of oscillator on angular distribution measurement

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Abstract . — The expression for angular distribution of diffraction radiation of oscillator (DRO) in single crystals is derived. The effect of the beam parameters on angular distribution density and oscillator kinetics are taken into account. The analysis of the conditions for DRO observation on the parametric radiation and bremsstrahlung “background” is made. Some experimental procedures for detection DRO are suggested.

1. Introduction.

For a charged particle channeled along a crystal axis or plane the projectile path is a result of correlated collisions with crystal atoms. The particle moves in an effective potential obtained by smearing the crystal potential along the axis or plane [1]. This motion is accompanied by a special type of radiation, so-called channeling radiation [2, 3]. X-ray radiation of a relativistic oscillator in a crystal is essentially modified under diffraction conditions for emitted photons. A new diffraction radiation is a result of coherent summation of two processes — photon radiation and photon diffraction. It has been called diffraction radiation of oscillator (DRO) [4].

Though predicted back in 1977 [5], DRO has not so far been observed experimentally. Its study is of considerable interest, since DRO may find application in treating different effects in the optics of a relativistic emitter moving in refracting media [6]. Besides, DRO angular distribution is a strong function of the charged particle beam characteristics, which may be used for monitoring high-quality beam parameters.

In this paper we analyze conditions of DRO observation in experiments on measurement of angular distribution of the radiation emitted towards the lateral diffraction maximum by relativistic electrons (positrons) passing through perfect single crystals under planar channeling. DRO should be detected against a “background” of parametric X–radiation (PXR) [7] and bremsstrahlung diffracted by the crystalline planes.
2. Spectral-angular distribution.

Suppose that charged particles cross a plane-parallel single crystal plate along a direction specified by the unit vector $e_3$ which does not coincide in the general case with a unit vector of the normal to the surface $N$ directed inwards. Let us further assume that the axis given by the vector $e_3$ crosses a family of crystallographic planes corresponding to the reciprocal lattice vector $\tau$ ($|\tau| = 2\pi/d$, $d$ — is the interplanar distance). In this case the wave length of photons with the wave vector $k_B = \omega e_3/c$ and frequency $\omega_B$, determined from the equality $|k_B + \tau| = |k_B|$ and satisfies the Bragg condition $\lambda_B = 2d\sin\theta_B$, where $\sin\theta_B = -e_3.\tau/|\tau|$. The spectral-angular distribution of the number of photons with the wave vector $k(|k| = \omega/c)$ and polarization $s(\sigma, \pi)$ emitted from the crystal towards the lateral maximum given by the vector $k_{B\tau} = k_B + \tau$ can be found by the general method described in reference [8].

$$\frac{dN_{ks}}{dk} = \frac{\alpha\Omega R^2_{\text{L}(B)}}{4\pi^2c} \left| \sum_{\mu=1}^{2} \sum_{n=-\infty}^{+\infty} (-1)^{\mu} \exp \left( -i\Omega L_0/c + ik_{\mu}r_{0\perp} \right) \cdot (e_{\tau s}a_{\mu n}) \right|^2 \cdot (1 - \exp (-iL_0/l_{\mu n}))^2,$$

where $\alpha = 1/137$,

$\Omega$ is the oscillation frequency in the laboratory coordinate system,

$L_0 = L/\gamma_0$, $L$ being the plate thickness, $\gamma_0 = e_3 N$,

$r_{0\perp}(r_{01}, r_{02})$ is the transverse radius vector of particle at the moment it enters the crystal ($r_{\perp}(t = 0) = r_{0\perp}$),

$e_{\tau \sigma} \parallel [k, \tau]$, $e_{\tau \pi} \parallel [k - \tau, e_{\tau \sigma}]$ — polarization unit vectors,

$R^2_{\text{L}(B)}$ is the coefficient defining the efficiency of emitted photon diffraction in crystal for the Laue ($\gamma_1 = k_{B\tau} N/|k_{B\tau}| > 0$) and Bragg ($\gamma_1 < 0$) diffraction geometry

$$R^2_{\text{L}} = C_s^2 \left| \frac{\chi_\tau}{2(n_2 - n_1)} \right|^2,$$

(2)

