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Gravity and density dependences of sand avalanches

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Résumé . — Nous démontrons en utilisant les résultats d'expériences faites en centrifugeuse (LCPC Nantes) ($10 - 1000 \text{ m/s}^2$) sur les avalanches de sable que la cohésion entre grain est négligeable, que la taille des avalanches et l'angle maximal de repos dépendent de la densité initial du tas, et qu'un angle de frottement interne peut être défini, dont la valeur correspond à celle mesurée par un essai triaxial de mécanique des sols. Ces données sont cohérentes avec un effet de dilatance qui dépend de la densité.

Abstract . — We demonstrate using centrifuge experiment ($10 - 1000 \text{ m/s}^2$) on sand avalanches that: i) grain cohesion is negligible, ii) the avalanche size and the maximum angle of repose depend on the initial pile-density, iii) an internal friction angle may be defined and corresponds to that measured with triaxial cell as assumed in soil mechanics. These data are coherent with a dilatancy effect which depends on density.

Recently much work has been done to understand the mechanism of sandpile avalanches [1-6]; this is mainly due to the pioneering model of Bak, Tang and Wiesenfeld [1, 2] (BTW) of self-organized critical state, which predicts large fluctuations of avalanche sizes and $1/f$ noise. On the other hand, most experimental investigations on real pile do not get such large fluctuations [3-5], except when very small piles are concerned [6]; furthermore, it has been found experimentally that the typical avalanche size scales as the pile volume and not as the pile surface [3-5], since one may define two different values of the free surface inclination before and after the avalanche has occurred. In order to interpret these facts, one may invoke the

effects of grain elasticity or of grains cohesion [7]. Recently however, one of us has proposed to make a parallel between sandpile-avalanche behaviour [5] and typical results of triaxial test in soil mechanics; (this test [5, 8] allow us to measure the effect of dilatancy and to relate it to the difference between the real pile density d and the so-called "critical" density d_c) [8]. Using such an analogy, he has been able to predict that the maximum angle of repose (MAR) should reflect mainly a unique friction coefficient ϕ , but be modified by dilatancy effect the size of which depends on the pile density. One way of separating dilatancy- and cohesion-effects is to vary gravity.

So, this work is devoted to study the physics of sandpile avalanches under gravity changes, using a centrifuge, to determine if the maximum angle of repose (MAR) and the avalanche size are governed by sandpile density, by grain cohesion or by grain elasticity and to demonstrate that sandpile avalanches may be understood within the same concepts as triaxial test results.

We have then prepared by pluviation different homogeneous parallelepipedic samples ($0.4 \times 0.4 \times 0.15 \text{ m}^3$) of the same "normalized" Hostun sand [9] at different densities and have performed on each of them a series of avalanches at different gravity fields using the 5.5 m-radius centrifuge of the L.C.P.C. in Nantes; this centrifuge allows us to apply to a 2000 kg mass a gravity field varying from 1 G to 100 G , ($G = 9.81 \text{ m/s}^2$). Due to sample- and centrifuge-sizes, the acceleration homogeneity is not better than 3%; vibrations inside the centrifuge may be neglected, but a special carenage has been designed to prevent any wind perturbation. Hostun sand [9] is a sieved ground silicate sand. The density range which may be achieved by varying the pluviation parameters (which are the flow and the fall height) is 1.4-1.7; but keeping the same pluviation parameters leads to the same pile density within less than 1%.

Fig. 1. — Sketch of the experiment which is loaded in the centrifuge.

The experimental avalanche device is sketched in figure 1: it is located in the centrifuge nacelle. It consists in a parallelepipedic container filled with Hostun sand, which can rotate slowly around a horizontal axis in order to create a set of successive avalanches; the flowing sand is collected in a box, where it is weighted. A video camera records the flow from the container so that we analyze the avalanche dynamics and measure the remaining sand height before and after each avalanche. We also measure the angle of rotation of the container. So, we can determine the angles of the free surface just before (θ_b) ($\theta_b = \text{MAR}$) and just after (θ_e) any avalanche, their difference $\delta\theta$ and the avalanche weight as a function of the gravity and of the density.

