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Weak ergodicity breaking and aging in disordered systems

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Abstract. — We present a phenomenological model for the dynamics of disordered (complex) systems. We postulate that the lifetimes of the many metastable states are distributed according to a broad, power law probability distribution. We show that aging occurs in this model when the average lifetime is infinite. A simple hypothesis leads to a new functional form for the relaxation which is in remarkable agreement with spin-glass experiments over nearly five decades in time.

In spite of fifteen years of dispute, the theory of equilibrium spin-glasses is not yet settled [1, 2, 3]. Dynamical effects are, however, likely to be the dominant aspect in experiments. One of the most striking aspects of the dynamics of spin-glasses in their low temperature phase is the aging phenomenon — a rather peculiar and awkward feature from the thermodynamics point of view : the relaxation of a system depends on its history. More precisely, if a system is field-cooled below its spin-glass temperature, the magnetization relaxation depends on the waiting time t_w between the quench and the switch off of the magnetic field [4, 5, 6]. Similar effects are observed on the viscoelastic properties of polymer melts [7], magnetic properties of HTC superconductors [8] and more recently on the relaxation after a heat pulse in Charge Density Wave systems [9]. Analytical fits of the magnetization relaxation as a function of time have been proposed. In [6], it is proposed that the initial « stationary » part of the relaxation is a power-law with a small (negative) exponent. For times longer than the waiting time, relaxation is well fitted by a « stretched » exponential decay, provided that an effective time is introduced [6]. In [4, 5], however, a stretched exponential of the « real » time was found for relatively short times. Many phenomenological theories have been devised to account for the stretched exponential decay [10-14], and for the slow part and aging [6, 15-18]. We however feel that the basic mechanism underlying aging has not been fully appreciated (see however [17]) — although it is, in our opinion, one of the constitutive properties of spin-glasses. The aim of this note is to suggest that aging is related to ergodicity breaking — which has, in these systems, a peculiar and perhaps unexpected meaning. We find on some simple models that — according to our definition, see below — « weak » ergodicity breaking occurs in the spin-glass phase, defined as the phase where the Edwards-Anderson parameter is non-zero. We see however no reason why the two should be linked in general, and suggest that the appearance of aging could be taken as an operational definition of the spin-glass transition.

We reproduce both the short time $(t \ll t_w)$ and the long time $(t \gg t_w)$ parts of the magnetization decay under quite reasonable hypotheses.

The analytical form of this decay is however different from all those previously proposed. We find in particular that :

$$m(t) = m_0 \exp\left[-\gamma/(1-x)\,(t/t_w)^{1-x}\right] \quad 0 \le x \le 1$$

for $t \ll t_w$ and a *power law decay*

$$m(t) \simeq (t/t_w)^{-\gamma}$$

for $t \gg t_w$. Note that the two regimes are strongly connected, since the above exponent is related to the parameters describing the initial part of the relaxation. This feature should be a stringent test of our theory. As we shall see, our theory is in remarkable agreement with both the « short » time experiments of [4, 5], and the very long time data of [6].

It is widely accepted that the energy landscape of a finite disordered system is extremely rough, with many *local minima corresponding to metastable configurations*, which we shall call loosely « states ». Since the energy landscape is rough, these local minima are surrounded by rather high energy barriers. We thus expect that these states act as « traps » which get hold of the system during a certain time τ.