(for an explicit form of the coefficient $R^2_{\text{L}}$ the reader should refer to [8]),

$C_\sigma = 1$, $C_\pi = \cos 2\theta_B$, $n_\mu s = 1 + \delta_{\mu3} + \alpha_1/2$ is the index of X-radiation refraction in a crystalline plate in the case of two-wavrefraction of photons by the atomic lattice, $\mu = 1, 2$

$$\delta_{1(2)s} = \frac{1}{4\gamma_1} \left\{ -\alpha_1 \gamma_1 + \chi_0 (\gamma_0 + \gamma_1) \pm \sqrt{(\alpha_1 \gamma_1 + \chi_0 (\gamma_0 - \gamma_1))^2 + 4\gamma_0 \gamma_1 \gamma_{s}^2} \right\},$$

(3)

$\alpha_1 = (|k - \tau|^2 - |k|^2)/|k|^2$ is the deviation from the exact Bragg condition, $r_s = r'_s + i r''_s = \chi_{\tau} \chi_{-\tau} C_\pi^2$,

$\chi_0 = \chi_0' + i \chi_0''$, $\chi_{\tau}$, $\chi_{-\tau}$ are the crystal complex susceptibilities, $k_{\mu s} = k - \tau + \frac{\omega \delta_{\mu s} N}{c \gamma_0}$ is the photon wave vector propagating in a crystal at a small angle with the vector $k_{B\tau}$,

$$a_{\mu s} = \frac{\Omega}{2\pi c} \int_{-\pi/\Omega}^{\pi/\Omega} dt \ v(t) \cdot \exp \left( i k_{\mu s} e_1 v \int_0^t dt' \theta_1(t') - i n \Omega t + i k_{\mu s} e_3 v/2. \left[ \int_0^t dt' \theta^2(t') - \theta_0^2 t \right] \right),$$

(4)

is the coefficient of Fourier expansion of the function periodic in $t$ with period $2\pi/\Omega$. 


\( v(t) = v \cos(\theta(t))e_3 + v \theta(t) \) is the particle velocity, 
\( \theta = |\theta(\theta_1, \theta_2)| \ll 1, \ \theta_1(t + 2\pi/\Omega) = \theta_1(t), \ \theta_2(0) = \theta_{01}, \ \theta_2 = \theta_{02}, \ \theta_0 = (\theta_{01}, \theta_{02}) \) is the particle entrance angle with respect to the axis given by vector \( e_3 \) (we assume the particle goes through the crystal along the periodic trajectory in the plane specified by the vectors \( e_1 \) and \( (\theta_{02}e_2 + \cos(\theta_0)e_3) \); \( e_1, e_2, e_3 \) stand for the basis vectors of the Cartesian coordinate system originating at the crystal surface irradiated with charged particles),

\[
l_{mn} = \frac{2c}{\omega} \left( \gamma^{-2} + \vartheta^2 + 2\delta_{\mu\nu} - \alpha_1 + \theta_0^2 - \frac{2\theta_{02}c}{\omega} e_2(k - \tau) - \frac{2n\Omega}{\omega} \right)^{-1}
\]

is the complex coherent length of radiation,

\[
\theta_0^2 = \frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} dt \theta^2(t).
\]

In equation (1) \( |(k - \tau)r_{0,\perp}| \ll \omega \vartheta r_{0,\perp}/c, \ \theta_0^2 \) is of the same order of magnitude as the maximum value of the square of the angle \( \theta_0 \) at which the particle penetrates the crystal, the modulus \( \left| \frac{2\theta_{02}c}{\omega} e_2(k - \tau) \right| \ll \theta_0 \vartheta \), where \( \vartheta \) is the angle between the wave vector of the emitted photon \( k \) and the vector \( k_{B_r} \) (the polar angle of radiation). For the following consideration we suppose that \( \omega \vartheta r_{0,\perp}/c \ll 1, \ \theta_0^2 \ll \gamma_{tr}^{-2}, \ 2\theta_{02} \vartheta \ll \gamma_{tr}^{-2} \), where \( \gamma_{tr} = \omega/\omega_L, \ \omega_L \) being the Langmure frequency of the crystal, are fulfilled.

