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On galvanomagnetic effects in layered conductors

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Abstract. — In the quasiclassical approximation we analyze the dependences of the resistivity $\rho$ and Hall field on the magnetic field orientation and intensity in layered conductors with quasi-2D energy spectrum of arbitrary form. The Shubnikov-de Haas amplitudes are shown to drop abruptly and the smooth part of magnetoresistance to increase at certain orientation of $H$ when the respective extreme (Fermi surface) FS cross section is self-intersecting. Anisotropy of $\rho$, depending essentially on FS topology, manifests itself most when the current flows across the layers.

In recent years, interest in physical properties of layer conductors has grown considerably. A wide class of superconductors - NbSe$_2$, TaS$_2$, dihalogenides of transition metals, cuprate-based metal - oxide compounds, organic conductors such as salts of tetrathiafulvalen (BEDT-TTF), halogens of tetrathiafulvalen (TSeT), etc. - represents layered structures with a sharp anisotropy of electrical conductivity in the normal (non-superconducting) states. They have an anomalously low electrical conductivity along the n direction normal to a certain plane. The experimentally observed Shubnikov-de Haas oscillation of magnetoresistance in organic conductors (see, for example [1-10] and Refs. there in) and metal type of electrical conductivity in most of them make it quite reasonable to believe that the well founded concept of charge carriers in metals is also applicable for describing electron properties of the layered conductors. Nevertheless the only way to prove the correctness of using the concept of elementary excitations, similar to conduction electrons in metals, is the solution of the inverse problem of reconstructing the Fermi surface (FS) from experimental data. In particularly, topology of FS, can be reliably determined by studying galvanomagnetic phenomena [11].

Let us consider a conductor with quasi-two dimensional electron energy spectrum, placed in d.c. uniform magnetic field $H$. A small curvature in the direction normal to layers is characteristic of all FS geometries of sharply anisotropic conductors. Therefore it is quite justified to make use in this case of the strong coupling approximation. It means that a coefficient $A_n$ in the expression

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for the energy of charge carriers

$$\epsilon(p) = \sum_{n=0}^{\infty} A_n(p_x, p_y) \cos(np_z/hq)$$

(1)

can be assumed to be rapidly decreasing with n and all $A_n$ at $n \geq 1$ to be much less than Fermi energy $\epsilon_F$. Here $h$ is the Planck's constant,

$$q = |a \times b|/a(b \times c),$$

(2)

and $a, b, c$ the basic vectors of the single crystal, forming the primitive elementary cell, with $a$ and $b$ lying in the layer plane ($x, y$).

A weak dependence of the energy on the electron momentum projection $p_z$ is due to the slow charge carriers motion across the layers. The velocity of this motion

$$v_z = \partial \epsilon/\partial p_z = -\sum_{n=1}^{\infty} (n/hq) A_n(p_x, p_y) \sin(np_z/hq)$$

(3)

is considerably smaller than the maximum drift velocity $v_F$ of charge carriers in the layer plane. The condition

$$v_z^{\max} = v_0 = \eta v_F \ll v_F$$

(4)

will be assumed below to be always satisfied.

The isoenergetic surface $\epsilon(p) = \epsilon$, depending on the type of the function $A_0(p_x, p_y)$, can be either a system of weakly corrugated cylinders (isolated or interconnected by thin links), or elongated closed surfaces in the momentum space, when $\epsilon$ is near to the energy band boundaries.

To determine the electric current density

$$j_i = \int d^3p 2e v_i f(p)/(2\pi h)^3 = \sigma_{ij} E_j$$

(5)

it is necessary to solve the Boltzmann equation for the distribution function of charge carriers

$$f(p) = f_0(\epsilon) - eE \psi \partial f_0/\partial \epsilon,$$

which within a linear approximation with respect to weak electrical field $E$ and $\tau$-approximation of collision integral $W[f] = -(f - f_0)/\tau$ has a rather simple form

$$\partial \psi/\partial t + \psi/\tau = v.$$ (6)

