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Critical phase transitions made self-organized : proposed experiments

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Abstract. — In Sornette, *J. Phys. I France* **2** (1992) 2065, a scenario for self-organized critically (SOC) has been proposed according to which SOC relies on a non-linear feedback of the order parameter on the control parameter(s), the amplitude of this feedback being tuned by the spatial correlation length ξ . Implementing such a feedback mechanism, it is possible in principle to convert standard « unstable » critical phase transitions into self-organized critical dynamics. Here, we analyze this idea in more detail and suggest to couple a standard experiment on critical phenomena with some probing radiation or some electronic feedback using a microprocessor or analog device which pushed the temperature or analog control parameter to that value where the susceptibility, the correlation length or the inverse of the decay rate is maximal. The practical realization of the feedback thus corresponds to an optimization of the response of the system under the action of a probe or a disturbance. We discuss liquid-vapor and binary demixion critical points, and briefly the He4 superfluid transition, magnetic systems, and superfluid transitions.

1. Introduction.

Self-organized critically (SOC) has been proposed as a unifying concept describing the dynamics of a vast class of open non-linear spatio-temporal systems, which evolve spontaneously towards a critical state characterized by (1) response functions obeying powerlaws and (2) a self-similar underlying geometry [1]. Notwithstanding a lot of work, the precise physical mechanisms underlying this phenomenon are still not understood : 1) we do not know *a priori* whether a given system submitted to some imposed external conditions will exhibit SOC or will present other kinds of behavior ; 2) apart from the existence of

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« powerlaws » characterizing the system behavior, which are often claimed to be the hallmarks of SOC, it is not understood how to qualify precisely a system as being self-organized critical. In other words, the key features which make a system self-organized critical are not understood and the different universality classes, if they exist, are not known. Since nature presents us with many realizations of systems with a self-similar geometrical or temporal organization [2-4], it seems worthwhile to explore further the SOC concept in an attempt to unravel the physical origin of such remarkable spatio-temporal organizations.

The most basic ingredient behind the concept of SOC is the notion of a feedback mechanism that ensures a steady state in which the system is marginally stable against a disturbance. However, the laws that govern this state are unknown. In reference [5], inspired by this idea of a general feedback mechanism, a precise implementation was proposed according to which the order parameter acts back onto the control parameter. In general, the mechanisms which control the strength of the feedback are extremely intricate and difficult to unravel. For instance, in the sandpile model [1] (see also [6]), it is a subtle function of the fluctuating field of local slopes which are intricately correlated at long range and at long times by the set of avalanches of all sizes. This feedback thus acts locally at various length and time scales and is a subtle cooperative process. In contrast, the physical mechanism described in reference [5] controlling the strength of the feedback process is much more straightforward since it relies on the global correlation properties of the order parameter in the system. It is the value of the correlation length ξ which provides the measure of the amplitude of the feedback process. In other words, the amplitude of the feedback is determined on the basis of *global* spatial correlation properties. These ideas have been illustrated on simple statistical lattice models [5], such as a simple dynamical generalization of the bond percolation problem, on the dynamical thermal fuse model recently introduced [7] and on the 2D Ising model. For instance, in the Ising spin model, the proposed procedure consists in the regulation of the temperature as a function of the value of the correlation length ξ of the spin flip fluctuations estimated geometrically by using the correspondence between the thermal Ising problem and the geometrical bond percolation model [8].

It is interesting to note that there is some similarity between the present approach and the idea of the feedback proposed by Ott, Grebogi and Yorke [9] to lock chaos onto a periodic behavior using the exponential sensitivity of a chaotic system to tiny perturbations. In the same way as for transforming unstable standard critical phase transitions into self-organized critical systems that we propose here, the trick is to make the chosen periodic orbit stable (which is *a priori* unstable) by applying a judicious perturbation to an available *global* system parameter. Recently, the validity of this rather artificial theoretical scheme has been demonstrated experimentally [10].

It could be felt that the above proposed global feedback mechanism for SOC [5] is *ad hoc* and also quite artificial. It is the purpose of the present work to propose realistic experimental situations in which these ideas can be worked out and tested. This would provide genuine experimental situations in which to study a certain class of SOC phenomena.

