Structural properties of planar random heap of hard discs

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Structural properties of planar random heap of hard discs

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Abstract. — We study here numerically the average angle of repose (θ_r) and the packing fraction (ρ_r) of a random (two dimensional) heap of hard discs falling (under gravity) ballistically on a (sticky) base line, where the first layer of discs gets quenched in random positions. We find $\theta_r \cong 52°$ and $\rho_r \cong 0.80$ for such heaps. We also study the pressure or load distribution on the base line of such a random heap.

The statistics of random packings of hard spheres (in three dimensions) or discs (in two dimensions) are being studied for many years, mainly in the context of investigations regarding the thermodynamic properties of simple liquids [1], glasses [2] and growth of defects [3]. Very recently, this hard sphere or disc packing problems are being studied in connection with the investigations on static and dynamic (flow) properties of granular media, in particular, of sandpiles [4]. Here, we are interested in studying the static (structural) properties of a two dimensional (planar) random heap of hard discs formed due to random deposition on a line under gravity. Specifically, we are interested in the values of the average angle of repose θ_r, the packing density ρ, the load distribution at the bottom layer of the heap, etc. In an earlier study [5], on the formation of the heap of hard spheres falling under gravity on a sticky surface (so that the first layer of hard spheres get quenched), we suggested that a heap can stabilize (in three dimension) due to the quenching of the first layer and the hard core repulsion of the spheres in successive layers, and that the regular hexagonal close packed (HCP) heap (with volume packing fraction $\rho_{HCP} \cong 0.74$ and angle of repose $\theta_{HCP} \cong 70.5°$) makes a discontinuous transition (due to thermal noise or random shaking) to the random close packed (RCP) structure with $\rho_{RCP} \leq 0.64$ and the angle of repose $\theta_{RCP} \cong 18.0°$. This angle of repose, for a heap of random hard spheres with smooth surfaces (no surface friction), is thus purely statistical in origin and has not been studied so far. It is important to know the magnitude of the average angle of repose (and the corresponding packing fraction) for random (stable) heaps (deposition) of hard spheres of discs falling on a (sticky) plane or line gravity. We study here these static structural properties of a heap of hard discs (in two dimensions), using computer simulation.

In our simulation, the hard discs are dropped on a sticky base of length L from an arbitrary height, at a randomly chosen horizontal position. The deposition process is ballistic in nature assuming that disc (of unit radius) are falling downward with zero kinetic energy. The first layer, being sticky, the discs get frozen in random positions, as they touch the base. For other discs touching discs settled in lower layers, the discs look for stable positions under gravity (magnitude of gravity is unimportant). If the horizontal coordinate of the centre of the incoming disc lies in between those of the neighbouring (contacting) left and right discs, then only it permanently sits there and this is the stable position of that disc. Otherwise it rolls and searches for the stable position. If it does not find any stable position within 0 and L it leaves the heap ultimately through successive rolling.

We measured the angle of repose and the packing density of the heaps grown in the process described above. For all random heaps grown, the edges (boundary) look (on average) like a straight line which is neither convex nor concave type. In order to have a quantitative measurement, we tried a polynomial fitting of the coordinates of the boundary points (upto a quadratic form: $y \approx ax^2 + bx + c$) and found the contribution of the quadratic term to be insignificant compared to the linear term $\left(a/b \sim 0\left(10^{-3}\right)\right)$ The angle of repose has therefore been measured by the linear fitting of the points (centres of discs) lying on the boundary line of the heap. In order to calculate the packing density we calculated the total area of the heap and the number of particles within the heap. From the value of the number of particles we calculated the area occupied by the discs. For the discs on the boundary (which are separately counted), the appropriate contribution is taken. Packing density is then obtained as the ratio of area occupied by the discs and the total area of the heap. Typically, we have formed the heap on a base of length $L = 48$. Our statisticsis is based on averages out about 100 such heaps.

For regular crystallographic deposition (when the first layer is deposited in a regular way), we obtained the packing fraction value $\rho_{\text{Comp}} \cong 0.906$, which is comparable to the geometrically calculated value [6] $\rho_{\text{Cryst}} = \pi/\left(2\sqrt{3}\right) \cong 0.907$. The angle of repose we obtained in this case is $\theta_{\text{HCP}} = 60°$, which is also the exact value. For random heaps, the average angle of repose $\theta_{\text{r}} = \left(\theta_{\text{Rnd}}\right)$ turns out to be about 52° ($\theta_{\text{Rnd}} = 51.75° \pm 3.00°$) and the average packing density $\rho\left(\rho_{\text{Rnd}}\right) \cong 0.80$ $\left(\rho_{\text{Rnd}} = 0.795 \pm 0.01\right)$. From geometric consideration, one gets [6] the minimum value of the packing fraction to be equal to $\pi/4 \cong 0.785$ (for a regular arrangement of discs in such a manner so that each disc makes the angle $\pi/4$ with both of its neighbouring lower left and lower right discs in contact. Our result (for the packing fraction) for random deposition is very near this minimum value. The (normalised) distribution ($p(\theta)$) of the angle of reposes (θ), for random deposition obtained from various samples, looks like a self-averaging (symetric) one and the most probable value ($\theta_{\text{r}} - 52°$) is indeed very close to the average value of the angle of repose (see Fig. 1). We also measured the average angle of repose θ_{r} for a typical random heap on a base or line of length $L = 200$ and found the best linear fit value of θ_{r} to be about 51°. This indicates that the angle of repose has an well defined (unique) value (around 52°) for planar heaps in two dimensions. The inset of figure 1 shows the (normalised) distribution $p'\left(\theta'\right)$ of angles $\left(\theta'\right)$, made with the horizontal line by the centres of two consequtive discs lying on the boundary line. This distribution does not look self averaging (symetric) and its most probable value is very close to 58°. Though, the most probable value is roughly around 58°, the average value turns out to be approximately equal to 51.6°, which is indeed very close to the average angle of repose obtained from the linear fit for the entire boundary.