Its solution

$$\psi_i = \int_{-\infty}^{t} \exp((t' - t)/\tau)v_i(t') dt'$$

(7)

makes it possible to represent the components of conductivity tensor in the following form [11]:

$$\sigma_{ij}(H, \eta) = \langle v_i \psi_j \rangle = \frac{2e^3 H}{c(2\pi h)^3} \int_{\epsilon(p) = \epsilon_F} dp_H \int_0^t dt v_i(t) \int_{-\infty}^{t'} dt' v_j(t') \exp \left( \frac{(t' - t)}{\tau} \right)$$

(8)

It can also be done in the Fourier representation of electron velocities:

$$\sigma_{ij}(H, \eta) = \frac{4\pi e^2 \tau}{(2\pi h)^3} \int m^* dp_H \sum_{k=-\infty}^{+\infty} v_i^{(-k)} v_j^{(k)} (1 + ik\Omega \tau)^{-1}$$

(9)
Here $e$, $\tau$ and $f_0(e)$ are the charge, the mean free time and Fermi distribution of conduction electron, respectively, $t$ and $\Omega = eH/m^*c$- the time of its motion and gyrofrequency in the magnetic field on the orbit $e(p) = \text{const}$, $p_H = pH/p = \text{const}$, $m^*(\epsilon, pH)$ is the cyclotron effective mass, $c$- the light velocity in vacuum, $T = 2\pi/\Omega$,

$$
v_j^{(k)} = T^{-1} \int_0^T v_j(t) \exp(-ik\Omega t)dt.
$$

In the base of a two-dimensional energy spectrum of conduction electrons the conductivity tensor components $\sigma_{iz}$ and $\sigma_{zj}$ go to zero, and $\sigma_{iz}$ ($\alpha, \beta = x, y$) depends only on the magnetic field projection $H_z = H \cos \theta$. In this case the electrical resistivity of the sample along layers, $\rho$, depends on $H$ and $\theta$ in a universal manner, namely

$$
\rho = \rho(H \cos \theta)
$$

One can easily prove it by making use of the equations of charge motion in the magnetic field

$$
\begin{align*}
\partial p_x/\partial t &= (v_y \cos \theta - v_z \sin \theta) eH/c; \\
\partial p_y/\partial t &= -(v_x \cos \theta) eH/c; \\
\partial p_z/\partial t &= (v_x \sin \theta) eH/c.
\end{align*}
$$

The $x$ axis is normal to the vector $\mathbf{H}$ and $\mathbf{n}$. In the case of weakly corrugated FS the angular dependence of the component $\sigma_{iz}$ in a broad $\theta$ range has to be only slightly different from the dependence $\sigma_{iz}(H \cos \theta)$ as long as the parameter $\eta$ remains small and only at certain angles $\theta = \theta_c$ the relative value of correction to the resistance of the type (10) can be of the order of unity. However, the dependence of $\sigma_{iz}$ and $\sigma_{zj}$ on $\theta$ will be substantially different because there are no zero order approximations in parameter $\eta$ in those components of the conductivity tensor.

1. An asymptotic expression for $\sigma_{zz}$

$$
\sigma_{zz}(H, \eta) = \frac{2e^2H}{c(2\pi\hbar)^3(Hq)^2} \sum_{n, l=1}^\infty n! \int_0^{2\pi\hbar} \cos \theta d\phi_H \int_0^T dt \int_{-\infty}^t dt' \exp \left( \frac{t' - t}{\tau} \right) \cdot A_n(p_H, t) \times
	imes A_1(p_H, t') \cdot \sin \left( \frac{nL}{\hbar q \cos \theta} (p_H - p_y (p_H, t) \sin \theta) \right) \sin \left( \frac{1}{\hbar q \cos \theta} (p_H - p_y (p_H, t') \sin \theta) \right)
$$

(12)

at $\eta \ll 1$ can easily be calculated if one makes use of the slow dependence of $p_y (p_H, t) = p_y (t) + \eta \Delta p_y (p_H, t)$ on $p_H$ at small $\eta$. Taking into account this slow dependence one gets only small corrections to $\sigma_{zz}$, which are of higher orders than $\eta^2$. Retaining only terms proportional to $\eta^2$ one gets the following asymptotic expression for $\sigma_{zz}$.