2. Non-linear feedback of the order parameter on the control parameter in experimental critical phase transitions.

The key idea we want to build on is to start from a well-known standard unstable critical phase transition and to imagine an experimentally realistic mechanism which can make it self-organized. In other words, we look for a physical mechanism which can couple the temperature regulation for instance to the order parameter or its fluctuations. The candidate experimental systems we consider are the critical liquid-gas points of single compounds, critical binary mixture consolute points, the normal-superfluid transition of helium 4, magnetic

systems and the normal-superconductor transitions. These problems have been studied extensively in the past by many physicists, at the experimental, numerical and theoretical level, in many different systems. We do not aim by any means at being exhaustive and will cite only a few references useful for our discussion. Most of our discussion will be centered on critical points associated with fluids. We will then indicate a list of other potentially interesting systems and problems.

2.1 LIQUID-VAPOR CRITICAL POINT OF PURE FLUIDS AND CRITICAL BINARY FLUIDS CONSOLUTE POINTS. — There exist many recent reviews of critical phenomena in fluids [11]. The best characterized systems are pure fluids near their liquid-vapor critical points and binary fluids near liquid-liquid consolute points [12]. Both types of critical points belong to the same universality class as the three-dimensional Ising model. Mixtures with three or more components have been studied and their liquid-liquid phase separation seems to be also related to the Ising model [13]. More recently, the study of critical phenomena in fluids has been extended to binary micellar solutions and multicomponent microemulsion systems (see Ref. [14] and references therein), which constitute interesting potential systems to implement our idea, due to their large « microscopic » molecular scale (100 Å or more).

Light and neutron scattering, electrical conductivity (when possible), transient electric birefringence and ultrasonic absorption are among the main probes used to study these systems. These experimental techniques are thus candidates for providing a physical mechanism which can couple the temperature regulation to the order parameter or its fluctuations. Indeed, let us consider light scattering for instance. The extreme dependence of the isothermic compressibility as a function of the temperature in the vicinity of the liquid-vapor critical point leads to a drastic increase of the scattering power of the solution (so-called « critical opalescence »). A light scattering measurement should thus give directly a reliable quantitative measure of the susceptibility and of the correlation length (see Appendix A) as long as the temperature T is such that the correlation length remains smaller or equal to the wavelength λ of the probing radiation. Then, with a suitable electronic regulation, this should allow one to guide the temperature to its critical value. In this temperature range, an optimization of the susceptibility, i.e. of the intensity of the scattered wave, allows the temperature to be attracted towards T_c , to within $|T - T_c|/T_c \approx (a/\lambda)^{1/\nu}$ (which is of order 10^{-4} typically), where a is a molecular scale (a few angströms for normal fluids). The larger the wavelength of the probing radiation, the closer the approach to the critical temperature. Of course, one will eventually be limited by the finite size of the sample. The idea of this approach is that the maximization of the scattered light organizes the system in a close vicinity of the critical point.

It is also worthwhile to consider other quantities which can be optimized and allow the system to be directed to its critical point. One of these quantities which is readily measurable is the dynamical correlation function of the system obtained from dynamical light scattering measurements [14, 15] (see Appendix B). In a way similar to the case of the susceptibility or the correlation length, monitoring the decay rate at fixed scattering wave number q can be used and exerting a feedback action on the temperature such that the decay rate $\langle \Gamma \rangle$ is decreased to zero will eventually bring the system close to its critical temperature.