3. Angular distribution.

The scalar product \( e_{r\mu}a_n \) (the index \( \mu \) is dropped here due to a negligible dependence of the Fourier component on the dispersion branch number) will further be written as a sum \( e_{r\mu} = a_{\perp} + e_{r\mu} a_{\parallel} \), the first term of which describes the oscillatory motion of the particle \( (a_{\perp} = a_{n1} \cdot e_1) \) and the second its linear motion in the crystal \( (a_{\parallel} = (a_{n1}, a_{n2}) \). In this case the part of equation (1), proportional to the modulus squared \( |e_{rs} a_{\parallel}|^2 \), should be interpreted as a spectral-angular distribution of PXR emitted by the oscillator and corresponding to the constant component of its velocity.

Let us use the harmonic oscillator with the transverse velocity

\[
v_{\perp} = v \cdot \theta_{01} \cdot e_1 \cdot \cos(\Omega \cdot t + \varphi_0) + v \cdot \theta_{02} e_2
\]

as a model of a relativistic oscillator. We choose \( \theta_{02} = 0, \ \varphi_0 = 0 \) as initial conditions. It is further assumed that the transverse velocity perturbation, determined by particle transverse oscillations in the channel, is small, i.e. the condition \( \frac{k v \theta_{01}}{\Omega} \ll 1 \) is met. Therefore, expansion can be carried out in this small parameter. Then for the Fourier-components \( a_n \) we obtain

\[
a_n^\perp = \frac{1}{2} \theta_{01} \frac{e_3}{c} v \left\{ J_{n+1}(\eta) + J_{n-1}(\eta) - \frac{\omega \theta_{01}^2}{16\Omega} [J_{n-1}(\eta) + J_{n-3}(\eta) - J_{n+1}(\eta) + J_{n+3}(\eta)] \right\},
\]

\[
a_n^\parallel = e_1 \frac{v}{c} \left\{ \left( 1 - \frac{\theta_{01}^2}{4} \right) J_n(\eta) - \frac{\theta_{01}^2}{8} \left( 1 + \frac{\omega}{2\Omega} \right) J_{n-2}(\eta) - \frac{\theta_{01}^2}{8} \left( 1 - \frac{\omega}{2\Omega} \right) J_{n+2}(\eta) \right\},
\]

\( (\eta = \omega t / \Omega) \).
where \( J_n(\eta) \) is the \( n \)-th order Bessel function, \( \eta = \frac{ke_1v}{\Omega} \). Consider dipole approximation. In this case the condition \( \eta < 1 \) should be fulfilled, then \( a_t^\perp = \frac{1}{2}\theta_{01}e_1\frac{v}{c}, \ a_0^\perp = e_3\frac{v}{c} \).

It follows from (1) that the magnitudes of frequencies \( \omega \) and angles \( \vartheta, \varphi \) (\( \varphi \) is the azimuth angle measured from diffraction plane specified by the vectors \( k_B \), and \( \tau \)) of the emitted photon with the wave vector \( \mathbf{k} \) are close to the values at which the real part of the coherent length of radiation \( l_{\mu n} \to \infty \). This condition can be conveniently rewritten as follows

\[
\alpha_1 = D - \chi_0'(\beta - 1) - \beta r_s'/D. \quad (9)
\]

Here \( \beta = \gamma_0/\gamma_1, \ D(\omega_B, \vartheta) = \gamma^{-2} + \gamma_r^{-2} + \vartheta^2 - 2\Omega/\omega_B \). To simplify the treatment, we have dropped \( \chi_0'', r_s'' \). Equation (9) satisfies the condition \( 1/l_{\mu n} = 0 \) either on the first diffraction branch if \( P = D + \beta r_s'/D > 0 \), or on the second one provided \( P < 0 \). For Laue diffraction, the number of a branch where DRO is emitted is determined by the sign of the quantity \( D \) (since \( \beta > 0 \)).