What is the distribution of these trapping times $\psi(\tau)$? A simple picture for the energy landscape is the following (Fig. 1, see also [11]) : there exists a « percolation » energy level f_0 below which « states » are disconnected ; f_0 is the minimal energy required to « hop » between any two states. In other words, we assume that the dynamics *between* the traps is very fast and that the probability to find the system between two metastable states is negligible.
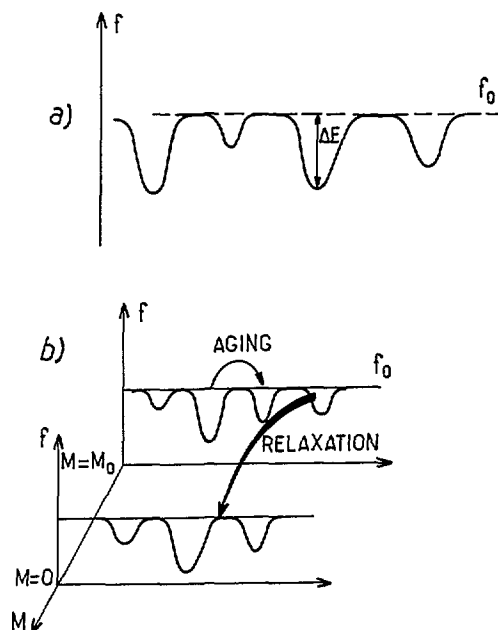
Fig. 1. — a) Schematic view of the energy landscape : holes are drilled below the reference energy f_0 which is the minimal energy needed to go from one metastable state to another. Note that this drawing is one dimensional : in reality *mountains of height* $\gg f_0$ *also exist between different states.* b) When the external field is cut, the landscape acquires a non-zero slope which drives the system towards zero magnetization states.

In this picture, the energy barrier ΔE is thus equal to the depth of the trap, $f_0 - f$. In the spin glass phase of both the Random Energy Model (REM) and of the SK model, the distribution of very low f's is exponential [1, 23, 11] :

$$P(f) = N/T \exp x (f - f_0)/T \qquad (1)$$

where x is a temperature dependent number between 0 and 1, f_0 is the reference level above which levels proliferate and N is a constant. In the REM, $x = T/T_g$ which is connected to the fact that the free energy landscape is temperature independent. This is not the case in the SK model, where $x(T)$ is non-trivial [1]. Its shape is well approximated by the « one-step » replica breaking scheme [5].

Assuming that $\Delta E = f_0 - f$, and that $\tau = \tau_0 \exp(\Delta E/T)$, one finds, using (1) and $\psi(\tau) \, d\tau = P(f) \, df$:

$$\psi(\tau) = \kappa \tau_0^x \tau^{-(1+x)} \qquad (2)$$

for large τ. [κ is a normalization constant]. Such a form is also obtained in many other models — for example the one-dimensional « random force model » [17, 20-22], with $x = TF/\sigma$ — where F is the average force and σ its variance.

It may be that for real spin-glasses the probability distribution of low free energy states decays more slowly for large negative f, for example as a power law :

$$P(f) \simeq f_0^\zeta (-f)^{-(1+\zeta)} \qquad (3)$$

with $\zeta > 0$. In this case, $\psi(\tau)$ reads :

$$\psi(\tau) = \Theta/\tau \, [\log \tau/\tau_0]^{-(1+\zeta)} \qquad (4)$$

where Θ is a constant. This will lead to logarithmic, as opposed to power law, decay. (4) would probably conform more to the ideas of Fisher and Huse [15].

The most interesting property of these distribution laws is the fact that $\langle \tau \rangle = \int_0^\infty d\tau \, \tau \psi(\tau)$ diverges, when $x \leqslant 1$ for equation (2) and for all finite ζ for equation (4).

Hence, the time needed to explore an infinite system is infinite. This is however a non-conventional scenario for ergodicity breaking, since the phase space is *a priori* not broken into mutually inaccessible regions in which local equilibrium may be achieved. We shall call this situation « weak » ergodicity breaking. It may be that « true » ergodicity breaking also occurs, i.e. the existence of many « pure states » between which infinite barrier stand. We shall carefully avoid this issue (the reader is referred to [19] for a recent experimental discussion), since the dynamics is by definition restricted to only one pure state, and for which the concept of « weak » ergodicity breaking is relevant.

Let us now see how this feature is deeply connected to the possibility of observing aging. Suppose that the system under study can be decomposed into $\mathcal{M}$ independent subsystems. These subsystems may be, for example, weakly coupled polycristalline « grains » or clusters of magnetic atoms — they are *not*, in our mind, « domains » growing with time as is assumed in references [15, 16] (although such domains may indeed be present *inside* each grain). In other words, we assume that an experimental sample is already a collection of a large number of micro spin-glasses, and hence that the measured quantities are disorder averages. We claim that performing dynamical experiments on mesoscopic samples, where $\mathcal{M}$ is small, would yield irreproducible results.