In practice, it may be difficult to distinguish whether T is slightly larger or smaller than T_c and this information is important in order to implement the feedback which, at each step, should produce a temperature increment from T to T' such that $\text{sign}(T' - T) = -\text{sign}(T - T_c)$ and such that the convergence to T_c be ensured. One should then use the following procedure devised to make the temperature converge automatically to T_c without knowing beforehand the exact value of T_c . Starting from some temperature T_1 , perform the measurement of χ , ξ or $\langle \Gamma \rangle$ for this temperature, that we note χ_1 , ξ_1 or $\langle \Gamma_1 \rangle$. Then, change

the temperature to a neighboring value T_2 and measure again the corresponding quantities χ_2 , ξ_2 or $\langle \Gamma_2 \rangle$. This allows us to construct the discrete gradients $(\chi_2 - \chi_1)/(T_2 - T_1)$, $(\xi_2 - \xi_1)/(T_2 - T_1)$ or $(\langle \Gamma_2 \rangle - \langle \Gamma_1 \rangle)/(T_2 - T_1)$. Then $\chi_1/(\partial\chi/\partial T)$ gives an estimate of the temperature difference $T_1 - T'_c$, where T'_c is the corresponding estimate of the critical temperature. The feedback process must thus act on the temperature such that this quantity $\chi_1/(\partial\chi/\partial T)$ is subtracted from T_1 . This process is nothing but the first step of Newton's iteration method which converges towards the temperature T such that $1/\chi(T) = 0$, which is by definition the critical temperature T_c . Iterating this procedure on one of the quantities χ , ξ or $\langle \Gamma \rangle$ thus allows it to converge rapidly to T_c .

Another more direct way to implement the feedback effect is to work with a system which tends to absorb the probing radiation. Suppose that an experimental apparatus is devised such that the system is always cooled down slowly. Starting from a temperature above T_c , the temperature decreases until the correlation length ξ becomes comparable to the wavelength λ . In this case, the absorption of light increases dramatically and begins to heat the system. In the limit of weak cooling power and weak light intensity, the system should then stabilize at a temperature close to T_c , as a result of the competition between cooling and heating induced by light absorption.

A general remark is in order concerning the times scales. As the temperature converges towards its critical value, the temperature regulation will have to take into account the increasing thermal inertia of the sample due to the increase of the specific heat :

$$C_v \sim |T - T_c|^{-\alpha} \quad \text{with} \quad \alpha = 0.109 \pm 0.002 \text{ for the 3D Ising universality class.} \quad (1)$$

This implies that, at constant heating power, the heating or cooling duration at each feedback step will increase in proportion to C_v . However, this time scale diverges more slowly than the fluctuation time scale $\langle \Gamma \rangle^{-1} \sim \xi \sim |T - T_c|^{-\nu}$ and should not be a drastic hindrance.

When the system has converged towards its critical point, it is important to be able to measure physical quantities which allow the SOC regime to be tested. All the above quantities can still be measured and the self-similar distribution of density fluctuations is reflected in the fluctuations of the scattered light. In addition, the field exponent δ can now be measured at T_c and corresponds to the response of the order parameter under the action of an external applied field :

$$\delta\rho \sim \delta p^{1/\delta} \quad (2)$$

This critical behavior can be attained for instance by measuring the decrease of the velocity of sound (soft mode) as a function of pressure amplitude as the temperature approached its critical value.

The choice of the experimental system should be made according to the accessibility of the chosen physical variables. For instance, microemulsions and micellar solutions [14, 15] are particularly suitable for direct microscopic observations of the spatial structure of the order parameter concentration field in the critical region since the microscopic molecular scale (the micelle size or the film persistence length [16]) in these systems is about two or three orders of magnitude larger than standard molecular scales.

2.2 SUPERFLUID TRANSITION OF LIQUID He4. — Let us first mention the superfluid transition of liquid He4, which belongs to the 3D X-Y universality class. This transition is well adapted to experimental tests, since among other advantages, it presents relatively small gravitational effects [17] which can easily be corrected for [18], allowing the critical singularities in the near vicinity of the critical point ($|T - T_c|/T_c$ as small as 10^{-8} can be reached) to be measured with

very high accuracy [19]. Also, the existence of a line of critical points (the λ -line) facilitates a systematic study. The accessible physical variables are, among others, the specific heat, the second sound velocity, the superfluid density.. Again, the optimization of the specific heat, of the second sound velocity and of the temperature derivative of the superfluid density allows the system to be attracted to its critical point. An interesting perturbation to study in the spirit of SOC is caused by a small velocity localized flow.