Considering the foregoing and confining ourselves to the Laue diffraction case, provided that the radiation maxima on different harmonics do not overlap, and assuming the frequency \( \omega \) found from \( 1/l_{\mu n} = 0 \) to be approximately equal to \( \omega_B \), we obtain the DRO angular distribution (for \( n = 1 \)) (see Ref. [9])

\[
N_{\text{ad,DRO}} = \frac{\alpha \omega_B\theta_{01}^2}{16\pi c \sin^2 \theta_B} \left( \mathbf{e}_r \mathbf{e}_1 \right)^2 C_s^2 |\chi_r|^2 F(\vartheta) \sum_{\mu = 1}^2 L_{\text{eff}}^{\mu} (\theta(D)\delta_{\mu 1} + (1 - \theta(D))\delta_{\mu 2}), \quad (10)
\]

where all the frequency-dependent terms in the right-hand side of equation (10) are taken for \( \omega = \omega_B, L_{\text{eff}}^{\mu} = L_{\text{abs}}^{\mu} (1 - \exp(-L_0/L_{\text{abs}}^{\mu})), L_{\text{abs}}^{\mu} \) is the photon absorption length

\[
L_{\text{abs}}^{\mu} = \frac{c}{\omega_B |\chi_0''| \beta} \frac{D^2 + \beta r_s'}{(D + \delta_s)^2 + r_s'' - \delta_s^2}, \quad (11)
\]

\[
\delta_s = r_s''/2\chi_0'', \quad F(\vartheta) = \frac{1}{(D^2 + \beta r_s')} + \frac{\Omega}{2\omega_B\sin^2 \theta_B} \frac{(D + \beta r_s'/D)^2}{(D + \beta r_s'/D)^2}. \quad (12)
\]

If the condition \( L_0 < L_{\text{abs}} = \frac{c}{\omega |\chi_0''| \beta} \) is fulfilled, the dependence of (10) on \( \vartheta \) is completely described by the angle function (12).

Consider the behavior of equation (10) at different values of the Bragg frequency \( \omega_B \). We assume to have a beam of charged particles of a fixed energy, then varying the value of \( \theta_B \), i.e. rotating the crystal with respect to the beam incidence direction, we shall obtain different patterns of DRO angular distributions (in [10] we described DRO angular distribution as a function of oscillator energy).

4. DRO angular distribution as a function of bragg angle at a fixed particle energy.

Note that the equation \( D = 0 \) defines the relation between the photon emission angle \( \vartheta \) and the photon frequency for the complex Doppler effect. It can be rewritten as

\[
\omega = \frac{2n\Omega}{\gamma^{-2} + \gamma_r^{-2} + \vartheta^2}. \quad (13)
\]
We would like to recall the basic parameters of the complex Doppler effect following from (13). The frequency \( \omega_\text{D} = \omega_\text{B}^2/(n\Omega) \) and the Lorentz-factor \( \gamma_\text{D} = \omega_\text{L}/(n\Omega) \) (below we shall consider only the first harmonic \( n = 1 \)) determine the point of the complex Doppler effect birth, at \( \gamma = \gamma_\text{D} \) the radiation frequency \( \omega = \omega_\text{B} \), the radiation angle \( \vartheta = 0 \). As \( \gamma \) increases, the upper and lower branches of the complex Doppler effect diverge from the point \( \omega_\text{D} \). When \( \gamma \gg \gamma_\text{D} \) the minimum frequency of the lower branch \( \omega_{\text{min}} = \omega_\text{D}/2 \), the maximum frequency of the upper branch \( \omega_{\text{max}} = 2\Omega\gamma^2 \). We assume that the Bragg frequency is \( \omega_\text{B} \gg \omega_{\text{min}} \). If \( \omega_\text{B} > \omega_{\text{max}} \), then \( D_0 = \gamma^{-2} + \gamma^{-2} + \gamma^2 - 2\Omega/\omega_\text{B} > 0 \) and the latter inequality being fulfilled only at \( \mu = 1 \). The DRO angular distribution is a single maximum strictly aligned with the vector \( k_\text{B} \). As \( \omega_\text{B} \) approaches \( \omega_{\text{max}} \), \( F(\vartheta) \) is of the order of \( 1/2\beta r_s^4 \), the angular width \( \simeq 2\sqrt{\beta r_s} \). In the region \( \omega_\text{B} < \omega_{\text{max}} \) the form of the DRO angular distribution density is considerably different, i.e. a single narrow maximum at the angle \( \vartheta = 0 \) transforms into two maxima (the azimuth angle \( \varphi \) is assumed to be a fixed one). The approximate position of the mentioned maxima in the angular distribution corresponds to the angle \( \vartheta_\text{D} \) obtained from a complex Doppler effect equation \( D_1|\omega=\omega_\text{B}| = 0 \). One of them corresponds to the second dispersion branch and is at angles \( \vartheta < \vartheta_\text{D}(D < 0) \), the other to the branch \( \mu = 1 \) and lies at angles \( \vartheta > \vartheta_\text{D}(D > 0) \),

