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The remanent magnetization in Spin-Glass models

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Abstract. — In this work we study the remanent magnetization in various spin glass models after the removing of a strong magnetic field. We compare the behavior of the remanent magnetization of infinite-range models and of more realistic short-range models. In both cases we find a reasonable agreement with a simple phenomenological law at low temperatures, especially in the case of continuously distributed exchange interactions. The relaxation of the internal energy shows a more complex behavior.

1. Introduction.

In spin glasses at low temperatures it is possible to observe a very slow approach to equilibrium. Indeed if we change the magnetic field at low temperature the magnetization moves very slowly toward the new equilibrium point, which sometimes is reached only after astronomical times. This phenomenon is called magnetic remanence and it has been the object of a very detailed study [1, 3]. When the decay of the remanent magnetization takes place, the internal energy of the system also decreases. This process has also been measured experimentally [2].

Recently particular interest has been raised by the aging phenomenon [4], i.e. by the dependence of the decay of the magnetic remanence on the previous story of the system. It has been argued [5, 6, 7] that the observed aging phenomena can be qualitatively understood in the framework of the usual mean-field theory based on replica symmetry breaking [8], while no apparent explanation can be found in the droplet model approach [9]. Unfortunately these arguments are not fully rigorous as far as we are unable to find the precise predictions of mean-field theory for the off equilibrium dynamics.

It is extremely interesting to compare the result for the remanent magnetization for short-range finite-dimensional models and for long-range models. This comparison is crucial. Indeed the mean-field approximation (which consists in neglecting correlations) becomes exact in the infinite-range models (e.g. the Sherrington-Kirkpatrick model or the random Bethe lattice). The correct form of mean-field theory predicts exactly what happens after the mean-field approximation is done and therefore it should give precise results in the infinite-ranged model. If the experimental data on real systems could be qualitatively understood in the framework of mean-field theory, numerical simulations on long-ranged models should give similar results. On the other hand if numerical simulations on long-ranged models would behave in a qualitatively different way from the experimental data, mean-field theory should be unable to explain the experimental data and new ingredients (e.g. corrections to mean-field theory, renormalization group) should play a vital role in the explanation.

In this paper we start this program by measuring with numerical simulations the decay of the remanent magnetization and the internal energy (the so called excess energy) in some infinite-ranged models and in some short-ranged models (three and four dimensions) in the simplest case where we start from a point very off equilibrium. More precisely we apply to our sample a very strong magnetic field (i.e. larger than the saturation field), we suddenly remove it and we observe the subsequent remanent decay of the magnetization. We do not address here the behavior of the remanent magnetization for weak applied fields [10, 11] and the related aging effect, which we hope to study in a future work and which should display a rather interesting phenomenology.

Although the behavior of the remanent magnetization, especially in the low temperature region is sensitive to the details of the model, we do not find qualitative differences between infinite-range models and short-range ones (and also with real experiments). We can thus conclude that it is possible to explain in mean-field theory the phenomenon of remanent magnetization. To find such an explanation is an extremely interesting and open task, which we hope is not out of reach.

2. General considerations on remanence.

The simplest numerical experiment in which remanent magnetization can be measured consists in starting from a system at thermal equilibrium at a temperature T and magnetic field h and suddenly change the magnetic field to a new value $h - \delta h$. If δh is not very small the system goes to a strongly non equilibrium state and the results should not depend on how well the system has been thermalized before changing the magnetic field, i.e. on the waiting time (t_W) spent in the initial conditions.

Only if δh is very small, the system is not carried too much out of equilibrium by the variation of the magnetic field and we become sensitive to small deviations of the system from perfect thermal equilibrium at the initial time. Only in this regime, which we do not attempt to study now, we observe variations of the decay in the magnetic remanence with the waiting time (t_W). As we have already stated in the introduction this very interesting regime is not covered by our present study and we limit ourselves to consider only the case of very large δh.

The object of our study is the quantity $m(t)$ that represents the remanent magnetization as a function of the time (the field is changed at $t = 0$). The phenomenon of remanent magnetization consists in the slow decay of the quantity $\delta m(t) = m(t) - m(\infty)$ at large times. Indeed at low temperatures and large times $m(t)$ slowly decays toward zero. The experimental behavior of the remanent magnetization in the low temperature region can be well fitted by the simple form [12]

$$m(t) = \text{const} \,.\, M\left(- T \ln (t/\tau)/T_c\right) , \tag{1}$$

where the function $M(u)$ seems to be approximatively an exponential for not too large values of its argument, τ is a microscopic time (typically in spin glasses τ is measured to be of the order of 10^{-13} seconds) and we stay in the regime where t is much larger than τ. The temperature T_c is the freezing temperature.

Also for the decay of the excess energy it has been found [13] that the following scaling law is satisfied

$$\dot{Q} \equiv \frac{\partial Q}{\partial t} \sim \frac{T}{t} M'(u), \tag{2}$$

where $\dot{Q}$ is the dissipated heat per unit time. Also the excess energy is a function of $T \ln (t/\tau)$.

We stress (to avoid misunderstanding with other theoreticians) that we are interested in the regime in which $m(t)/m(0)$ is not too small. It is quite possible that in a really asymptotic regime (e.g. where $m(t)/m(0) < 10^{-4}$) the behavior of $m(t)$ is dominated by the rare ferromagnetic regions which by chance are present in spin glasses (these regions could give a contribution to $m(t)$ that decays extremely slowly, i.e. as $\ln (t)^{-3/2}$). It seems however that this regime sets in at very large times and we are not interested in it.

Only if

$$M(u) = \exp(\alpha u), \tag{3}$$

we have a perfect power law decay, i.e.

$$m(t) = \text{const}.\ (t/\tau)^{-\alpha T/T_c} = \text{const}.\exp(-\alpha T/T_c \ln (t/\tau)), \tag{4}$$

It is quite possible that the function $M(u)$ decays faster than an exponential for larger values of u and a stretched exponential behavior sets in at very large times. Indeed, in some cases there are deviations from a simple power law and it seems to be an approximation valid only for short times. We believe that the scaling law equation (1) is valid on much more general grounds and theoretical arguments which support it can be found in references [15, 14]. In this paper we concentrate our attention on the region of not too large times, i.e. where a simple power law can be observed.

Also if we are not at very small temperatures we can fit the remanent magnetization as :

$$m(t) = \text{const}.\ (t/\tau)^{-a(T)} = \text{const}.\exp(-a(T) \ln (t/\tau)). \tag{5}$$

Experiments are done in the region where t/τ is quite large and not too small values of $m(t)$. Therefore remanent magnetization can be observed only if $a(T)$ is relatively small (more precisely if $a(T) \ln (t/\tau)$ is not much larger than one).

Experimentally one finds in the low temperature region (at least for temperatures smaller than $0.3\ T_c$ [14], where T_c is the measured critical temperature, defined as the point where the non linear susceptibility diverges) that $a(T)$ is remarkably linear and extrapolates to zero at zero temperature.

In a particular material AuFe (5-6 %), which is one of the RKKY spin glasses most intensively studied, one finds [16]

$$a(T) = \alpha T/T_c, \tag{6}$$

with $\alpha = 0.41$ and, quite remarkably the time τ is very weakly temperature dependent and it is of the same order of magnitude as the microscopic time for single spin flip ($\tau \sim 10^{-13}$ seconds).