2.3 MAGNETISM. — Magnetism provides another very large class of systems exhibiting critical phase transitions. The richness and the complexity of magnetic systems arise from the many terms of different symmetries, spatial forms and magnitudes which enter realistic Hamiltonians for magnetic materials [20]. A useful accessible physical variable is the order parameter and its fluctuations obtained for instance by neutron scattering. The problem is then somewhat analogous to that of the measurements of the density fluctuations in fluid systems by light scattering.

2.4 SUPERCONDUCTIVITY. — The superconductivity transition is another very interesting example [21], especially so since the recent discovery of high T_c -superconductors (see [22] for instance and Refs. therein). The drastic change of the resistivity at the critical temperature offers a natural way to exert a feedback on the temperature parameter in order to attract the system to its critical point. In superconductors of type II, the (Abrikosov vortex phase to normal) transition at h_{c2} is also second order and can be used for our problem. In this case, we have to deal with complex systems of vortex lines and their relaxation as a function of some disturbance. This system has recently been proposed as being related to SOC [23]. A feeling for this analogy is obtained by using the sandpile picture proposed by de Gennes (see [21] p. 83) to describe the metastable « critical states » of a superconductor submitted to a magnetic field of less than the value necessary to reach the Abrikosov regime, in which the vortex lines are pinned by random impurities. Also, polycrystals or superconducting networks, made of grains or elements having different transition temperature T_c distributed according to some fixed quenched distribution, constitute systems related to the thermal fuse model discussed in [5] (see also [7]) in the sense that a global connectivity constraint will control the states of the individual elements. Here, it concerns the percolation of a subset of grains or elements which become superconducting [24] yielding a vanishing value of the total resistance of the system. We can also make the problem more complex and consider that each grain or element is coupled to a Peltier cell and that this cell is polarized in such a way that it pumps heat out of the grain or the element and thus decreases its temperature when the resistivity is finite (high temperature phase of the superconductor grain or element), and *vice versa* when the resistivity is vanishing (low temperature phase). Possible relations with Josephson junction arrays can also be considered. The normal-superconductor transition is particularly interesting due to the great sensitivity of the superconducting properties on external perturbations, such as electrons, phonons, light or particle fluxes. Paradoxically, note that a great sensitivity implies a better control for the feedback process.

A more direct way to reach, in a self-organized way, the normal-superconducting critical point could be to use the heating power of the Joule effect itself in the vicinity of the critical point to counterbalance a systematic slow cooling imposed on the sample. Indeed, suppose that a constance voltage is applied on the system. Then, the heating power of the Joule effect is inversely proportional to the sample resistance. Therefore, it should become large only very close to the critical temperature and could then stabilize the temperature by counteracting the systematic weak cooling. Another way to implement the feedback is in the manner described above for the critical binary mixtures, using the critical fluctuations of the order parameter observed recently [25]. Indeed, such fluctuations, which produce small transient regions of the

superconducting phase and which give rise to an anomalous increase in the normal-state conductivity known as paraconductivity, are direct probes of critical phenomena in general and a stringent signature of the approach to the superconducting critical point. Their recent observation in 3D systems [25] makes feasible an experiment using these fluctuations as a probe to direct the temperature to its critical value.

2.5 CHEMICAL FLUIDS UNDER PRESSURE IN POROUS SYSTEMS. — These ideas could also be applied to the study of chemical fluids under pressure in porous materials (see [26] for instance). A competition appears between hydraulic fracturing controlled by the fluid pressure which tends to create pores by rupture and chemical deposition which tends to close the pores. Here again, the fluid pressure is transmitted across the system only above the global percolation threshold. Experiments in the laboratory could be designed to test these ideas on porous rocks in pressure cells filled with chemical agents.

3. Concluding remarks.

We have considered experimental systems which could be used to implement the idea of a global feedback mechanism for Self-Organized Criticality (SOC), which proceeds by a non-linear dynamical feedback of the order parameter onto the control parameter, this dynamics being controlled by the value of the spatial correlation length.