\[
\vartheta_{1,2} = \sqrt{\vartheta_\text{D}^2 \pm \sqrt{\beta r_s^2/4}} \frac{2 \sin^2 \vartheta_\text{B}}{\Omega/\omega_\text{B}}. \tag{14}
\]

The maximum value of the angular function is still of the order of \( 1/2\beta r_s^4 \).

5. Effect of the beam parameters on angular distribution density.

The foregoing is valid only under ideal conditions when a spread of charged particles over energy \( \Delta \gamma/\gamma \) and an efficient angular spread

\[
\theta_{\text{eff}} = \sqrt{\theta_A^2 + 2|n| e_2|\theta_0| \vartheta}
\]

\((n = k_\text{B}r_\gamma/|k_\text{B}r_\gamma|, )\) satisfy the inequality

\[
\frac{\Delta \gamma}{\gamma} + 2(\theta_{\text{eff}} \gamma)^2/3 \ll \frac{\gamma^2 \sqrt{\beta r_s}}{3} \tag{16}
\]

For experiments on the majority of accelerators \( \frac{\Delta \gamma}{\gamma} > 2(\theta_{\text{eff}} \gamma)^2/3 \) (for \( \gamma \) of the order of 100), therefore, to evaluate angular distribution density when inequality (16) is invalid, one should average (10) over the beam energy spread. In our further calculations we assumed the beam to have normal energy distribution with dispersion \( \sigma_b = \frac{\Delta \gamma}{\gamma \sqrt{\ln 4}} \) and that it averaged

\[
F(\vartheta) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_b} \exp \left( \frac{\Delta \gamma}{\gamma 2\sigma_b^2} \right) F \left( \vartheta, \frac{\Delta \gamma}{\gamma} \right) d \left( \frac{\Delta \gamma}{\gamma} \right).
\]
6. Oscillator kinetics.

As far as the DRO maximum intensity is concerned, one should conduct an experiment at beam energies of 1 - 100 MeV. At such beam energies and at X-radiation energies limited to at most several tens of KeV, the PXR is attenuated due to its threshold character. Below \( \gamma_\text{tr} \) the PXR intensity drops with decreasing energy as \((\gamma/\gamma_\text{tr})^4\). Therefore, DRO can be observed on the PXR "background". At such energies a particle is a quantum oscillator. That is why in further calculations we should allow for the coefficient of population \( Q_n \) of the \( n \) th energy level with a transverse motion energy \( \varepsilon_{n,\perp} \) determining the chosen frequency of transition \( \hbar \omega = \varepsilon_{n,\perp} - \varepsilon_{m,\perp} \) between the levels \( n \) and \( m \). The value of \( \theta_{01}^2/4 \) in (10) can be treated in terms of the quasiclassical theory of the quantum oscillator as an estimate of the value of \( |X_{nm}|^2 \Omega^2 / c^2 \), where \(|X_{nm}|^2 \) is the dipole momentum of the radiative transition \( n \rightarrow m \).