Each subsystem i ($= 1, \dots \mathcal{M}$) can be in a certain number of « states » $S(i) \simeq S$ which we shall suppose roughly independent of i. The total number of states is thus $\simeq S^{\mathcal{M}}$ Note that in the SK model, S depends on temperature, with in particular $S(T_g) = 1$.

The system is field cooled to a certain temperature $T < T_g$. Since a magnetic field H is on, the system evolves within a « pool of states » which carry a net magnetization — but cannot reach states with a different value of this magnetization : this would cost an extensive (free) energy. The free energy distribution of these states is characterized by a certain $x(T, H)$ or $\zeta(T, H)$ — depending on which of the two distributions (1) or (3) is more adapted. [For convenience, we shall adopt in the following the distribution (1), the corresponding results for distribution (3) can be obtained from those given below by defining a time dependent effective parameter $x(t)$ according to $x(t) = (1 + \zeta) \log(\log t/\tau_0)/\log(t/\tau_0)$ which goes to zero as $t \to \infty$].

The magnetic field is left on a time t_w. During this time, the subsystems start exploring their phase space. The deepest trap available for a given subsystem i is characterized by a trapping time $t(i)$ such that :

$$S \int_{t(i)}^{\infty} d\tau \, \psi(\tau) \simeq 1 \,. \tag{5}$$

Equation (5) simply means that out of S trials, a value of τ larger or equal to $t(i)$ has been encountered roughly only once. (A more rigorous justification can be found in e.g. [21]). Thus, from (5), $t(i) \simeq \tau_0 S^{1/x}$. This longest trapping time of course fluctuates from one subsystem to the other, with a distribution given by $\mathcal{F}(t, S)$ such that

$$\mathcal{F}(t, S) \simeq \tau_0^x S/t^{1 + x} \tag{6}$$

for $t \gg S^{1/x}$. [The *maximum* relaxation time available in the whole system is thus, similarly, $t(\mathcal{M}) \simeq S^{\mathcal{M}/x}$.]

Let us now estimate the order of magnitude of the longest trapping time τ_{max} actually encountered during a time t_w, when τ_{max} is much smaller than $t(i)$. If $N(t_w)$ denotes the number of different « states » visited by the subsystem after t_w, τ_{max} is obtained through :

$$N(t_w) \int_{\tau_{max}}^{\infty} d\tau \, \psi(\tau) \simeq 1 \tag{7a}$$

$$t_w = \sum_{j = 1, N(t_w)} \tau_j \simeq N(t_w) \int_{0}^{\tau_{max}} d\tau \, \tau \psi(\tau) \tag{7b}$$

which gives, for $x < 1$ and for all finite ζ, $\tau_{max} \simeq t_w$. This is the key point in our picture : the deepest state encountered traps the system during a time which is comparable to the overall waiting time. Schematically, states such that $\tau \ll t_w$ are thus, for $x < 1$, ergodically probed, whereas those for which $\tau \gg t_w$ are barely visited. This is not so for $x > 1$: from (7a, b) one finds that the typical value of τ_{max} is $\tau_0^{1 - 1/x}(t_w)^{1/x} \ll t_w$.

We are now in a position to describe the observed relaxation laws quantitatively. When the magnetic field is switched off, the free energy of the metastable states are uniformly shifted by an amount $+ MH$ (Fig. 1b). The subsystems will evolve independently towards states carrying no magnetization, which requires a collective change of a finite fraction of the spins. The system must leave the metastable state that it had entered when the field was on. However, the *system may also continue to probe deeper and deeper wells while relaxing* ; its age t_w is thus *time dependent* with : $t_w(t) = t_w + t$. This feature was recognized as a crucial ingredient for a consistent analysis of the experimental data [6].