The main idea, that can be outlined looking back at the above list of systems and problems, is the notion that the practical realization of the feedback corresponds to an *optimization* of the response of the system under the action of a probe or disturbance. It would be quite interesting to generalize this idea to the systems and numerical models which have been proposed as examples of SOC. It is indeed an intriguing idea that the attracting nature of the critical point in SOC could be seen as an optimization process in complex systems [27].

Finally, it is interesting to note that the physical examples which have been listed may have useful practical applications. Indeed, stable self-organizing critical systems constitute a new class of smart materials [28] (the term « smart » implies the quality of the material to adjust itself in response to an external perturbation), offering potentially interesting properties, for instance, for furtivity problems, or for detection applications (using their high sensitivity with respect to external perturbations).

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Appendix A.

Light scattering by critical fluids and determination of the susceptibility and the correlation length.

A total intensity measurement can be made which monitors the scattered wave intensity $dh/d\Omega$ in a solid angle $d\Omega$ or the total intensity of light $I_1(L)$ which is transmitted across the sample of thickness L in the direction of the incident plane wave. $I_1(\Omega, L)$ is given by

$$I_1(L) = I_0 \exp \{ - h(\Omega = 4 \pi) L \} , \quad (A1)$$

valid for a correlation length ξ less than the wavelength λ . The extinction coefficient is

$$h(\Omega) = \int_{\Omega} (dh/d\Omega) d\Omega . \quad (A2)$$

The integral is carried over the solid angle Ω and the intensity $dh/d\Omega$ scattered in the solid angle $d\Omega$ is given by [29]

$$dh/d\Omega = \frac{\omega^4}{32 \pi^2 c^4} \left(\frac{\partial \varepsilon}{\partial \rho} \right)_T \left[\frac{1}{\rho} \left(\frac{\partial p}{\partial \rho} \right)_T \right]^{-1} \frac{(1 + \cos^2 \theta)}{1 + q^2 \xi^2} \quad (\text{A3})$$

One often finds an alternative definition according to which

$$h(\Omega = 4 \pi) = 2/\ell_e, \quad (\text{A4})$$

where ℓ_e is the so-called elastic mean free path [30]. The scattering wavenumber q reads

$$q = (4 \pi / \lambda) \sin (\theta / 2). \quad (\text{A5})$$

λ is the wavelength, ω is the wave pulsation, c is the wave velocity in the medium, θ the scattering angle, $d\Omega = 2 \pi \sin \theta d\theta$ is the differential solid angle in the direction $d\theta$, ε is the dielectric coefficient, ρ is the density, p is the pressure and ξ is the correlation length of the density fluctuations. Note that the susceptibility $\chi = -\rho^{-1} \partial \rho / \partial p|_T$ diverges at the critical point as

$$\chi \sim |T - T_c|^{-\gamma}, \quad (\text{A6})$$

with $\gamma = 1.240 \pm 0.002$ for the 3D Ising universality class [31] relevant for these systems. Therefore, the extinction coefficient increases dramatically on the approach to the critical temperature and the transmitted wave along the initial direction is very rapidly damped out. The term $(1 + \cos^2 \theta)/(1 + q^2 \xi^2)$ in equation (A3) presents a sharp maximum for $\theta = 0$, i.e. most of the wave is scattered near the forward direction. The $(1 + q^2 \xi^2)^{-1}$ dependence in equation (A3) is called the law of Ornstein and Zernike [11a]. The q -dependence of the intensity $dh/d\Omega$ scattered in the solid angle $d\Omega$ gives access to the correlation length ξ whereas its magnitude is proportional to the susceptibility $\chi = -\rho^{-1} \partial \rho / \partial p|_T$ (also called osmotic compressibility for a binary mixture).