We also studied the pressure or load distribution (due to the weights of the discs in successive layers) on the bottom layer or line. The nonuniformity in the pressure distribution in the last layer comes from the typical modulations or "arches" in the structure of the layers (see Fig. 2). The load distribution (on the base line) in a typical random heap is shown in figure 2, where

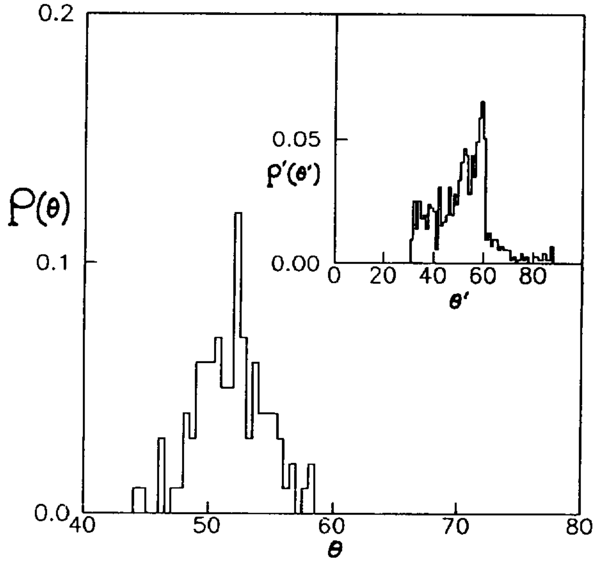

Fig. 1. — Normalised distribution $p(\theta)$ of the average angle of repose θ_r obtained from the linear fit of the centres of the discs on the boundary. The inset shows the normalised distribution $p'(\theta')$ of the inclinations θ' of the segments between two conseqtutive discs on the boundary.

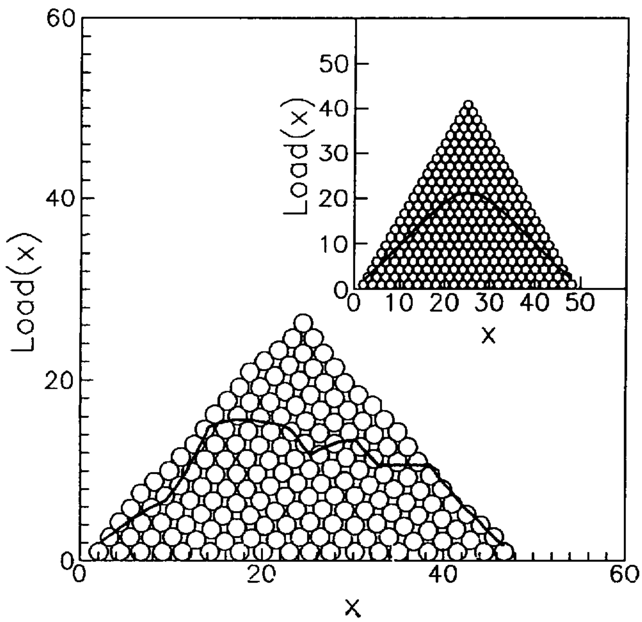

Fig. 2. — Load distribution (shown by continuous line) on the base line for a typical random heap. Inset shows the same for an ordered heap.

the heap is also shown. The inset shows the same for the ordered (crystallographic) heap. This load distribution shows that the maximum pressure is not necessarily at the centre of the base line.

We have investigated in this paper the average angle of repose θ_r and the packing fraction ρ_r of a random two dimensional heap of hard discs falling under gravity (and rolling down to its meta-stable position) without any kinetic energy on a (sticky) base line. We find $\theta_r \sim 52°$ and $\rho_r \sim 0.80$ for a random heap, compared to $\theta_r = 60°$ and $\rho_r \sim 0.91$ for a regular heap. The load distribution at the bottom of the heap has also been studied and we find that the maximum pressure is not neccessarily at the centre of the heap, unlike that for a regular heap.

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