Two cases must then be distinguished :

A) *Short waiting times :*

Suppose first that the waiting time t_w is much shorter than $\tau_0 \, S^{1/x}$, defined by equation (5). In this regime, the phase space is effectively infinite. The probability $P(\tau, t_w)$ to find a given subsystem in a state characterized by τ can easily be shown to be equal to $r(\tau/t_w)\, \tau \psi(\tau)$, with $r(u) = 1$ for small u, and decaying to zero for $u \gg 1$. The detailed form of $r(u)$ would depend on the « geometry » of phase space — i.e. the relative positions and interconnections between the different metastable states. In the simple case where all states are equally accessible, one finds $r(u) = 1/u$ for $u \gg 1$.

The very curious feature in the $x < 1$ phase *is that $\tau \psi(\tau)$ is a priori not normalisable,* since $\langle \tau \rangle$ is infinite. $P(\tau, t_w)$ can thus only be normalised when the waiting time t_w is finite, and then reads :

$$P(\tau, t_w) = A r(\tau/t_w)\, (t_w)^{x-1}/\tau^x \tag{8}$$

with $A = \int_0^\infty du\, r(u)/u^x$ [$= 1/1 - x$ if one takes a simple step for $r(u)$]. Note that the *microscopic* time τ_0 has entirely disappeared from equation (8) ! This would not be the case for $x > 1$: in this case, one simply finds $P(\tau, t_w) \propto \tau \psi(\tau)$, which in fact corresponds to the ergodic result (indeed, $\tau \psi(\tau)$ is proportional to the Boltzmann weight).

Assuming a simple exponential decay from single states ([1]), we thus find, for the magnetization relaxation rate :

$$dm/m = -p \langle 1/\tau \rangle \, dt = -dt\, [p/Z] \int_0^\infty d\tau \, 1/\tau \, P(\tau, t + t_w) \exp(-t/\tau) \tag{9}$$

where $Z = \int_0^\infty d\tau \, P(\tau, t + t_w) \exp(-t/\tau)$. p is the probability for the magnetization of the subsystem to relax when leaving a τ-trap, rather than continue aging among magnetized states. p thus measures « how easy » it is to find states with small overlaps [24].

Aging during relaxation is taken into account by the appearance of $t + t_w$ in the above expression. Formula (9) is easily transformed into :

$$dm/m = -t^{-x}(t + t_w)^{x-1}\, G(t/t + t_w)\, dt \tag{10}$$

where $G(u)$ is another slowly varying function, related to $r(u)$, such that $G(0) = p\Gamma(x)/A$. Note that equation (10) is close in spirit, although not equivalent, to the treatment of references [6, 7], where a description of aging during relaxation was proposed. In the following we shall neglect the variations of G, and study (10) in the short and long time regimes.

For $t \ll t_w$, one finds

$$m(t) = m_0 \exp[-\gamma/(1-x)\, (t/t_w)^{1-x}] \simeq m_0(1 - \gamma/(1-x)\, (t/t_w)^{1-x} + \cdot \quad) \tag{11}$$

where $\gamma = p\Gamma(x)/A$. For $t \gg t_w$, aging during relaxation becomes the predominant feature and we find :

$$m(t) \simeq (t/t_w)^{-\gamma} \tag{12}$$

([1]) Note however that our central result, equation (10), is unchanged — up to a redefinition of G — if $\exp(-t/\tau)$ is replaced by any function of t/τ decaying faster than t^{-1}

(assuming that $G(1) = G(0)$). Three remarks must be made :

i) our theory reproduces the ubiquitous « stretched » exponential decay, which is seen to be only valid at *short* times $t \ll t_w$.

One should also note that based on the exponential distribution of free-energies (Eq. (1)), De Dominicis *et al.* [11] have proposed a dynamical model for which a stretched exponential decay is found for *long* times, with an exponent x (while we find $1 - x$, see Eq. (11)). However, our approach is very different since we assume that the hopping rate from one state to another is determined by the « starting state » and not, as in [11], by the « destination » state. Furthermore, no aging effects were found in this model.

ii) (11) and (12) show that the two limits (short and long times) are intimately connected.

iii) Note that in this regime, $m(t, t_w)$ is a function of the ratio t/t_w only. In particular, we find that the short time decay, for $t \ll t_w$, is *not* stationary. This is at variance with the theory of references [15, 16], where the equilibrium (stationary) dynamics is predicted for $t \ll t_w$. In our model, this is only the case when $t_w \gg \tau_0 S^{1/x}$ (see below). Moreover $m(t_w, t_w)$ is still of order of m_0.

