On superluminal barrier traversal

A. Enders, G. Nimtz

Classification
Physics Abstracts
41.10H — 03.50D — 04.20C

Short Communication

On superluminal barrier traversal

A. Enders and G. Nimtz

II. Physikalisches Institut der Universität zu Köln, Zülpicher Str. 77, 5000 Köln 41, Germany

(*Received 22 June 1992, accepted in final form 10 July 1992*)

Abstract. — We report on microwave measurements of the barrier traversal time of electromagnetic wave packets in an undersized waveguide. This time was significantly shorter than the ratio L/c, with L the barrier length and c the velocity of light in vacuum. Consequently the transport velocity for the wave packet inside the barrier has to be superluminal, i.e. larger than c.

The existence of superluminal velocities which carry information like the signal velocity can not be ruled out in the framework of the Maxwell equations. Some theoretical papers discuss superluminal transport properties for electromagnetic modes [1-3]. Of special interest here are so-called evanescent modes which are solutions of the Helmholtz wave equation with a purely imaginary eigenvalue for the wave vector k. The corresponding eigenfunctions give an exponential decay in space and have no spatial phase dependence, therefore they can't be regarded as waves at all. They occur e.g. in the case of undersized waveguides. The eigenvalue equation for the lowest, so-called H_{10}-mode of a rectangular waveguide with cross-section $a \times b$ with $a < b$ is given by [4] :

$$k^2 = \left(\frac{2\pi}{\lambda_{\text{vac}}}\right)^2 - \left(\frac{2\pi}{2b}\right)^2 = \left(\frac{2\pi\nu}{c}\right)^2 - \left(\frac{\pi}{b}\right)^2 = \left(\frac{2\pi}{c}\right)^2 (\nu^2 - \nu_c^2). \qquad (1)$$

If the condition $\lambda_{\text{vac}} > 2b$ holds for the vacuum wavelength λ_{vac} at the corresponding frequency ν, the wave vector k is purely imaginary which is the « undersized » waveguide situation. The threshold $2b$ for λ_{vac} or the corresponding frequency value ν_c is known as cut-off condition. In analogy to the quantum tunneling process of a particle through a barrier, the waveguide can be regarded as a one-dimensional (1-dim) barrier for the electromagnetic (e.m.) wave ; in both cases there is an imaginary wave vector either for the matter or the e.m. wave inside the barrier. This mathematical analogy has been further interpreted and elaborated in several publications [5, 6].

The transport properties of e.m. waves in form of wave packets are calculated by a Fourier superposition of stationary solutions. For the barrier traversal, such an analysis has been given in detail by Hartman [7]. He has shown that under certain conditions (« thick » barrier) the form of the transmitted wave packet is substantially the same as the one of the incident packet and the traversal time is independent of barrier thickness. Furthermore the Fourier integration can be restricted to a certain frequency interval without changing these results : if all the frequency components mainly forming the wave packet behind the barrier have purely imaginary wave vectors within the barrier, higher frequency components having real wave vectors inside can be neglected. In short, the treatment is as follows : if a wave packet $F_{(t)}$ has travelled through a certain region in space, its new position and shape in time, $F'_{(t)}$, is given by the convolution of $F_{(t)}$ with the time response of that region. According to Fourier analysis the result $F'_{(t)}$ may be given by the following equation ($A_{(\nu)}$ is the inverse Fourier transform of the initial reference wave packet, e.g. represented by a Gaussian or similar distribution $A_{(\nu)}$ mainly situated within the integration interval $[\nu_1, \nu_2]$, and describes the incident pulse shape in the frequency domain ; $T_{(\nu)}$ is the frequency dependent transmission function within $[\nu_1, \nu_2]$) :

$$F'_{(t)} = \int_{\nu_1}^{\nu_2} A_{(\nu)} T_{(\nu)} e^{i2\pi\nu t} d\nu . \tag{2}$$

Without the boundary value problem at its in- and outputs the transmission function of a waveguide of length L below cut-off is given by $T_{(\nu)} = e^{ikL}$. Since k is purely imaginary (Eq. (1)), $T_{(\nu)}$ is a real quantity altering only the weighing of the integrand in equation (2) but not shifting the phase. Consequently the wave packet's shape will change more or less, depending on the special structure of $T_{(\nu)}$, but not its position in time. Making the barrier longer no additional time will be required for the transfer of a wave packet-superluminal transport velocities could be achieved if the time delay due to phase shifts at the boundaries is low enough. In this Short Communication we will demonstrate that exactly these conditions can be realized experimentally by the use of undersized waveguides.