The scaling laws equation (1) and equation (2) may be correct at very low temperatures, only in models where the distribution of the coupling constant strength is continuous. Indeed if the coupling among spins may take only a discrete number of values (and also the spins are discrete, e.g. Ising spins), there will be a minimum (in absolute value) non zero microscopic field. At very low temperatures all spins will orient themselves in the direction of the microscopic field, or will randomly move if the microscopic field is zero (spins *fou* or idle spins). The probability that they will not orient with respect to the temperature will be proportional to $\exp(-\beta\,\Delta E)$, with ΔE proportional to the minimum microscopic field.

In this situation we expect that $m(t)$ at finite small temperature should coincide with $m(t)$ at zero temperature in the region of time $t < \exp(\beta\,\Delta E)$. This result is clearly incompatible with the scaling law equation (1). Therefore for systems with discrete values of the couplings some violations should be present at low temperature. We mention here two possibilities : a) The function $a(T)$ goes to zero faster than linear at low temperatures (e.g. quadratically). b) The characteristic time τ diverges at low temperature, e.g. as $\exp(\beta\,\Delta E)$.

However it is quite possible (and we will see that this happens in some cases) that the scaling law is correct in a wide range of temperatures. We stress that we stay in the situation where we start very far away from equilibrium, which corresponds to removing a magnetic field larger than the saturation field, i.e. a very strong magnetic field. In the opposite regime, very weak changes in h, where aging is present, the relaxation can be very different and a much slower approach towards equilibrium is expected.

3. Theoretical results for infinite range models.

We consider a simple spin glass of Ising type where the Hamiltonian at zero magnetic field is given by

$$H = -1/2 \sum_{i,k=1,N} J_{i,k}\,\sigma_i\,\sigma_k\,, \tag{7}$$

where N is the number of spins and the spins σ take the values ± 1.

The model depends on the choice of the probability distribution of the J's. Short-ranged models are obtained when the J are different from zero only if the points i, k are nearest neighbor, or (more generally) are at a finite distance.

In the case of infinite-ranged models there is no *a priori* distance among the points. A simple form of the infinite-ranged model (the so called fixed coordination number [19]) can be obtained when for each i exactly z $J_{i,k}$ are different from zero. In other words, for each point there are z neighbors. If these pairs are chosen random, the model is defined on a random lattice with fixed coordination number z.

Each instance of the Hamiltonian depends on the connection matrix (i.e. which J's are not zero) and of the values of the non zero J's. In the random lattice case we must specify the distribution probability ($P(J)$) of the non zero J's, which for simplicity we assume to be symmetric under the exchange of J into $-J$. Various choices are possible (e.g. $J_{i,k} \pm 1$, with equal probability, or a continuous distribution).

Generally speaking in the termodynamic limit the mean-field approximation (i.e. to neglect the correlations) is valid in these random lattices and the high-temperature expansion can be resumed in a closed form. These models can be studied using the replica approach. The mean-field approximation is exact in these models so that the appropriate form of mean-field theory should describe correctly these models. In the present model it is widely believed that the approach to mean-field theory, based on the breaking of replica symmetry (using a hierarchical matrix), produces the correct results also in the low-temperature phase [18]. One finds a transition from a replica symmetric high-temperature phase to a replica broken low-temperature phase. The critical temperature can be found to be given by

$$(z-1)\int \mathrm{d}P(J)\,(\mathrm{th}\,(\beta_c J)^2) \equiv \overline{(z-1)\,(\mathrm{th}\,(\beta_c J)^2)} = 1\,, \tag{8}$$

which for large values of z reduces to

$$(z-1)\,\overline{J^2}\,\beta^2 = 1\,. \tag{9}$$

Indeed in the large z limit all thermodynamic quantities depend only on $\overline{J^2}$. When z goes to infinity (if we set $z = N - 1$ we get precisely the usual SK model) we recover the results of the SK model. From the analytical point of view in the limit z going to infinity the computations are much simpler as far as the central limit theorem can be used. This technical, but unfortunately inevitable point, implies that our control of the usual SK model (i.e. $z = \infty$) is much better than the one for finite z. Indeed at the present moment computations have been done only in the framework of the $1/z$ expansion [17] or near the critical temperature [20]. However it is widely believed that the statics for this model may be computed in the same framework of broken replica theory as it happens in the more usual SK model.

Another model which appeared in the literature is the hypercube model [21] in which the spins are assigned to the vertices of a D dimensional hypercube. The total number of spins (N) is 2^D and the coordination number z is just D. This model should coincide with the usual SK model in the limit D going to infinity, having the advantage that the time needed to compute the Hamiltonian (and to perform one Monte Carlo cycle) grows like $N \ln(N)$ and not like N^2, as in the SK model. The price we pay for this fast convergence is that we have in some quantities finite-size corrections which go to zero like $1/D$ or equivalently likely $1/\ln(N)$, which is much slower that in the usual SK model.

In the broken replica approach one finds that in the glassy phase at low temperature there are many equilibrium states of the system, the total magnetization of each of these states differs from the other and is of order $1/N^{1/2}$ at $h = 0$ [22]. In the replica approach all these states have a similar free-energy density, and they must be considered as equilibrium states. The time for jumping from one of these states to another is supposed to increase as $\exp(N^{\omega})$, with ω close to 1/3 [23]. If we increase the magnetic field some of these states will become metastable (i.e. they will get an increase in the free-energy density) and new states will become the ground states. Changing the magnetic field the system should go a sequence of first-order microtransitions where the discontinuities in the magnetization density are proportional to $N^{-1/2}$

If we do a very small change of the magnetic field (less than $N^{-1/2}$) the susceptibility gets two contributions $\chi = \chi_R + \chi_I$ where : a) χ_R is the susceptibility inside one state (it is given by $\beta(1 - q_{EA})$, which has a peak at the critical temperature, b) χ_I is the irreversible susceptibility which is the contribution to the susceptibility of the possibility of the system of jumping from one equilibrium state to another.

In the mean-field approximation the times needed for a transition from one pure state to another are exponentially large. Let us consider the case where we start from an equilibrium state in presence of a magnetic field δh, which we remove at time zero. If the magnetic field δh is infinitesimal, the system will remain in the original state for not exponentially large times. In this situation, if we measure the time dependence of the magnetization after a change in the magnetic field, we get only the reversible contribution (the so called linear response magnetization, which we denote by $m_L(t)$) unless we go to exponentially large times. In this situation $m(t)$ should go for very large times (which remain smaller than $\exp(N^{\omega})$) to a non zero value (obviously proportional to h). The precise form or the decay of $m_L(t)$ in this region of large times is a very interesting problem on which some results have been obtained. These results are certainly valid for fields δh of order $N^{-1/2}$. Indeed in this case the difference in variation in the total free energy of the systems between different states is of order 1 and the set of equilibrium states does not change when h changes. In the opposite case, where the field δh remains finite in the infinite volume limit, the difference in free energy between the true ground state and the previous ground state becomes macroscopically significant (it is of order $(\delta h)^2 \chi_I/2$). The old ground state becomes a metastable state. It is usually believed that in the infinite-ranged models (and also in realistic models) the dynamics at finite temperature is such that no metastable states are present (i.e. the internal energy density reaches a value near its equilibrium value in a time which does not diverge with N). Therefore the old ground state, which is now a metastable state, should decay toward a stable state in a finite time, which quite likely diverges exponentially when δh goes to zero.