We have fitted some of the experiments of [6] with the function obtained from the integration of (10), with a constant G. This leaves three fitting parameters : m_0, x, and γ. (Note however that the shape of the curve is only parametrized by x).

As can be seen from figure 2, the agreement is remarkable in view of the broad scale of times which is probed : t varies from 0.1 to 3 500 min, and the waiting time t_w is equal to 31.3 min. For $T = 0.6\,T_g$, we find $x = 0.76$ and $\gamma = 0.062$; using $A = 1/(1 - x)$, this corresponds to p of the order of 1/3. Note in particular that an algebraic distribution of lifetimes (Eq. (3)) appears to be more appropriate than the logarithmic distribution, equation (4).
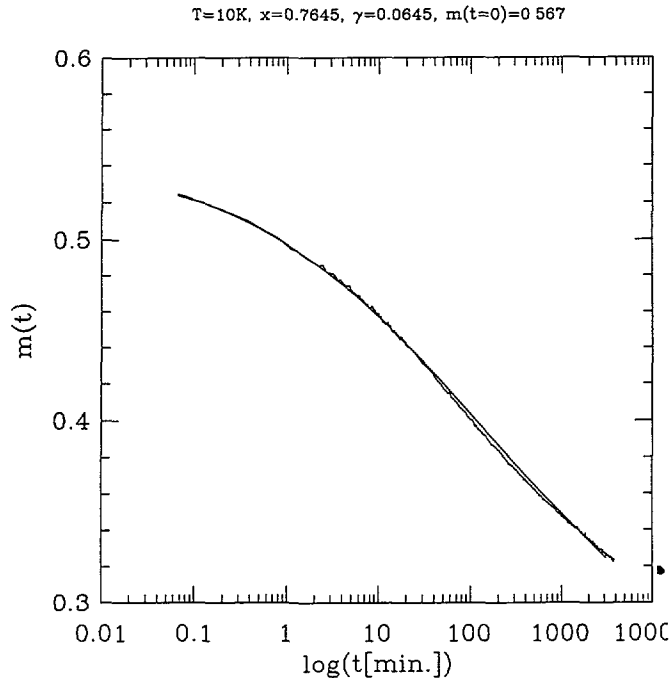

Fig. 2. — Fit of one of the aging experiment on $CdCr_{1.7}In_{0.3}S_4$ spin glass [6]. The waiting time is 31.3 min under 15 Gauss, and the temperature is $10\,K \simeq 0.6\,T_g$. The set of parameters used was $m_0 = 0.567$ (in units of the field cooled magnetization), $x = 0.76$ and $\gamma = 0.0645$.

Other temperatures and waiting times are equally well reproduced by our theory. A more extensive presentation of these fits, and of the very interesting field and temperature dependence of the parameters, will be published elsewhere [25]. Note finally that the experiments of [5] have been analyzed in terms of a stretched exponential decay. This is nicely explained by our theory, since in these experiments, the decay time is less than or comparable to t_w.

In order to emphasize the peculiarity of the region $x < 1$, one should note that for $x > 1$, where the distribution (8) becomes normalisable, one finds $m(t) \simeq m_0 [1 - (t/\tau_0) + \cdot \quad]$: the magnetisation has thus drop substantially after a *microscopic* time τ_0, and not after the macroscopic time t_w.

B) *Long waiting times and « interrupted aging »*

Let us now consider the other limit $t_w \gg \tau_0 S^{1/x}$, which corresponds to the case where most systems have reached equilibrium. Using (6), we find that (8) is replaced by :

$$P(\tau) = S\tau_0^x/\tau^{1+x}[\tau/S^{1/x}\tau_0] \simeq S^{1-1/x}(\tau_0)^{x-1}/\tau^x \tag{13}$$

for $\tau \ll \tau_0 S^{1/x}$

Hence, for $t \ll \tau_0 S^{1/x} \ll t_w$, we find that

$$m(t) = m_0(1 - \gamma/(1-x)(t/\tau_0 S^{1/x})^{1-x} - \cdot \quad) \tag{14}$$

which is independent of t_w.