In what follows we shall consider DRO emitted due to the radiative transition between the levels \( 1 \rightarrow 0 \), then the DRO intensity is proportional to \( \int_0^{E_0} Q_1(z)dz \), \( Q_1 \) being population of the first level of the transverse motion energy.

The relation between the population \( Q_1 \) and the crystal thickness is described by the system of equations:

\[
\frac{\partial Q_n}{\partial t} = -\Gamma_n Q_n + \sum_m W_{nm} Q_m,
\]

where \( W_{nm} \) is the probability of a non-radiation transition from the level \( m \) to the level \( n \), \( \Gamma_n = \sum_m W_{nm} \);

\[
W_{nm} = \frac{2 \pi}{\hbar} \sum_{N,p} \langle O | < p_0 | V_{nm} | p > | N > | N | < p | V_{mn} | p_0 > | O > \delta \left( E_N - E_0 + \varepsilon_\parallel(p) - \varepsilon_\parallel(p_0) + \varepsilon_n - \varepsilon_m \right).
\]

Here \( < O | \) and \( < N | \) are ground and excited states of the crystal, \( E_0 \) and \( E_N \) are corresponding energies, \( p, p_0 \) are the starting and final momentum of the electron, parallel to the channeling plane, \( V \) is the potential of electron - crystal interaction, the indices \( m \) and \( n \) imply that the matrix element is taken over the wave eigenfunction of the electron transverse motion in the channel. Summation is made over the crystal excited states \( < N | \) and the momenta \( p \).

In the expression for the \( \delta \) - function argument one can neglect the differences \( E_N - E_0 \) and \( \varepsilon_n - \varepsilon_m \), expand the quantity \( \varepsilon_\parallel(p) - \varepsilon_\parallel(p_0) \) over parameter \( |p - p_0| \), assuming the momentum transfers to be small

\[
\varepsilon_\parallel(p) - \varepsilon_\parallel(p_0) = \frac{(p_2 - p_{02})^2}{2m\gamma} + (p_3 - p_{03})c
\]

and ignore the term \( \frac{(p_2 - p_{02})^2}{2m\gamma} \). Eventually we have for \( W_{mn} \):

\[
W_{mn} = \frac{4e^4n_a}{\hbar^2} \int dq_1 \sum_\tau M_{mn}(q_1 - \tau_1/2) \cdot M_{mn}(\tau_1/2 - q_1) \cdot \tilde{F} (q_1 - \tau_1/2, \tau_1/2 - q_1),
\]

the function \( \tilde{F} \) is expressed in terms of the electron density form factor \( F(q) \) and the amplitude of crystal atoms thermal oscillations \( u \), \( n_a \) is the crystal atoms density, \( e \) is an electron charge,

\[
\tilde{F}(q, q') = \int dq \ (Z^2 - Z F(q) - Z F(q') - F(q) F(q')) . \left( e^{-(q-q')^2u^2/2} - e^{-q^2u^2/2} - e^{-q'^2u^2/2} \right) + (F(q + q') - 1/Z F(q) F(q')) e^{-(q+q')^2u^2/2}
\]
Here q is the vector with the coordinates \((q_1, q_2, 0)\), and \(q'\) is the one with \((-q_1, -q_2, 0)\), \(Z\) is the nucleus charge number, \(M_{mn}(k)\) are the matrix elements:

\[
M_{mn}(k) = \int \psi_n(x) e^{-ikx} \psi_m(x) \, dx.
\]

The wave eigenfunctions of the Schrödinger equation for the potential \(U(x) = U_0/\chi^2(\alpha x)\) are taken as wave functions of transverse motion. The constants \(U_0\) and \(\alpha\) are chosen so that the potential should be close to an interplanar potential obtained from the Moliere model. In figure 1 the populations \(Q_n\) are plotted against the depth of penetration into the crystal. The calculations were done for the 50 MeV electrons channeled in a Si single crystal along the (100) planes. The starting populations were calculated for a beam penetrating the crystal at an angle of \(2.3 \times 10^{-4}\) rad and an angular spread of \(1 \times 10^{-4}\) rad.