This discussion strongly suggests that in the region where $\chi_I \neq 0$, i.e. in the region where replica symmetry is broken, some kind of remanent magnetization is expected, but its precise form is quite unclear.

The main analytic results obtained up to now for the dynamics near equilibrium for the infinite-ranged model is on the large time behavior of the decay of magnetization in the region of linear response : one finds for large (but not too large) times [24]

$$m_L(t) = \text{const} \,.\, \delta h \,.\, (\tau/t)^{-\nu} + m_I(t) , \tag{10}$$

where the irreversible magnetization is a constant for t much smaller than $\exp(N^{\omega})$. The exponent ν has been computed in the limit where $N \to \infty$; it takes the value 0.5 at the critical temperature and it decreases to about 0.25 at $T = 0$.

Quite recently a very interesting result has been obtained for the decay of the remanent magnetization (with parallel dynamics) when we start from infinite magnetic field (i.e. all spins aligned in the direction of the field) at $t = 0$ and we decrease it suddenly to zero field. Here one finds [25]

$$m(t) = \text{const} \,.\, (\tau/t)^{-c} + m_{\infty} , \tag{11}$$

where m_{∞} is 0.18 and the exponent c is 0.47. It is interesting to note that at exactly zero temperature the magnetization does not go to zero. Indeed the internal energy density goes to a value which is definitively higher than the correct one. We expect that at small non-zero temperatures $m(t)$ should behave as at zero temperature for small times, while the term corresponding to m_{∞} should start to decay. Indeed it would be extremely interesting to generalize these results to finite temperature.

In both cases (infinitesimal or infinite magnetic field at time zero) one has computed analytically the fast decay (exponents being always not far from 1/2) in the transient region before the start of the slow decay. The numerical study of this slow decay (on which no analytic results are unfortunately available) will be the subject of numerical simulations reported in this paper. In order to simplify the analysis and to decrease the number of parameters involved we will consider here only the case where the magnetic field at time zero is very large (strictly speaking infinite). This regime can be found experimentally measuring the decay of the saturated remanent magnetization.

It is certainly worthwhile to study the dependence of the decay of the magnetization on the value of the magnetic field especially for low fields. In this case we should start to see the effects of aging.

4. Numerical simulations of short-range models.

Results on the remanent magnetization for the two-dimensional and three-dimensional Ising spin glass can be found in the literature [27, 26]. It was found that the decay of the remanent magnetization in a wide range of temperatures follows a power law decay

$$m(t) \sim t^{-\alpha} \tag{12}$$

In this studies the remanent magnetization evolves from a very strong non-equilibrium configuration in which all spins are pointing up (which corresponds to a stable configuration for an infinite applied homogeneous field). Over a wide range of temperatures α increases linearly with the temperature. This means that the scaling law equation (1) governs the decay of the remanent magnetization. In case of the internal energy a slow decay is also observed

$$E(t) - E(\infty) \sim t^{-\beta} \tag{13}$$

$E(\infty)$ is the equilibrium energy and the excess energy is defined as $\Delta E = E(t) - E(\infty)$. Fitting numerical data from simulations to equation (13) we are able to extract $E(\infty)$, β and the excess energy. Results already obtained for Ising spin glasses in two and three dimensions show that the decay of excess energy agrees reasonably with the scaling law equation (13) [28].

One alternative way in order to test the scaling law equation (1) is to make fall into the same curve several decays of the remanent magnetization corresponding to different temperatures (the same applies in case of the excess energy). This can be achieved by plotting the logarithm of the remanent magnetization *versus* $T \ln (t)$ where T is the temperature and t is the Monte Carlo time (1 Monte Carlo step corresponds to a sweep over the whole lattice). The fact that all the decays of the remanent magnetization seem to fall into the same curve means that for $t = 1$ all of them have the same initial remanent magnetization and the parameter τ of the scaling law equation (1) corresponds approximately to one Monte Carlo step (then one Monte Carlo step would be the equivalent of the time in which a single spin flips, i.e. from 10^{-11} seconds to 10^{-14} seconds for several spin glasses).

We have let the system relax up to approximately 10^4 Monte Carlo steps. In order not to have a proliferation of points in the plots, we have measured the remanent magnetization and the excess energy every 10 Monte Carlo steps. After, they are averaged over intervals of time (successive powers of 2) in order to distribute them uniformly in a logarithmic time scale. All our Monte Carlo simulations make use of the heat-bath algorithm. Because it is computationally faster we have used discrete ± 1 couplings in all cases (only in the random lattice and for comparison with the discrete case we used a continuous distribution of couplings).

Let us present the results we have found for the decay of the remanent magnetization and the excess energy for two different short-range models (Ising spin glass in three and four dimensions).

In figures 1a and 1b we show the decay of the remanent magnetization and the excess of the internal energy for the three-dimensional binary Ising spin glass at four temperatures ranging from $T = 0.25$ to $T = 1$ (in this model we assume $T_c = 1.2$ [29, 30] for the freezing temperature). A very large size $L = 39$ has been simulated where L is the lattice size. We can see in figure 1a that all the points fall in the same straight line when plotting the remanent magnetization *versus* $T \ln (t)$. Only for very low temperatures ($T = T_c/4$) the scaling law equation (1) seems to be in trouble due to the existence of an energy gap. In this case we find a reasonable agreement with the scaling law equation (1) and the remanent magnetization decays like a power law as given by equation (4) with $\alpha \simeq 0.4$. This value is in qualitative agreement

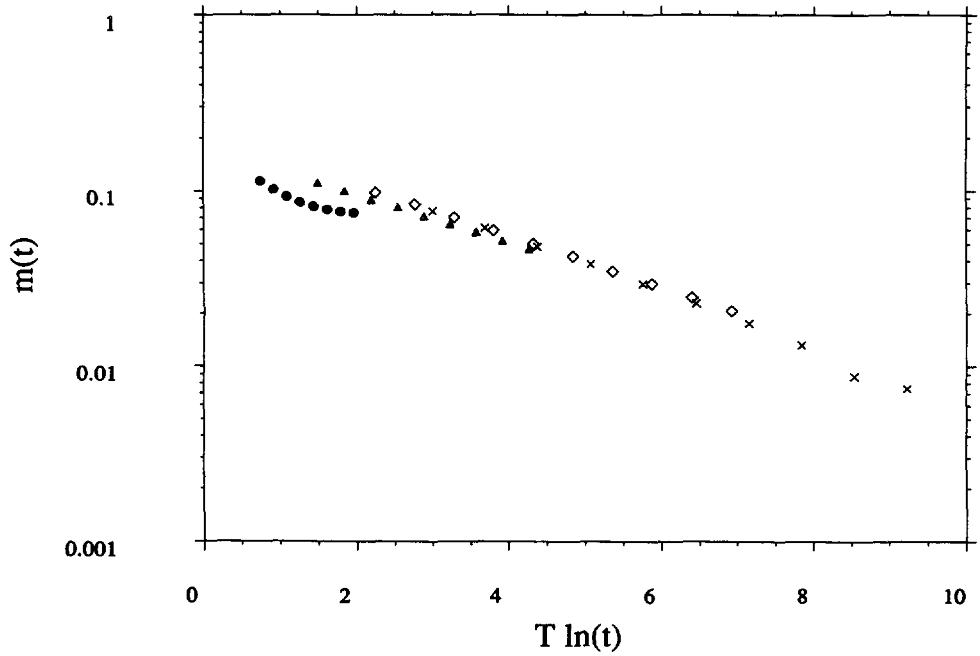

Fig. 1a. — Decay of the remanent magnetization in the binary 3-d Ising spin glass with $L = 39$ and 1 sample at four different temperatures : (•) $T = 0.25$, (△) $T = 0.5$, (◇) $T = 0.75$, (×) $T = 1.0$.