For $\tau_0 S^{1/x} \ll t \ll t_w$, one observes those subsystems such that $t(i) < t_w$, for which equilibrium is reached. From the theory of Lévy sums (see e.g. [21]), one may show that the deepest traps of these subsystems, for $x < 1$, entirely dominates the partition function. In other words, *the probability to find the subsystem i in its deepest state is of order* 1. The fraction $m(t)$ of subsystems still carrying their initial magnetisation is thus, for $\tau_0 S^{1/x} \ll t \ll t_w$,

$$m(t)/m_0 \simeq \int_0^\infty d\tau \left\{\tau_0^x S/\tau^{1+x}\right\} \exp(-t/\tau) \simeq S(\tau_0/t)^x \ll 1 \tag{15}$$

where we have used the probability distribution (6). Hence aging essentially stops (or becomes inobservable) when $\tau_0 S^{1/x} \simeq t_{w_{max}}$, beyond which the finite size of the phase space appears. « Interrupted aging » has indeed been observed in Charge Density Waves [9], and can be used to measure the « complexity » Ω of the system, which is naturally defined as $\Omega = \text{Log } S \simeq x \text{ Log } (t_{w_{max}}/\tau_0)$. Let us finally mention that aging of low frequency a.c. susceptibility can be understood using an extension of equation (9) :

$$\chi(\omega, t_w) = \int_0^\infty d\tau \, P(\tau, t + t_w) \, 1/(1 + i\omega\tau) \tag{16}$$

where a simple Debye form is assumed for each state [25]. In particular, we find that $\chi''(\omega, t_w) = \phi(\bar\omega = \omega t_w)$ with $\phi(\bar\omega) \simeq \bar\omega$ for small $\bar\omega$, and $\phi(\bar\omega) \simeq \bar\omega^{x-1}$ for large $\bar\omega$, in agreement with Koper and Hilhorst [16].

We have thus proposed a phenomenological model which allows us to understand, on very general grounds, how the phenomenon of aging sets in : the phase space must posses a large number of metastable states, which a broad, scaleless, distribution of lifetimes. When the average lifetime of these metastable states diverges, all the physical observables are dominated by the properties of the *deepest state* that the system was allowed to probe in its

quest for the « true » equilibrium state. Aging stops either when the distribution of lifetimes is truncated and is a finite size effect, or when the distribution of lifetimes decays more rapidly than τ^{-2} A more formal description of our results could be given, along the lines of reference [11], using a Master equation in phase space.

The most important quantity of our theory is the index x, which naturally appears in the SK or REM models of spin-glasses. x gives information on the distribution of free-energies in the spin-glass phase, and the present model is a suggestion for its experimental determination, which would be of great interest. The behaviour of x and the disappearance of aging as T_g is approached determines the nature of the freezing at the glass point : in the Random Energy Model [23], $x(T_g) = 1$, which means that the phase space is broken into many metastable states of comparable « size ». In the SK model, on the other hand, $x(T_g) = 0$ (one valley dominance) [1] and the number of states $S(T \rightarrow T_g)$ is tends to 1 : aging is then interrupted for microscopic times.

Let us finally emphasize that our model is not restricted to spin-glasses and may thus be useful to understand aging in other systems such as polymer glasses [7] and Charge density waves [9].

Acknowledgements

References

[1] MÉZARD M., PARISI G., VIRASORO M., « Spin Glass Theory & Beyond » (World Scientific, Singapore, 1986).

[2] BINDER K., YOUNG A. P., Rev. Mod. Phys. 58 (1986) 801.

[3] FISHER D. S., HUSE D., Phys. Rev. Lett. 56 (1986) 1601 ; Phys. Rev. B 38 (1988) 373.

[4] LUNDGREN L., SVEDLINDH P., NORDBLAD P., BECKMANN O., Phys. Rev. Lett. 51 (1983) 911 ; NORDBLAD P., LUNDGREN L., SVEDLINDH P., SANDLUND L., Phys. Rev. B 33 (1986) 645.