The container has been kept horizontal during the sample preparation by pluviation. The sample preparation lasts about 5 hours and is much longer than the real time needed at large gravity. So, we have decided to record a whole series of avalanches for each sample preparation, even if the real pile density was only well defined for the first avalanche of each sample.

It is known that sandpile exhibits history effect so that the i^{th} avalanche may depend not only on the initial pile density but also on the intertwining of gravity changes and avalanches so that a precise experimental protocol has to be defined. One way to proceed is to increase gravity to a given value G_1 and to perform a first series A_1 of few (N_1) avalanches; then to stop the container rotation, to change gravity to G_2 and to produce a second series A_2 of N_2 avalanches; and so on till the sand level reaches the container bottom. Our protocol is then defined by the set of $(\dots, G_i, A_i, N_i, \dots)$. Furthermore, looking for a protocol dependence, we have tried two different procedures: for the first five samples which cover the whole range of possible densities (1.42-1.68), we have proceeded in three steps, imposing an increase of gravity $G_1 < G_2 < G_3$ at each step; G_1, G_2, G_3 turn out to vary slightly with the samples but in the ranges $G_1 = 20G - 25G$, $G_2 = 40G - 50G$, $G_3 = 100G$, with N_1, N_2, N_3 equal to 2-4 depending on the run. We report θ_b and $\delta\theta$ data obtained with in this procedure in figure 2. The parameter g which is plotted as the x axis of these figures has been called "gravity" g since it is mostly the gravity G_i really applied during the avalanche sequence A_i , but it takes also into account memory effects by including in g a small part which depends on the number j of the avalanche in the sequence A_i containing N_i avalanches ($g = G_i + (j - (N_i + 1)/2) \times G$). So that chronology effects are visualized: labelling j the j^{th} avalanche of the sequence A_i , containing N_i avalanches ($N_i = 2$ - to - 4), we remark that this notation keeps the mean g value of the sequence at the real applied G_i . This is peculiarly useful in the case of loose packing for which we observe an increase of θ_b all along the two avalanche sequences at $G_1 = 250 \text{ m/s}^2$ and $G_2 = 500 \text{ m/s}^2$ (Fig. 2a) demonstrates that loose and dense packings do not lead to the same characteristics of the first avalanches (as far as these behaviours are reproducible).

In the second protocol, G_i has been chosen to decrease: we have performed $N_1 = 8$ avalanches at $G_1 = 100G$ then $N_2 = 2-4$ avalanches at $G_2 = 2G$. We report in figure 3, θ_b and $\delta\theta$ data corresponding to this last procedure as a function of the number of the avalanche (from 1 to 12, gravity G_i is printed in superscript). This second set of experiments was devoted also to check the reproducibility of the results and we have repeated this experiment twice on the two extreme packing densities. Indeed, figure 3a demonstrates that the data are reproducible (within a given uncertainty).

Comparing now figure 2a with the left hand part of figure 3a (i.e. the data at $100G$) confirms that our experiments are reproducible. We conclude also that the evolutions of θ_b are very similar in both cases so that it depends mainly on the initial density of the pile, but does not really depend on the value of the gravity within the investigated range ($20G$ - $100G$). Comparing the results at $100G$ of the figure 3a with those at $2G$ on the right hand part of this one proves the existence of a history effect when decreasing the gravity, which will be discussed later. Let us now come back to the first part of these results: θ_b evolves spontaneously as a function of

Fig. 2. — Avalanche characteristics, for Hostun sand packed at different initial densities, as a function of the gravity field, according to protocol 1 (see the text). a) The maximum angle of repose θ_b . The dashed line of figure 2a corresponds to the mean θ_b asymptotic value, which is also about the friction angle of the “critical state” of soil mechanics as measured from triaxial test [5, 8, 9]. b) The avalanche size $\delta\theta$.

the number of the avalanche in the series and its evolution depends on the pile initial density but converges eventually to a common limit value which is about 34° , independent of the initial density; but, the precise evolution depends on this density (θ_b is continuously increasing for the loose packings ($d = 1.45$), remains constant (within the experimental fluctuations) for intermediate packings and decreases for the highest initial densities).