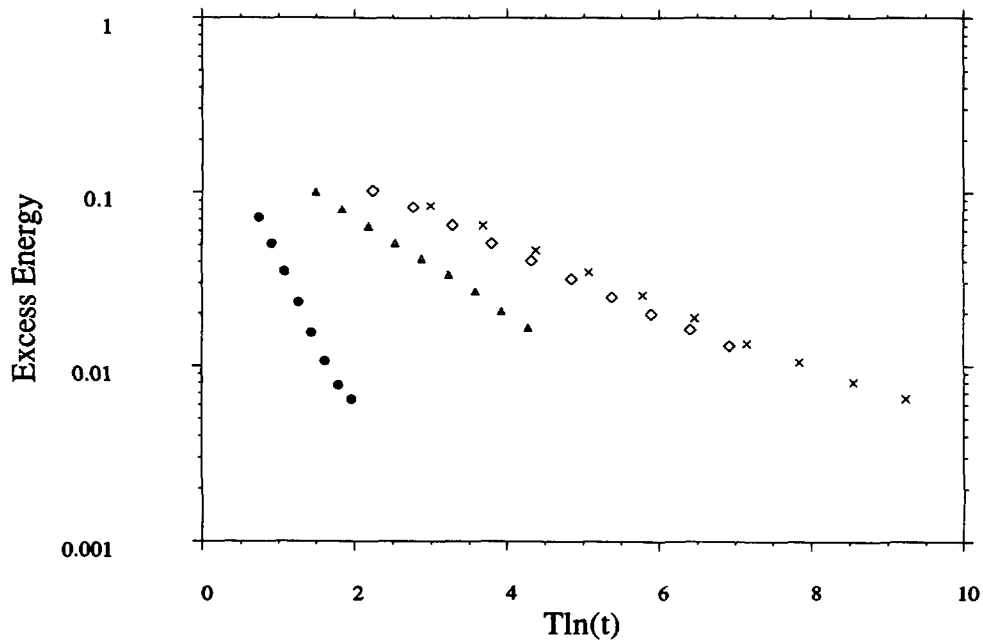

Fig. 1b. — Decay of the excess energy in the binary 3-d Ising spin glass with $L = 39$ and 1 sample. The same temperatures and symbols like in figure 1a.

with experimental data reported on RKKY spin glasses. In figure 1b, we show the decay of the excess energy plotted *versus* $T \ln(t)$. As soon as T is less than 0.5 the scaling law is strongly not fulfilled. This is a consequence of the discrete strength of the couplings. Indeed, the energy relaxation becomes nearly temperature independent at low T.

Figures 2a and 2b show the same plots as in figures 1a and 1b in the four-dimensional Ising spin glass with binary couplings and lattice size $L = 10$. In this case we know that $T_c \simeq 2.02$ [31] and we have studied four different temperatures ranging from $T = 0.5$ to $T = 2.0$. As soon as T is approximately less than $T_c/4 \simeq 0.5$ the data begins to violate the scaling behaviors equations (1) and (2). Again the scaling law is strongly not fulfilled for the excess energy because of the energy gap as we commented in the previous paragraph. Near the critical temperature some small deviations from the scaling laws are also expected (due to possible next order T^2 corrections). Nevertheless, we obtain for α in equation (4) a value close to 0.5. This faster decay of the remanent magnetization suggests that equilibration will be faster in 4d Ising spin glasses than in the 3d case, a result widely accepted [29].

We can summarize the results we have found in case of finite-range models. All of them, and also the results previously obtained by other people for the two-dimensional Ising spin glass, seem to fit well the experimental scaling laws equations (1) and (2) in a wide range of temperatures. If the temperature is near the critical one we can expect small deviations from the scaling laws due to higher T^2 corrections. Moreover, if the temperature is too low we expect that the scaling laws will not be satisfied because of the existence of an energy gap in the case of discrete couplings. This happens at a temperature $\simeq 0.2\, T_c$ when the system is not able to relax and to surmount the energy barriers. Results recently shown in [28] show a remarkable agreement with the scaling laws equations (1) and (2) in case of a continuous strength of the

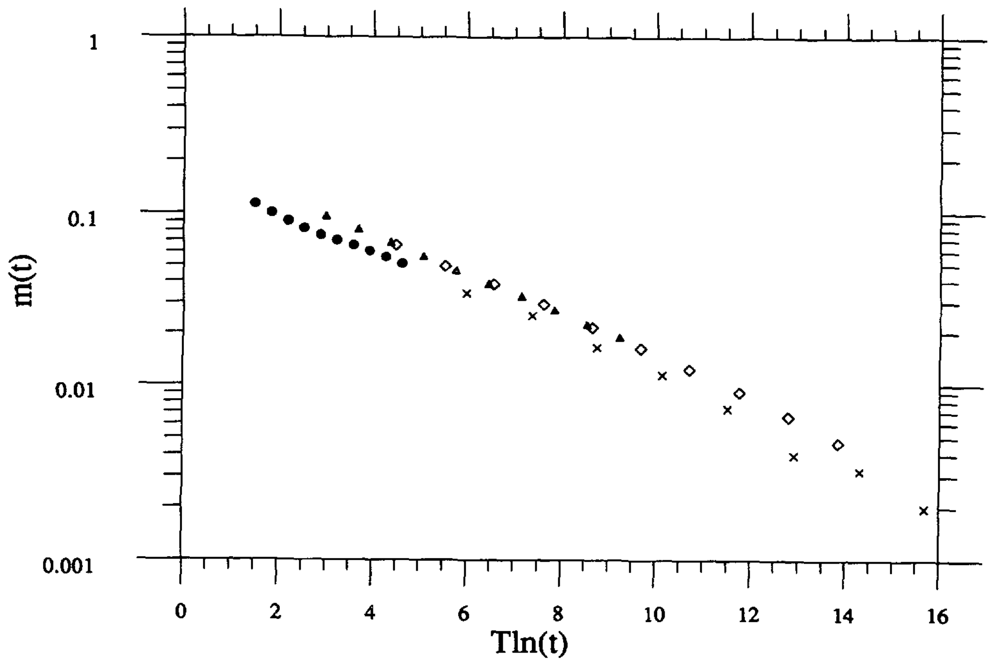

Fig. 2a. — Decay of the remanent magnetization in the binary 4-d Ising spin glass with $L = 10$ and 8 samples at four different temperatures : (•) $T = 0.5$, (△) $T = 1.0$, (◇) $T = 1.5$, (×) $T = 2.0$.