[5] CHAMBERLIN R. V., Phys. Rev. B 30 (1984) 5393 ; HOOGERBEETS R., LUO W. L., ORBACH R., Phys. Rev. B 34 (1986) 1719.

[6] ALBA M., OCIO M., HAMMANN M., Europhys. Lett. 2 (1986) 45 ; J. Phys. Lett. 46 (1985) L-1101, for a review, see M. Alba, J. Hammann, M. Ocio, Ph. Refregier, J. Appl. Phys. 61 (1987) 3683 in « Recent Progress in Random Magnets », D. H. Ryan editor (McGill University, 1992).

[7] STRUIK L. C. E., Physical Aging in Amorphous Polymers and Other Materials (Elsevier, Houston, 1978).

[8] ROSSEL C., in Relaxation in Complex Systems and related topics (Plenum Press, New York, 1990).

[9] BILJAKOVIC K.. LASJAUNIAS J. C., MONCEAU P., LÉVY F., Phys. Rev. Lett. 67 (1991) 1902 ; BILJAKOVIC K., LASJAUNIAS J. C., MONCEAU P., Phys. Rev. Lett. 62 (1989) 1512.

[10] PALMER R., STEIN D., ABRAHAMS E., ANDERSON P. W., Phys. Rev. Lett. 54 (1984) 958 ; see also D. Stein, « Chance & Matter », J. Souletie, R. Stora, J. Vannimenus Eds. (North Holland, 1986).

[11] DE DOMINICIS C., ORLAND H., LAINÉE F., *J. Phys. Lett.* **46** (1985) L-463.
[12] MONTROLL E. W., BENDLER J. T., *J. Stat. Phys.* **34** (1984) 129.
[13] CAMPBELL I. A., *J. Phys. Lett.* **46** (1985) L-1159.
[14] RAMMAL R., *J. Phys.* **46** (1985) 1837 ;
    CASTAING B., SOULETIE J., *J. Phys. I France* **1** (1991) 403.
[15] FISHER D. S., HUSE D., *Phys. Rev. B* **38** (1986) 386.
[16] KOPER G. J. M., HILHORST H. J., *J. Phys. France* **49** (1988) 429 ;
    see also H. J. Hilhorst, in Fundamental Problems in Stat. Mech. VII, H. Van Beijeren Eds.
        (Elsevier, 1990).
[17] FEIGELMAN M. V., VINOKUR V., *J. Phys. France* **49** (1988) 1731 ;
    DOTSENKO V. S., FEIGELMAN M. V., VINOKUR V., Spin Glasses and Related Problems, Soviet
        Scientific Reviews, vol. 15 (Harwood, 1989).
[18] SIBANI P., HOFFMANN K., *Phys. Rev. Lett.* **63** (1989) 2853 ;
    SIBANI P., *Phys. Rev. B* **35** (1987) 8572.
[19] LEDERMAN M., ORBACH R., HAMMANN J., OCIO M., VINCENT E., *Phys. Rev. B* **44** (1991) 7403.
[20] BOUCHAUD J. P., COMTET A., GEORGES A., LE DOUSSAL P., *Ann. Phys.* **201** (1990) 285.
[21] BOUCHAUD J. P., GEORGES A., *Phys. Rep.* **195** (1990) 127 ;
    see also Appendix A of BOUCHAUD J. P., OTT A., LANGEVIN D., URBACH W., *J. Phys. II France*
        1 (1991) 1465.
[22] DERRIDA B., POMEAU Y., *Phys. Rev. Lett.* **48** (1981) 627.
[23] DERRIDA B., *Phys. Rev. B* **24** (1981) 2613 ;
    DERRIDA B., TOULOUSE G., *J. Phys. Lett.* **46** (1985) L-223.
[24] ORBACH R. (unpublished) argues that p can be related to the « branching ratio » of the ultrametric
        tree in Parisi's solution. Note that he also obtains an algebraic decay of $m(t)$ at large times.
[25] BOUCHAUD J. P., VINCENT E., HAMMANN J., OCIO M., in preparation.