These behaviours may be understood within the framework of the “critical” state model of avalanches [5]: let us assume that the pile density d near the free surface evolves spontaneously due to successive avalanche processes till it reaches the so-called “critical” state density eventually. In this case the limit value of θ_b (i.e. 34°) will be equal to the internal friction angle ϕ measured at large deformations by a triaxial cell apparatus, so that it will be independent of d . From the literature, we find that $\phi = 33\text{--}35^{\circ}$ for Hostun sand which is the value found here. Furthermore, this theoretical approach agrees also with the facts that θ_b should increase continuously when the initial pile is loose and decrease when the initial pile is dense, which confirms then the “critical” state analysis.

However the “critical” state analysis does not explain the existence of macroscopic events when the pile is loose since this model predicts a small and continuous evolution of the pile slope instead of the observed macroscopic variations of the slope inclination $\delta\theta = 1^{\circ}$ as exhibited in figures 2b and 3b for $d = 1.42, 1.45, 1.47$. Moreover, the two first events obtained with each

Fig. 3. — Avalanche characteristics, for Hostun sand packed at different initial densities, as a function of the avalanche number in a same series, according to protocol 2. a) The maximum angle of repose θ_b . b) The avalanche size $\delta\theta$.

very loose packings correspond to a variation of the slope free surface of about 1° each without any flow since we have found by measuring the weight of the flowing sand using the box device of figure 1 that these events do not correspond to a real flow (no sand has been recovered in this box after these events); this demonstrates that these two first events correspond to a macroscopic change of the mean volume of 5% so that the mean pile density after these two events is about 1.6. To the best of our knowledge, this is the first evidence of such a macroscopic densification process. This phenomenon cannot be understood in the framework of the "Granta-Gravel" model [5] of sand avalanches where one assumes that the density controls the avalanche process completely, thus proves the existence of at least an additional controlling parameter, which allows the densification process to be a subcritical bifurcation in general. This might perhaps be understood within the framework of Evesque and Sornette [10] on the dynamical system approach of the deformations in non-cohesive soil which suggests that deformation localizations in soil are subcritical bifurcations in general. We want however to emphasize that this discontinuous densification does not deny the role plays by density, which is really one of the controlling parameters of the avalanche size as shown in figures 2b and 3b; it only demonstrates that it is not the only one.

Turning now to the variations of the avalanche size, (cf. Figs. 2b and 3b), we note first that a correlation exists during the first avalanches of each series between the initial density and the avalanche size $\delta\theta$ (the larger the density the larger the avalanche); for instance we find first avalanche sizes of about $\delta\theta = 4^\circ$ for the densest packing instead of 1° or 2° for the

looser packings. This feature is in agreement with our approach based on soil mechanics and on triaxial test results. We note also that after this transient regime a limit regime is observed which exhibits size fluctuations, but which leads to a mean avalanche size $\langle\delta\theta\rangle$ of about 2° . This 2° value compares well with the other data from the literature [3-6], obtained with other devices on different silicate granular materials (glass spheres of different diameters, sand of different size). As $\langle\delta\theta\rangle$ does not depend on the gravity as shown by figure 2b, we may conclude that cohesion effect between grains is negligible.