![Graph](image.png)

Fig. 1. — Level population as a function of depth of electron penetration into the crystal, the beam angular spread \(-10^{-4}\) rad, the angular at which the beam penetrates the crystal \(-2.3 \times 10^{-4}\) rad. 
\(Q_1\) — population of the first level, \(Q_2\) — population of the second level et al.

7. Experimental procedure.

Except for DRO parametric X-radiation and bremsstrahlung diffracted by atomic planes are also emitted into diffraction maximum. For \(\omega_B \gtrsim \omega_{\text{max}}\) the DRO intensity will be compared to the angular density of bremsstrahlung at the angle \(\theta = 0\), for \(\omega_B < \omega_{\text{max}}\) one should try to discover a relatively narrow DRO maximum on the PXR “background”, where the distribution
The maximum is attained at an angle \( \theta_{\text{ph}} \approx \sqrt{\gamma^{-2} + \gamma_{\text{tr}}^{-2}} \). The calculations are meant for an accelerator whose basic properties are given in reference \[11\]: the electron energy 50 MeV, the beam angular spread 10^{-4} \text{ rad}, the energy spread for 50 MeV \( \frac{\Delta \gamma}{\gamma} = 0.5\% \). We suggest the following experimental procedure: the beam moves under planar channeling conditions, e.g. between the (100) planes, the radiation being recorded at an angle equal to the two Bragg angles relative to the longitudinal velocity of \( e^- \) The X-radiation is diffracted by the (110) planes (see Fig. 2). The Bragg angle is changed by the crystal rotation in the diffraction plane. One measurement will be made at \( \omega_B \approx 2\Omega \gamma^2 \), two others at \( \omega < 2\Omega \gamma^2 \). In the experimental geometry under consideration \( (e_{x+e_{1}})^2 \approx 1 \), \( (e_{x+e_{1}})^2 \approx \theta^2 \sin^2 \varphi T g^2 \theta_B \), i.e. the emitted DRO will actually be \( \sigma \)-polarized, the bremsstrahlung has a mixed polarization. The calculation is done for \( \varphi = \pi/2 \), the PXR acquires \( \sigma \)-polarization (see Ref. [12]). The crystal thickness should be chosen so that, on the one hand, the ratio between DRO and bremsstrahlung angular densities as well as between DRO and PXR should be maximum, and on the other hand, the crystal thickness should be greater than the extinction length to fulfill the dynamic diffraction condition. The intensities of bremsstrahlung, PXR and DRO are respectively proportional to \( L_0^2 \), \( L_0 \) and \( \int_0^{L_0} Q_1(z)dz \). The enhancement of the DRO intensity with increasing \( L \) is less pronounced than that of the bremsstrahlung and PXR intensities since \( Q_1(z) \) is a decreasing function. The crystal thickness was chosen to be \( 1 \times 10^{-3} \text{ cm} \).

Fig. 2. — Geometry of an experiment to observe the diffraction radiation of oscillator.

The basic parameters for the corresponding measurements and maximum values of DRO, PXR and bremsstrahlung are tabulated in table I. The calculations were done for a silicon single crystal. The averaging over the beam spread was performed for the case when the inequality \( (16) \) does not hold, therefore the table contains the column with the value \( 2\gamma^2 \sqrt{\beta} \tau_{\text{tr}}^2 / 3 \). The angular distributions corresponding to the first and second rows of a table are pictured in figures 3a, 3b. As a result of beam energy spread for case \( \omega_B < \omega_{\text{max}} \) two peaks merge together. The bremsstrahlung was calculated from the formula [12].
Table I. — Basic parameters and maximum values of DRO, Bremsstrahlung and PXR angular distributions for three different experimental procedures.