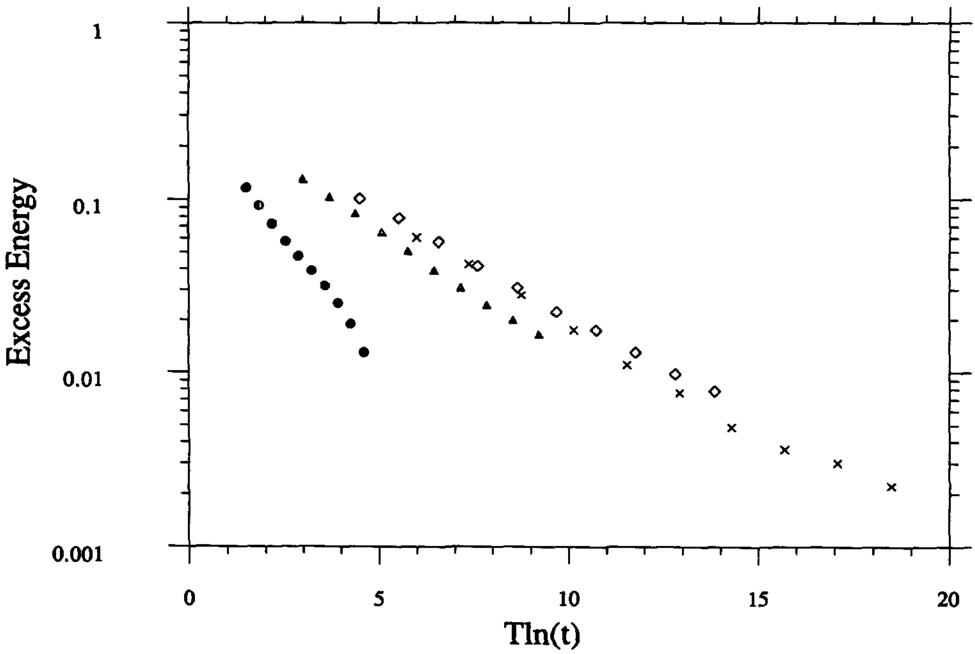

Fig. 2b. — Decay of the excess energy in the binary 4-d Ising spin glass with $L = 10$ and 8 samples. The same temperatures and symbols like in figure 2a.

couplings for the two and three-dimensional spin glasses. As was already discussed in section 3, a strong initial transient is expected to dominate the decay of the remanent magnetization for an increasing period of time as we decrease the temperature. Indeed, for discrete couplings, at zero temperature there will be always a finite remanent magnetization and a finite excess energy and the system will remain trapped in one metastable state. Concerning the energy decay, it is more complex than for the remanent magnetization. The exponent at four dimensions is similar (always for not too small T) to that of the remanent magnetization while in three dimensions it is always (within the scaling region) slightly higher (e.g. around 1.1 times greater).

Once we have outlined the main behavior of the decay of the saturated remanent magnetization and the excess energy for several finite-range models, we can test if the same features are also common to other infinite-ranged spin-glass models. Among them we are particularly interested in the Sherrington-Kirkpatrick (SK) model [32]. Nowadays we have a good theoretical control of the statics and the dynamics in the linear response regime of the SK model and it could be a starting point to understand theoretically long-time dynamics in finite-range models. We will see that several infinite-range models (i.e. SK model, random lattice model and hypercubic cell model) show the same behavior like short-ranged ones.

5. Numerical simulations of infinite-ranged models.

General considerations on the analysis of data already presented in the fourth paragraph of the previous section also apply to this section.

We begin the analysis of long-ranged models studying the random lattice with fixed coordination equal to z. In this case, we present data only for the decay of the remanent magnetization.

Figure 3a shows the decay of the remanent magnetization for $z = 4$ in case of discrete ± 1 couplings and $N = 100\,000$. Four different temperatures ranging from $T = 0.3$ up to $T = 1.25$ have been studied (from Eq. (8) we know $T_c \simeq 1.5$) and the results are in approximate agreement with the scaling law equation (1).

For comparison, figure 3b shows the analog of figure 3a in case of a continuous distribution of couplings (Gaussian) of unit variance. A size $N = 100\,000$ and six different temperatures ranging from 0.15 up to 0.5 have been studied. Even though $T = 0.15$ is a very low temperature the fact that the couplings are continuous avoids the presence of idle spins in the system. The agreement with the scaling law equation (1) is very good but the decay function $M(u)$ differs from an exponential.

Figure 3c shows the decay of the remanent magnetization for case $z = 6$ and discrete ± 1 couplings. For a size $N = 50\,000$ data are shown for five different temperatures from $T = 0.35$ up to $T = 1.0$ ($T_c = 2.08$). There is better agreement with the scaling law equation (1) than in figure 3a because a greater connectivity reduces the energy gap.

The second interesting infinite-ranged model is the hypercubic cell. We have studied a very large size $N = 32\,768$ corresponding to $D = 15$. This model has connectivity equal to its dimension. The bonds are discrete ± 1 with zero mean and variance equal to $1/\sqrt{D}$. Because the connectivity is so great, in the particular case $D = 15$ the energy gap is reduced. For this reason we can extend the scaling behavior equation (1) down to lower temperatures. We present numerical results for a wide range of six temperatures (from $0.15\,T_c$ up to $0.75\,T_c$ with $T_c = 1$ in the $D \to \infty$ limit). Both scaling laws equations (1) and (2) fit well the data (Figs. 4a and 4b) but for the excess energy the scaling law equation (2) breaks down at higher temperatures ($T \simeq 0.25$) than does the scaling law for the remanent magnetization.

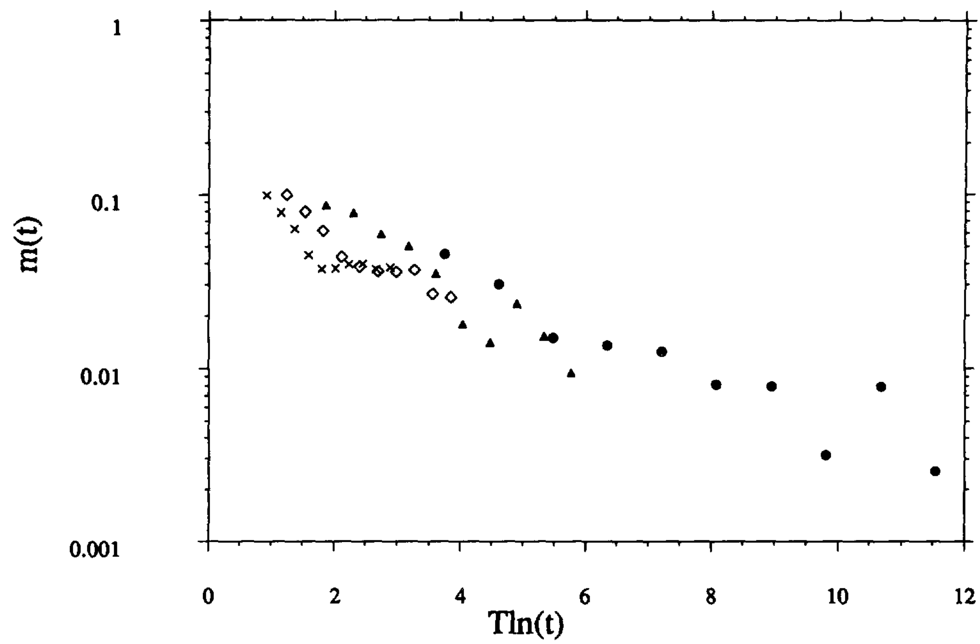

Fig. 3a. — Remanent magnetization in the random lattice model with discrete ± 1 couplings and coordination number $z = 4$. Data are shown for $N = 100\,000$ and 1 sample. The symbols are : (•) $T = 1.25$, (Δ) $T = 0.625$, (◇) $T = 0.42$, (x) $T = 0.3$.