In the same spirit, we note that the limit value of θ_b fluctuates within 2° . This width compares well to the mean limit size of the avalanche $\langle\delta\theta\rangle = 2^\circ$ as found from figures 2b and 3b data, so that the physics of the avalanche seems to be governed by an erratic process as if the internal friction angle could not be defined within an accuracy better than 2° . We can try and test this hypothesis and look at triaxial test curve to demonstrate the existence of such fluctuations in the internal friction angle ϕ . It is accepted that the scatter of the experimental data from a triaxial test to an other one allows to measure the friction angle within an accuracy of 2° , since large deformations without yieldings and localizations in a homogeneous sample are extremely difficult to obtain; however, if we examine carefully each triaxial test curve, we find that the stress-strain experimental curves are smooth in general; its fluctuations correspond to $\delta\phi = 0.5^\circ$. So it seems to us that we cannot really conclude whether the erratic size of the avalanche is due to a random distribution of the friction angle as a function of the strain (or of space), or if these $\langle\delta\theta\rangle$ fluctuations in avalanche experiment may result from dynamical amplifications of initial fluctuations.

In any case we note that we have not yet succeeded to get avalanche sizes tending to 0, so that we cannot yet approach the conditions under which critical fluctuations of avalanche size on large samples may be observed [5]. However, we have repeated the experiment reported by Held *et al.* [6] using a similar device (conic pile and very gentle pluviation on the pile top). We have obtained similar results: we confirm then that one may obtain critical fluctuations of the avalanche size for piles small enough; but there is a crossover to classical fluctuations, with $\delta\theta = 2^\circ$, when the pile slope is long enough, i.e. when it exceeds 30 to 40 bead diameters ($2R$). We have measured the critical exponent governing the probability distribution of avalanche masses $P(m)$ in the critical regime and have found the same behaviour (i.e. $P(m) = m^{-1.6}$). We notice also that the limit ratio $L_M/2R$ between the slope length L_M and the bead size $2R$ which defines the crossover between the critical behaviour and the subcritical one is such as $\langle\delta\theta\rangle L_M = 2R$ (where R is the grain size). As $\langle\delta\theta\rangle$ is also the accuracy $\delta\phi$ with which the friction angle ϕ may be measured with a slope-stability experiment in the macroscopic regime, this relationship $\langle\delta\theta\rangle L_M = 2R$ confirm the analysis of the crossover is verified.

We now analyse the right hand part of the data of figures 2a and 3a. These data demonstrate the existence of a history effect when one decreases the gravity field. This history effect is larger for the first events and for denser piles and it diminishes as and when avalanches run; however it does not vanish after a given number of avalanches since the limit value of θ_b is 38° at $2G$ instead of the 34° found previously. We note also this history effect may create events as large as 8° (Fig. 3b). In order to understand these features one shall probably invoke that the successive avalanches modify the density of the pile at the surface in order to reach an equilibrium corresponding to θ_b , but shall add also that a strong gravity field modifies the distribution of contacts more deeply in the pile so that the pile may resist to a larger shearing. This last effect is responsible probably of the history behaviours as exhibited by figure 3. But the determination of the exact hysteretic processes undoubtedly requires much more data.

As a conclusion, we think that these experiments demonstrate the existence of a density effect which is partly in agreement with triaxial data [5] and with a "critical" state analysis [5]: in particular, we have shown that the denser the packing the larger the maximum angle of

repose and that the first avalanche size $\delta\theta$ depends on the packing density. We have found that a limit regime is reached after a few avalanches. It is characterized by a typical avalanche size $\langle\delta\theta\rangle = 2^\circ$. These results seem to us very stimulating since they demonstrate that the typical limit avalanche size ($\langle\delta\theta\rangle = 2^\circ$) is gravity independent, and that a macroscopic discontinuous densification occurs in the case of very loose packings. These two effects were not known and not forecast by soil mechanics.

As a final remark, a part of the motivation behind this work was motivated by the research of a parameter controlling the avalanche size; it succeeded clearly to demonstrate that density is one of them (as we thought), but we have not yet been able to make the avalanche size to vanish and to get a state of critical bifurcation, obeying the BTW model, as we were hoping. We are then reduced to seek for an other controlling parameter; so we hope that this work will stimulate further developments of avalanche experiments in centrifuge.

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