Previously we have investigated the attenuation and the phase behavior of such a « photon barrier » as well as properties of resonant tunneling with respect to the analogies to quantum mechanical tunneling [8]. The measured properties of the cut-off sections were in perfect agreement with equation (1). Here we are going to present detailed traversal time data of opaque barriers where « opaque » is defined by the condition $|ikL| \gg 1$ and k being purely imaginary. Consequently multiple reflections inside the barrier are avoided due to the high attenuation and only the single section traversal is measured.

Experimental procedure.

Rectangular waveguides (X-band and Ku-band) were used, their cross-sections being $10.16 \times 22.86\ mm^2$ and $7.90 \times 15.80\ mm^2$, respectively. The cut-off frequencies are 6.56 GHz for the larger and 9.49 GHz for the smaller waveguide, the latter being used as the barrier below cut-off, see figure 1. The microwave measurements have been performed in the frequency domain with a HP8510B network analyzer (NA) system. The measured transmission functions $T_{(\nu)}$ are then transformed into the time domain. This direction of transformation has the advantage that « clean » conditions can be established : the wave packet can be superposed from frequency components being all below cut-off so that *a priori* the cut-off conditions are fulfilled (ν_1, $\nu_2 < \nu_c$, see Eq. (2)). On the other hand the conditions (frequency range and length of barrier) were chosen such that the barrier was not « too thick » : in that case a transmitted pulse in the time domain would contain quantitatively important frequency components around or above the cut-off frequency [7]. The restriction to integrate only frequency components below cut-off in equation (2) would be no longer applicable.

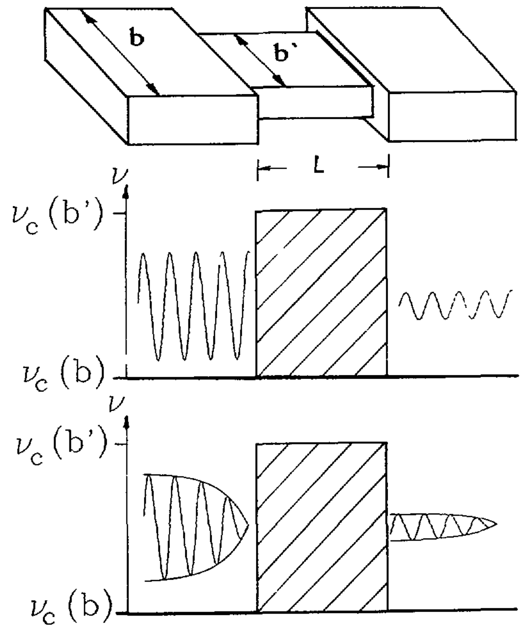

Fig. 1. — Experimental set-up with transitions from a larger waveguide to a smaller one (top) ; illustrations of the stationary measurements as done with the NA system (middle) and (below) amplitude modulated measurement conditions as done with the TA system.

First a calibration technique was performed using a line, a reflect and a matched calibration standard in X-band waveguide technique (so-called LRM-calibration) [9]. This eliminates all systematic measurement errors up to the connection of the two X-band waveguides where the Ku-band waveguide barriers were inserted ; consequently the frequency dependence of the total barrier transmission coefficient $T_{(\nu)}$ was measured (frequency range 8.2 to 9.2 GHz being at least 300 MHz below cut-off, 801 measuring points corresponding to a 1.25 MHz point to point distance). In figure 2 the phase shift of this arrangement is displayed for four different waveguide lengths (L = 40, 60, 80 and 100 mm) of the barrier and each measuring point. It is evident that the phase shift is equal for all four of them within an accuracy of better than $\pm 1°$ in phase. Only in the lower frequency range there are larger, but statistical deviations. Here the level of the transmitted signal for the longer barriers is so low that the accuracy of the phase detector in the NA decreases rapidly. It should be pointed out that a « normal » wave has a 1° phase shift in vacuum already after a distance of $L = 0.1$ mm ! The L-independence of the phase already shows that the measured phase shift can only be caused by the boundary conditions at the in- and outputs (« transition effects ») — the phase shift within the barrier is zero.