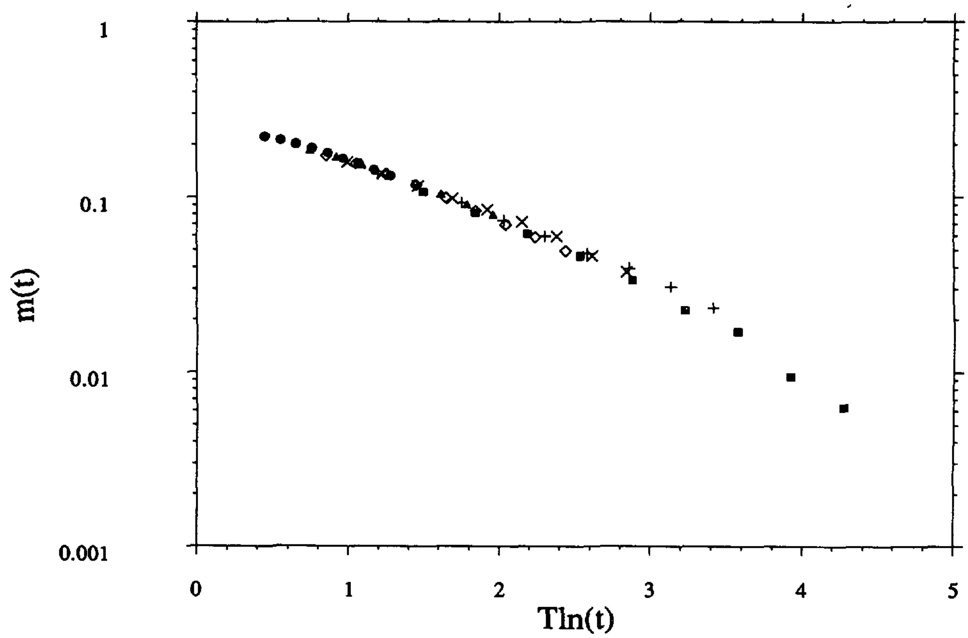

Fig. 3b. — Remanent magnetization in the random lattice model with continuously distributed Gaussian couplings and coordination number $z = 4$. Data are shown for $N = 100\,000$ and 1 sample. The symbols are : (•) $T = 0.15$, (△) $T = 0.25$, (◇) $T = 0.286$, (×) $T = 0.33$, (+) $T = 0.4$, (□) $T = 0.5$.

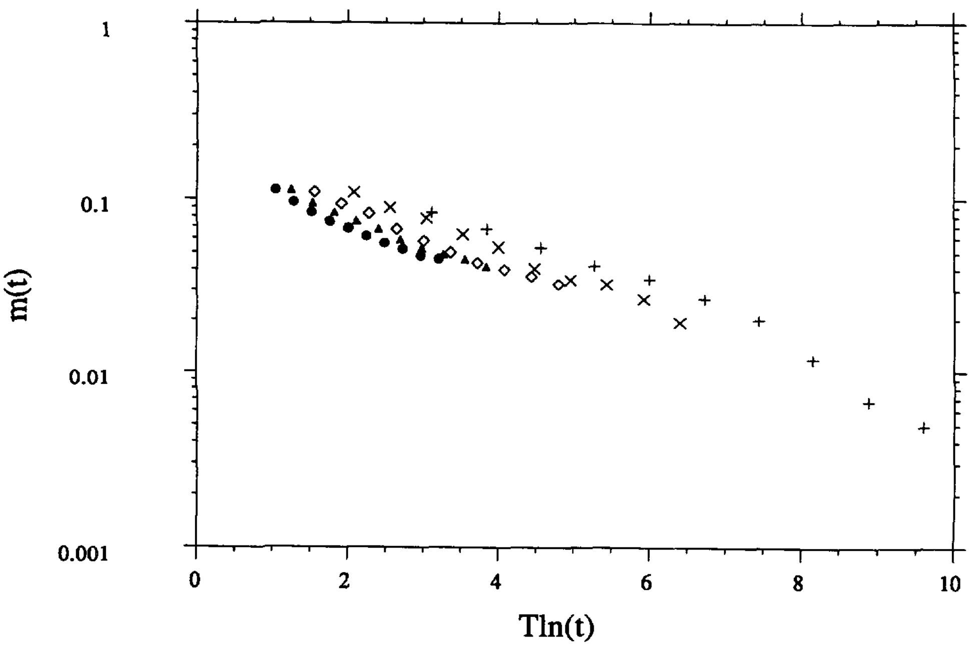

Fig. 3c. — Remanent magnetization in the random lattice model with discrete ± 1 couplings and coordination number $z = 6$. Data are shown for $N = 50\,000$ and 1 sample. The symbols are : (•) $T = 0.35$, (△) $T = 0.41$, (◇) $T = 0.52$, (×) $T = 0.69$, (+) $T = 1.04$.

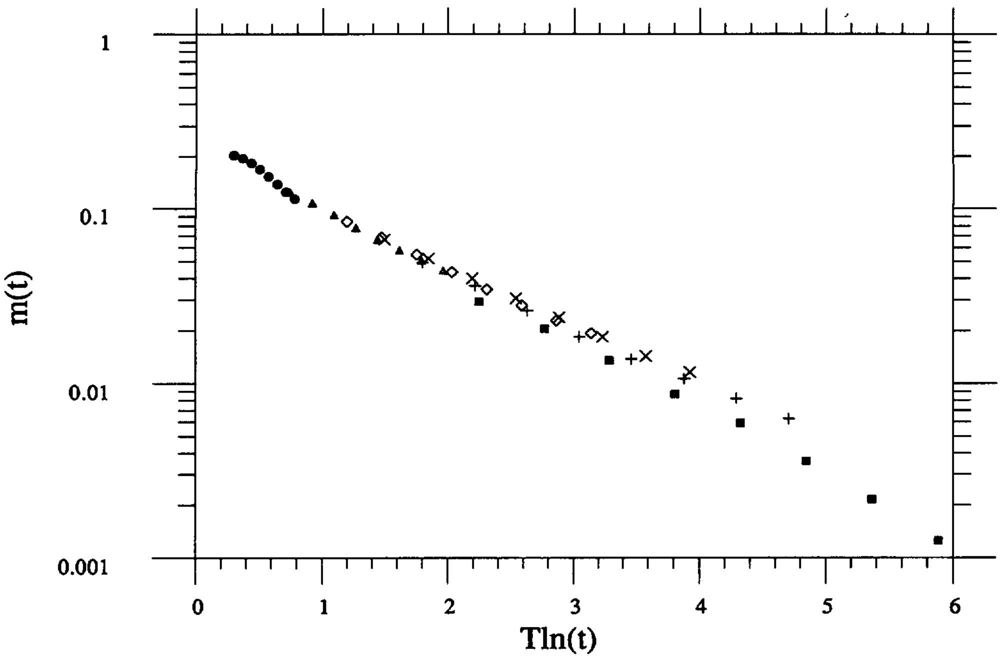

Fig. 4a. — Remanent magnetization in the hypercube model with discrete ± 1 couplings ($D = 15$). Data are averaged over 10 samples. The symbols are : (•) $T = 0.1$, (△) $T = 0.25$, (◇) $T = 0.4$, (×) $T = 0.5$, (+) $T = 0.6$, (□) $T = 0.75$.