| N° | \( \theta_B \) (°) | \( \omega_B (\times10^6) \) | \( |x_0'| \times10^6 \) | \( L_{\text{abs,chi}} \) | \( \gamma^2 \sqrt{\beta r_s} \times10^4 \) | \( \gamma_0 \), \( \theta_D \ \text{rad} \) | \( N_{\text{ad,DRO}}^{\sigma} \times10^4 \) \( \vartheta = \vartheta_D \) | \( N_{\text{ad,Br}}^{\sigma} \times10^4 \) \( \vartheta = 0 \) | \( N_{\text{ad,PXR}}^{\sigma} \times10^4 \) \( \vartheta = \vartheta_{\text{ph}} \) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.1 | 87.51 | 0.126 | 9.16 | 4.82 | 2827.7 | 0 | 5.74 | 0.287 | 0.233 |
| 2 | 5 | 37.05 | 0.704 | 0.69 | 26.4 | 1192.7 | 0.012 | 1.71 | 0.333 | 0.539 |
| 3 | 10 | 18.60 | 2.80 | 0.09 | 108 | 598.6 | 0.020 | 0.248 | 0.438 | 0.988 |

(*) The frequencies were calculated for diffraction plane \( (220) \).

\[
N_{\text{ad,Br}}^{\sigma} = \frac{\alpha \omega_B L_0 L_{\text{eff}} |x_0'^2 C_s^2 \theta_s^2}{16 \pi \sin^2 \theta_B \left( (\theta^2 + \gamma^{-2} - \chi_0)^2 + \beta r_s' \right)} + \frac{\alpha \theta_s^2 L_0 |x_0'^2 C_s^2}{\pi \sin^2 \theta_B \sqrt{\beta r_s' (\theta^2 + \gamma^{-2} - \chi_0 - \sqrt{\beta r_s'})^2}} + \frac{\alpha \theta_s^2 L_0 |x_0'^2 C_s^2}{\pi \sin^2 \theta_B \sqrt{\beta r_s' (\theta^2 + \gamma^{-2} - \chi_0 + \sqrt{\beta r_s'})^2}}
\]

(17)

where \( \theta_s^2 \) is a root-mean-square angle of multiple scattering of charged particles per unit length, 
\( \theta_s^2 = (E_s / (mc^2 \gamma))^2 \frac{1}{2 L_R} \), 
\( E_s = \sqrt{4 \pi 137 mc^2} \), 
\( L_R \) is the radiation unit length,

\[
L_{\text{eff}} = 2 L_{\text{abs}}^{1s} \left( 1 - L_{\text{abs}}^{1s} / L_0 \left( 1 - \exp \left( -L_0 / L_{\text{abs}}^{1s} \right) \right) \right).
\]

The first term in (17) is bremsstrahlung emitted by charged particle moving with a velocity exceeding that of the generated radiation, the second term is bremsstrahlung associated with photons whose refractive index is under the unity and efficiency of X-radiation reflection from atomic planes is at its maximum.

8. Conclusion.

As follows from table I and figures 3a, 3b the DRO can be detected in experiments on the measurement of the angular distribution. In particular, we suggest that keeping energy and angle \( \theta_B \) unchanged, one should vary the charged particle motion from straightforward to oscillatory by rotating the crystal. Then the crystal should be rotated in the diffraction plane, thereby changing the angle \( \theta_B \). This results in different angular distributions.

One may also conduct an experiment using a narrow slit positioned along the azimuth angle \( \varphi = O(\pi) \) as a radiation collimator. For \( \theta_B \lesssim \pi / 4 \) the PXR becomes linearly polarized \( (s = \sigma) \) and is not emitted at the azimuth angles \( \varphi = O(\pi) \), the DRO is also linearly polarized \( (s = \sigma) \) but is emitted at all values of \( \varphi \).
Fig. 3. — Angular distribution of radiation for the 50 MeV electrons channeled in a Si single crystal along the (100) planes. a) $\omega_B = 81.51$ keV, 1 – DRO, 2 – Bremsstrahlung, b) $\omega_B = 37.05$ keV, 1 – DRO, 2 – PXR.

References