Two regimes must be distinguished :

i) For $\xi < \lambda$, $h(\Omega = 4 \pi) = 2/\ell_e$ scales as

$$h(\Omega = 4 \pi) \approx C \frac{\omega^4}{32 \pi^3 c^4} \left(\frac{\partial \varepsilon}{\partial \rho} \right)_T \left[\frac{1}{\rho} \left(\frac{\partial p}{\partial \rho} \right)_T \right]^{-1} \sim |T - T_c|^{-\gamma} \quad (\text{A7})$$

where C is a numerical factor. The measure of h , either for total transmission or for light scattering at specific angles, thus provides a direct measurement of the susceptibility [32]. In order to attract the system towards T_c , it suffices to construct an electronic device, for instance coupled to a Peltier cell, which modifies the temperature such that the susceptibility, i.e. the scattered light signal, is maximized.

ii) As T approaches T_c , ξ becomes eventually larger than λ . This regime is characterized by a large turbidity and multiple scattering processes. In this multiple diffusive regime, the transport of the wave energy through the solution is best characterized by the so-called transport mean free path given by [30]

$$2/\ell_t = \int_{\Omega = 4 \pi} (dh/d\Omega)(1 - \cos \theta) d\Omega. \quad (\text{A8a})$$

Note that equation (A8) differs from equation (A2) only *via* the factor $(1 - \cos \theta)$ in the integrand, which puts a zero weight to the forward scattering and gives a maximum weight to

the backward scattering. This additional factor takes into account the fact that the forward scattering does not correspond to a true scattering event [30]. Note that this factor suppresses the logarithmic divergence at $\theta = 0$ in $h(\Omega = 4\pi) = 2/\ell_c$. Performing the integral in expression (A8a) leads to

$$\frac{2}{\ell_t} \approx C' \frac{\omega^4}{c^4} \left(\frac{\partial \varepsilon}{\partial \rho} \right)_T \left[\frac{1}{\rho} \left(\frac{\partial p}{\partial \rho} \right)_T \right]^{-1} \left(\frac{\lambda}{\xi} \right)^2 \quad (\text{A8b})$$

to leading order in λ/ξ . C' is a pure number. Equation (A8b) predicts that $2/\ell_t$ becomes proportional to $\chi \xi^{-2} \sim |T - T_c|^{-\gamma+2\nu} = |T - T_c|^{+\eta\nu}$, using the scaling relation $\gamma = (2 - \eta)\nu$, where $\eta \approx 0.03$ is the anomalous exponent of the correlation function of the order parameter and $\nu \approx 0.63$ is the exponent of the correlation length for the 3D Ising universality class. The exponent $\eta\nu \approx 0.02$ is positive and extremely small, which indicates that $2/\ell_t$ saturates and even begins to decrease slowly again when T becomes extremely close to T_c . This saturation effect has been observed experimentally [32]. In this regime where ℓ_t is smaller than the size L of the sample, total intensity measurements can still be made and provide an estimate of the correlation length ξ , using the experimental methods developed recently in the multiple scattering regime (see for instance [33] and references therein).

Appendix B.

The decay rate.

The correlation function of the scattered electric field decays exponentially with a characteristic decay rate $\langle \Gamma \rangle$ given by [34]

$$\langle \Gamma \rangle = \frac{k_B T}{16 \eta_s} q^3 f \left(\frac{1}{q\xi} \right) \quad (\text{B1})$$

where η_s is the macroscopic shear viscosity and

$$f \left(\frac{1}{q\xi} \right) = \frac{2}{\pi} \left[\frac{1}{q\xi} + \frac{1}{q^3 \xi^3} + \left(1 - \frac{1}{q^4 \xi^4} \right) \tan^{-1} q\xi \right]. \quad (\text{B2})$$

The hydrodynamic mean diffusion coefficient D_H is obtained as the limit of $\langle \Gamma \rangle/q^2 = D_H[1 + 3q^2 \xi^2/5 + \dots]$ as q goes to zero and reads

$$D_H = c(k_B T)/(6\pi\eta_s\xi), \quad \text{with } c \approx 1. \quad (\text{B3})$$

When ξ becomes comparable to λ , multiple scattering events become significant and the correlation function of the scattered electric field no longer decays exponentially [35]. D_H and ξ can still be extracted but one has to use novel multiple scattering techniques and fit the experimental results with computations which take the multiple scattering processes fully into account [35]. Thus, monitoring the decay rate at fixed q can be used and exerting a feedback action on the temperature such that $\langle \Gamma \rangle$ is decreased will eventually bring the system close to its critical temperature.

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