$$
\begin{align*}
\sigma_{zz}(H, \eta) &= \tau \frac{2e^2 m^* \cos \theta}{(2\pi\hbar)^3 \hbar q} \int_0^{2\pi} d\phi \int_{-\infty}^\phi d\phi' \exp(\phi (2\phi' - \phi)) \sum_{n=1}^\infty n^2 A_n(\phi) A_n(\phi) \times
	imes \cos(\chi_n(\phi) - \chi_n(\phi')); \\
\chi_n(\phi) &= np_y(\phi) \frac{\tan \theta}{\hbar q}; \gamma = (\Omega t)^{-1}; \phi = \Omega t; \phi' = \Omega t'.
\end{align*}
$$

(13)

Although components $\sigma_{xz}$ and $\sigma_{zx}$ contain only terms linear in $v_x$ they are, nevertheless, of the order of $\eta^2$, since $v_o$ and $p_o$ (at $\eta \to 0$) are independent of $p_H$ so that at any magnetic field
there are no terms proportional to $\eta$ in the expressions
\[
\sigma_{z\alpha} = -\frac{2e^2\tau}{(2\pi\hbar)^3\hbar} \int_0^{2\pi} \int_0^{2\pi} \int_{-\infty}^{\infty} d\phi m^* \gamma d\phi \int_{-\infty}^{\infty} d\phi' \exp (\gamma (\phi' - \phi)) \sum_{n=1}^{\infty} nA_n (p_H, \phi') \times \\
\times v_{\alpha} (p_H, \phi) \sin \left( \frac{n (p_H - p_y (p_H, \phi') \sin \theta)}{\hbar \cos \theta} \right);
\]
\[
\sigma_{z\alpha} = -\frac{2e^2\tau}{(2\pi\hbar)^3\hbar} \int_0^{2\pi} \int_0^{2\pi} \int_{-\infty}^{\infty} d\phi m^* \gamma d\phi \int_{-\infty}^{\infty} d\phi' \exp (\gamma (\phi' - \phi)) \sum_{n=1}^{\infty} nA_n (p_H, \phi') \times \\
\times nA_n (p_H, \phi) \sin \left( \frac{n (p_H - p_y (p_H, \phi) \sin \theta)}{\hbar \cos \theta} \right).
\]
\[
\text{(14)}
\]
\[
\text{(15)}
\]
Therefore, when the current flows across the layers ($j \parallel n$) the resistivity can be written as
\[
\rho_{||} = \sigma_{z\alpha}^{-1}
\]
\[
\text{(16)}
\]
to a sufficient accuracy. This resistivity steadily grows with magnetic field and reaches the saturation level at sufficiently high field intensities. Since $\varepsilon (p)$ is an even function one can reduce $\sigma_{z\alpha}$ in the limit of strong magnetic fields ($\gamma \ll 1$) to the following form:
\[
\sigma_{z\alpha} (\infty, \eta) = 4e^2\tau m^* (2\pi\hbar)^{-3}(\hbar)^{-1} \cos \theta \sum_{n=1}^{\infty} \left( n \int_0^{\pi} A_n (\phi) \cos (\chi_n (\phi)) d\phi \right)^2
\]
\[
\text{(17)}
\]
\[
\sigma_{z\alpha} (\infty, \eta) = 4e^2\tau m^* (2\pi\hbar)^{-3}(\hbar)^{-1} \cos \theta \sum_{n=1}^{\infty} \left( n \int_0^{\pi} A_n (\phi) \cos (\chi_n (\phi)) d\phi \right)^2
\]
\[
\text{(18)}
\]
Therefore even if $A_n = 0$ for $n \geq 2$ the asymptotic expression for $\sigma_{z\alpha}$ in strong magnetic fields at some $\theta$ values equal to $\theta_c$ is proportional not to $\eta^2$ but to some higher power of $\eta$. In the latter case the higher terms of expression of $\sigma_{z\alpha}$ in $\gamma$ become rather substantial, and as long as strong inequality $(A_{n+1} \ll A_n = A_{n}^{\max} (p_z, p_y))$ is hold, the resistivity $\rho$ even for $\gamma \ll 1$ is a growing function of magnetic field at
\[
(\eta^2 + A_z^2 / A_{\alpha z}^2) \ll \gamma^2 \ll 1
\]
\[
\text{(19)}
\]
and saturates at higher magnetic field than in the case $\theta \neq \theta_c$. This brings to a substantial anisotropy of magnetoresistance when the magnetic field deviates from the normal to the layer planes.