With these measured phase and amplitude data of $T_{(\nu)}$, the corresponding discrete Fourier transforms were performed into the time domain (the attenuation data are not shown in the figures because they are essentially represented by equation (1), i.e. the influence on them by the boundary effects at the waveguide narrowing transitions is negligible). Both for the barrier responses as well as for the reference conditions without the Ku-band waveguide these transformation were done. The NA function of bandpass transform to the time domain was used with window function on maximum which means that the frequency data are weighted with a Kaiser-Bessel function to give lowest side lobe levels in the time domain. This is a more

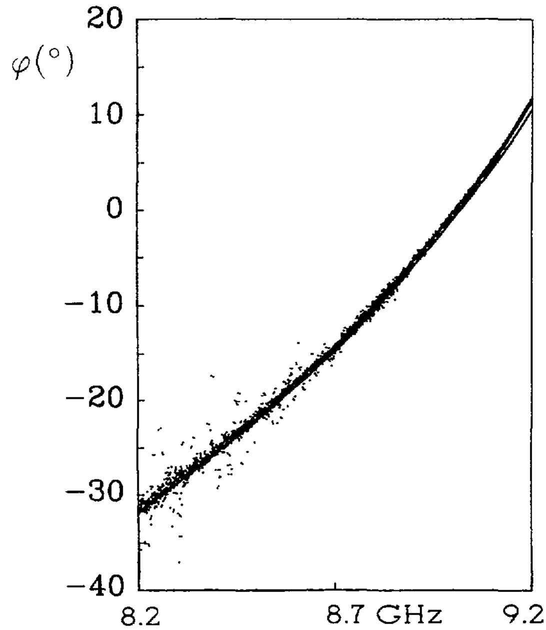

Fig. 2. — Measured phase shift φ *vs.* frequency of the total barrier transmission function $T_{(\nu)}$ (small waveguide including the transitions to the larger ones). All measured frequency points are presented for four lengths (L = 40, 60, 80 and 100 mm) of the cut-off section.

suited function $A_{(\nu)}$ in equation (2) than a truncated Gaussian one to construct a well-defined time-domain pulse shape, it does not influence the conclusions given below. In the following figures 3, 4 these pulse responses are normalized to the same unit amplitude (au) ; their absolute values at maximum are 0.999756 for $L = 0$ which is the reference value at the 5 ns time position in the figures, 0.06084 for the 40 mm barrier, 0.01323 for 60 mm, 0.00292 for 80 mm and 0.00064 for 100 mm. Notice, that the deformation of the pulses behind the barrier is negligible — it is not necessary to differentiate between the time point of the maximum of the pulse or its center of gravity.

Results and discussion.

In figure 3 the time domain pulse for the longest barrier (100 mm) is displayed. Obviously the pulse through this cut-off waveguide travels faster than that of a corresponding pulse in vacuum (130 ps compared to 333 ps). In figure 4 it can be seen that the time response of the different barrier lengths is nearly identical. The measured time delay of about 130 ps can only be caused by the waveguide transitions as has been already deduced from the phase measurements displayed in figure 2. The pulse transit through the barrier itself seems to be instantaneous — a direct consequence of our measurements together with equation (2) and the superposition principle of the linear Maxwell equations, nevertheless a very curious result. The linearity can, in our view, not be questioned in this framework. For a further proof, we have performed direct measurements in the time domain mode with a HP70820A transition analyzer (TA) system [8]. This eliminates the need for the transformation from equation (2). These delay time measurements could be performed only for the rising edge of a pulse (see Fig. 1