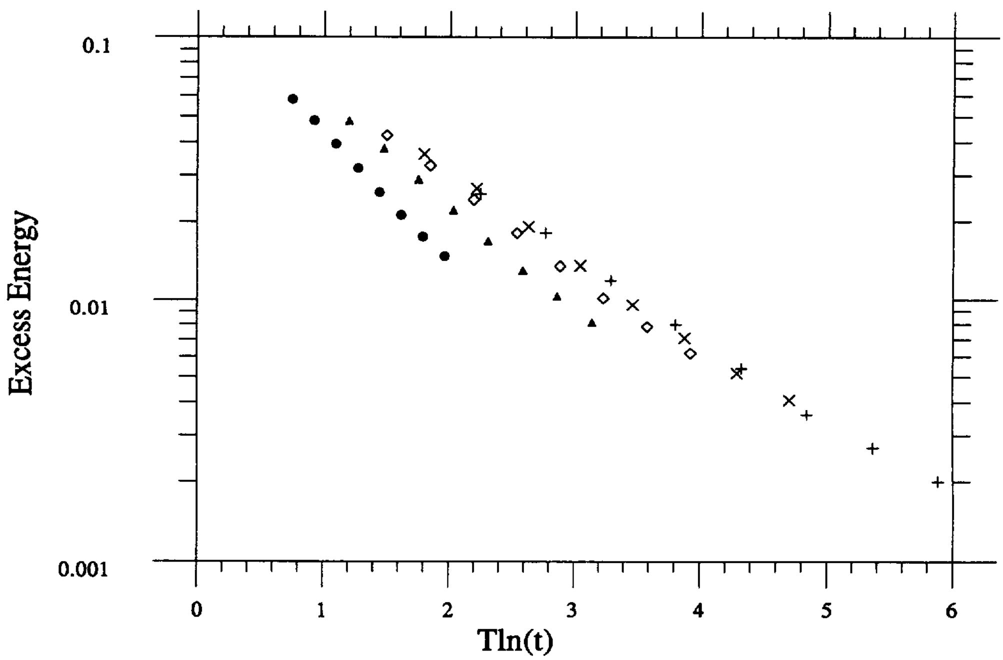

Fig. 4b. — Excess energy in the hypercube model with discrete ± 1 couplings ($D = 15$). Data are averaged over 10 samples. The same temperatures and symbols like in figure 4a. However, data is not shown for $T = 0.75$ because of large error bars when approaching the mean-field critical temperature $T_c = 1$.

We present now the results obtained for the SK model. In the literature some results can be found for the decay of the remanent magnetization and the excess energy in the SK model. W. Kinzel studied [33] the decay of the remanent magnetization and the excess energy in the SK model for several sizes up to $N = 1\,024$ at $T = 0.4$ ($T_c = 1$). His simulations were done using a Gaussian distribution of couplings and for the particular case $N = 512$ he obtained power law decays for both the remanent magnetization and the excess energy. For instance, for the remanent magnetization he obtained the result

$$m(t) = m(\infty) + At^{-\alpha} \tag{14}$$

with $m(\infty) \simeq 0.012$ and $\alpha \simeq 0.38$. Also Fisher and Huse [34] have performed numerical simulations with $N = 512$ spins for the case of a binary distribution of couplings at a slightly higher temperature $T = 0.5$. The power law behavior equation (14) fitted the data well and they found the same residual remanent magnetization $m(\infty) \simeq 0.012$ but the exponent of the decay seemed to be neatly higher $\alpha \simeq 0.6$ in their case. We have studied the decay of the remanent magnetization in case of binary couplings with $N = 512$ spins at $T = 0.5$ and $T = 0.4$. For $T = 0.5$ we obtain $m(\infty) \simeq 0.011$ and $\alpha \simeq 0.63$ in agreement with [34]. In case $T = 0.4$ we obtain $m(\infty) \simeq 0.016$ and $\alpha \simeq 0.58$. The exponent α is different to that obtained by Kinzel $\alpha \simeq 0.38$ because he used a Gaussian distribution of couplings.

In respect to the existence of a non-zero remanent magnetization in the infinite time limit it was argued by Kinzel that $m(\infty) \simeq O\left(\frac{1}{\sqrt{N}}\right)$ because the size of the system is finite and all samples relax from the same initial configuration in which all the spins are pointing up. If this were the case, we would expect that making the size of the system four times greater the magnitude of the residual remanent magnetization in the infinite-time limit shoud be approximately the half. In figure 5 we show the decay of the remanent magnetization for the SK model with $N = 512$ (500 samples) and $N = 2\,048$ (40 samples) at temperature $T = 0.4$ for the case of a binary distribution of couplings. Both of them fit the algebraic power law equation (14). In case $N = 2\,048$ we obtain $m(\infty) \simeq 0.011$ and $\alpha \simeq 0.48$. Our data for $m(\infty)$ (within error bars) seems to be compatible with the fact that the remanent magnetization goes to zero with the size like $O\left(\frac{1}{\sqrt{N}}\right)$. Moreover, the exponent of the decay α varies very much with the size.

In table I we present the parameters for the decay of the remanent magnetization and the excess energy at $T = 0.2$, 0.4, 0.6 in the SK model with size $N = 2\,048$ and 40 samples for each temperature. They are fitted with the usual power laws

$$m(t) = m(\infty) + at^{-\alpha} \tag{15}$$

$$E(t) = E(\infty) + bt^{-\beta} \tag{16}$$

Data for the remanent magnetization at temperatures $T = 0.2$, 0.4, 0.6 are plotted in figure 6 *versus* $T \log t$ (in case of $N = 2\,048$ which is the largest size we have studied). This is the largest size we have been able to study in the SK model. We think that the agreement with the scaling law equation (1) would be better for larger sizes since in this case $m(\infty)$ would be smaller and the exponent α closer to the result in the infinite size limit. The decay of the internal energy shows a more complex behaviour and the $T \log (t)$ behavior is not so clear as in other models.