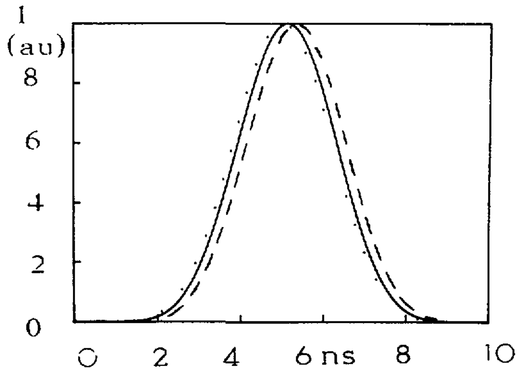

Fig. 3.

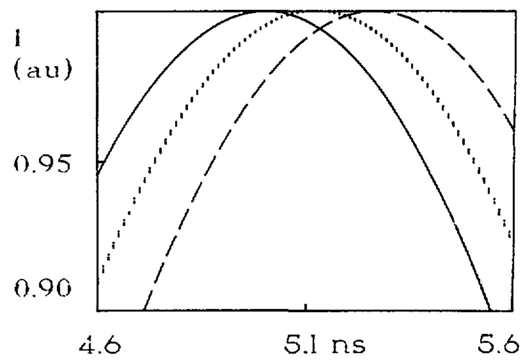

Fig. 4.

Fig. 3. — Reference pulse ($L = 0$ mm, dotted line) ; pulse transmitted through the barrier of length $L = 100$ mm (solid line) with about 130 ps time delay ; 333 ps time delay of reference pulse travelling at c over a distance of 100 mm in vacuum (dashed line).

Fig. 4. — Reference pulse ($L = 0$ mm, solid line) ; pulses of **all four** measured barrier lengths (four dotted lines which are nearly identical) ; pulse transmitted through the barrier of 100 mm length and an additional, « normal » (28.4 mm) X-band waveguide section (dashed line) which is operated above cut-off. The latter serves as a test for the time resolution accuracy under these measurement conditions.

below) for technical reasons and there was a significant contribution of above cut-off frequencies which limited the determination of the time delay. The generation and adequate shaping of pulses in the microwave region is far from trivial (the pulse is not allowed to have significant intensities from frequencies above cut-off even behind the barrier though the barrier shifts the frequency intensity spectrum upwards). Nevertheless the obtained results from these direct time domain measurements are consistent with the ones presented here namely that the pulse shapes are nearly identical before and behind the barrier including the possibility of a superluminal barrier traversal. Once more considering figure 4 the time delay differences between the different barrier section lengths are hardly resolvable. Looking at the numerical values and considering the experimental accuracy, especially in phase (see Fig. 2), we can conclude that the barrier traversal velocity obtained by this experimental procedure and evaluation is faster than the vacuum velocity of light.

Acknowledgements.

We are very grateful for helpful discussions with H. Spieker, P. Busch, F. Hehl, P. Mittelstaedt, R. Pelster and D. Stauffer and essential technical support by Hewlett-Packard (H. Aichmann, M. Hechler and W. Strasser).

References

[1] FOX R., KUPER C. G. and LIPSON S. G., *Proc. R. Soc. Lond. A* **316** (1970) 515.
[2] BOSANAC S., *Phys. Rev.* **A 28** (1983) 577.
[3] BAND W., *Found. Phys.* **18** (1988) 549.
[4] JACKSON J. D., Classical Electrodynamics (J. Wiley & Sons, New York, 1974).
[5] MARTIN T. and LANDAUER R., *Phys. Rev.* **A 45** (1992) 2611.
[6] RANFAGNI A., MUGNAI D., FABENI P. and PAZZI G. P., *Appl. Phys. Lett.* **58** (1991) 774.
[7] HARTMAN T. E., *J. Appl. Phys.* **33** (1962) 3427.
[8] ENDERS A. and NIMTZ G., submitted to *Phys. Rev. Lett.*
[9] EUL H.-J. and SCHIEK B., *IEEE-MTT* **39** (1991) 724.