In order to solve questions concerning the existence of a remanent magnetization in the infinite time limit for the SK model it would be very interesting to solve the dynamical evolution of the SK model in the very off-equilibrium regime (where the fluctuation-

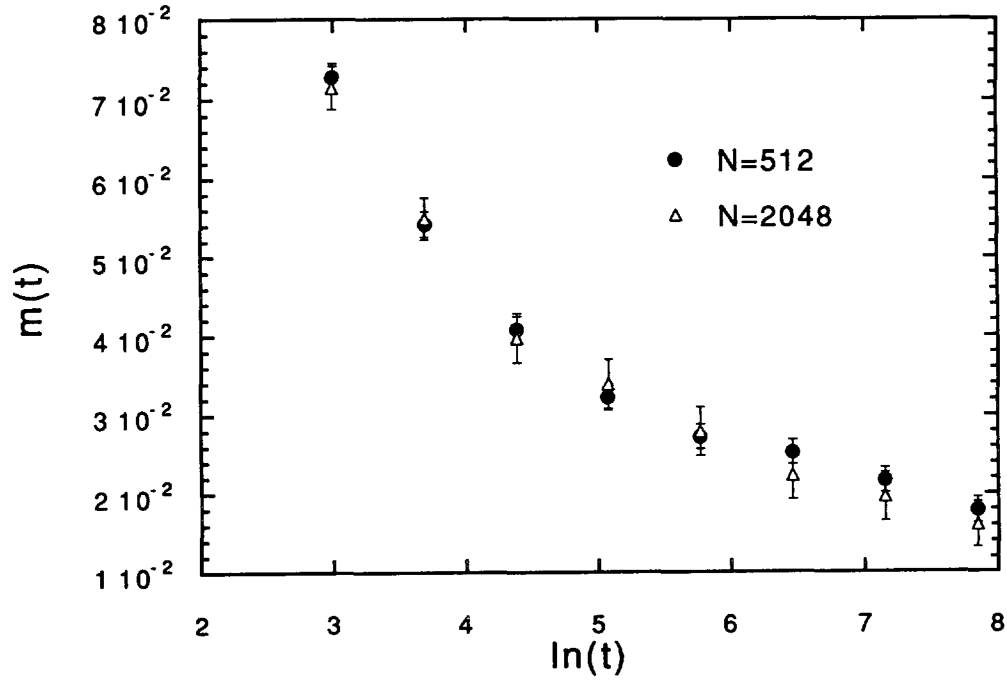

Fig. 5. — Remanent magnetization in the SK model with discrete ± 1 couplings at $T = 0.4$. Data is shown for two sizes $N = 512$, 2048 and 500 and 40 samples respectively. Remanent magnetization is measured each 10 Monte Carlo steps in a total run of 2 560 and averaging over 8 exponentially increasing periods of time.

Table I. — *Decay parameters for the* SK *model with* $N = 2\,048$.

T	0.2	0.4	0.6
$m(\infty)$	0.024	0.011	0.0072
α	0.29	0.48	0.745
$E(\infty)$	-0.74	-0.725	-0.674
β	0.61	0.73	0.84

dissipation theorem does not hold) at finite temperature. As we have said before, an important step in this direction has been already done in the zero temperature regime and for parallel dynamics [25].

6. Conclusions.

We have studied numerically the decay of the remanent magnetization and in some cases also the decay of the excess energy in the off equilibrium regime for several short-ranged and infinite-ranged spin glasses. We have found reasonable agreement with some phenomenologi-

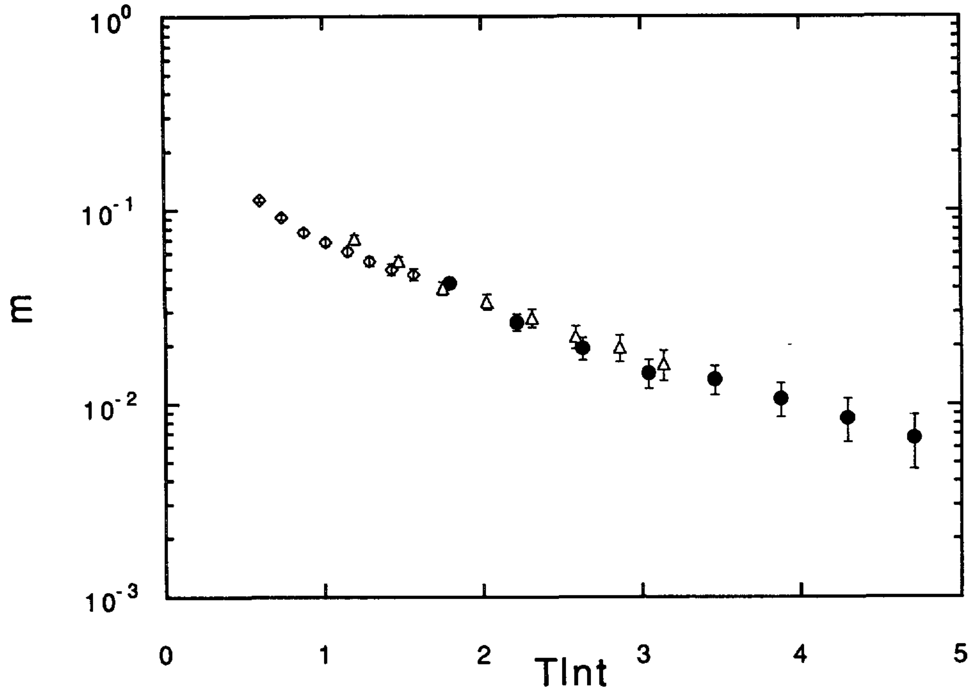

Fig. 6. — Remanent magnetization in the SK model ($N = 2\,048$) at three different temperatures. Data is averaged over 40 samples. The symbols are : (•) $T = 0.6$, (△) $T = 0.4$, (◇) $T = 0.2$.

cal laws obtained in the experimental observation of the relaxation after the application of a saturating field in some RKKY and insulating spin glasses.

In both cases (finite and infinite-ranged models) the $T \log(t)$ behavior is well observed for the decay of the remanent magnetization down to a certain low temperature ($\simeq 0.2\, T_c$). This supports the idea of a relaxation activated by energy barriers. In case of the decay of the excess energy its behavior is more complex and discrepancies begin to appear at low temperatures higher than for the remanent magnetization. Main discrepancies in our results are because of the discrete ± 1 values of the couplings which have been used in all models because they make simulations computationally faster. But in real spin glasses the strength of couplings is continuous and our results altogether with those obtained in earlier numerical studies suggest that the phenomenological law equation (1) is well reproduced in the simulations and could be explained in a future within mean-field theory. As important step and feasible step in this direction would consist in solving the dynamical equations for the SK model in the off-equilibrium regime at finite temperature.

Even though our numerical simulations in case of the SK model have been done for a sequential dynamics of the lattice we expect to obtain the same behavior for parallel dynamics. Only the value of the possible residual magnetization $m(\infty)$ and the exponent α will vary. This is because $1/N$ differences in the thermodynamic magnitudes for both models in the infinite-size limit [35] imply finite variations for the parameters of the decay. Also different values for the exponent α of the decay of the remanent magnetization are found doing simulations with a continuous distribution of Gaussian couplings or with a binary discrete one (i.e., if we compare data extracted from Ref. [33] and our results shown in Fig. 5 for $N = 512$). In this case also the thermodynamics of the SK model is the same except $1/N$ corrections. This suggests that different $1/N$ corrections in the thermodynamics imply different values for the parameters of the decays equations (15) and (16).

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