If the current flows in the layer plane
\[
\hat{j}_\alpha = \tilde{\sigma}_{\alpha\beta} E_\beta; \quad \tilde{\sigma}_{\alpha\beta} = \sigma_{\alpha\beta} - \sigma_{z\alpha} \sigma_{z\beta} / \sigma_{z\alpha}
\]
\[
\text{(20)}
\]
a considerable growth of the resistivity with increasing $H$ is possible if there are thin links between cylinders with the thickness of order of $\eta$. Such links give rise to open FS section oriented along $p_z$. In this case $\sigma_{yy}$ involves a term of the order of $\sigma_{0\eta}$ and the resistivity can be written [11] as
\[
\rho = \rho_0 \left( 1 + \eta^{-2} \cos^2 \phi \right)
\]
\[
\text{(21)}
\]
where \( \phi \) is an angle between the \( j \) vector and the \( x \) axis,

\[
\sigma_0 = \sigma_{yy}(0, \eta) \quad \text{and} \quad \rho_0 = \sigma_0^{-1}
\]

Let us assume that the difference between the numbers of electrons and holes per unit volume, \( N \), is nonzero.

Making use of equation of charge motion (11) one can easily show that at \( \gamma \ll 1 \) and in the absence of open electron trajectories, the following relations are valid:

\[
\begin{align*}
\sigma_{xx} &= \frac{\langle e^2 p_y^2 \rangle}{(eH \tau \cos \theta)^2}; \\
\sigma_{xy} &= \sigma_{yy} \cos \theta - \sigma_{xx} \sin \theta = \frac{Ne c}{H}; \\
\sigma_{yx} &= (\sigma_{yx} + \sigma_{xy}) \tan \theta + \sigma_{zz}(\tan \theta)^2 = \frac{\langle e^2 p_z^2 \rangle}{(eH \tau \cos \theta)^2}; \\
\sigma_{yz} &= \sigma_{zy} = \left( c p_x(t) \int_{-\infty}^{+\infty} dt' \exp \left(-\frac{|t-t'|}{\tau} \right) v_z(t') \right)
\end{align*}
\]  

(22)

and the resistivity, for example, for the current parallel to \( x \), has the following asymptotic form:

\[
\rho = \rho_{xx} \approx \sigma_{yy} \left( \frac{H \cos \theta}{N ec} \right)^2 = \frac{\langle p_z^2 \tau \rangle}{(Ne^2 \tau)^2} + \left( \frac{H \cos \theta}{2Ne c} \right)^2 \times 
\]

\[
\times \left( \frac{(\sigma_{yz} - \sigma_{zy})^2 - (\sigma_{xy} + \sigma_{yz} - 2\sigma_{zz} \tan \theta)^2}{\sigma_{zz}} \right)
\]

(23)

Resistance anisotropy is manifested only in small corrections proportional to \( \eta^2 \) which depend rather substantially on the orientation of the magnetic field with respect to the crystallographic axes. If \( \sigma_{yz} - \sigma_{zy} \approx 2\gamma \sigma_{yz} \), and \( \sigma_{yz} \) is nonzero, then at some values of \( \theta = \theta_c \) and magnetic fields satisfying the condition (19) when \( \sigma_{zz}(H) \gg \sigma_{zz}(\infty) \), the resistivity is growing function of the magnetic field. In this case the resistivity saturation level is reached with growing \( H \) only in a rather high field, when \( (\sigma_{zz}(H) - \sigma_{zz}(\infty)) \approx \sigma_{zz}(\infty) \), that is when at least the condition \( \gamma^2 \ll \left( \eta^2 + (A_2/A_1)^2 \right) \) is satisfied. A maximum in the angular dependence \( \rho(\theta) \) appears at the same \( \theta \) as those at which \( \rho_{||} \) is maximal. If \( \sigma_{yz}(H) = \sigma_{zy}(H) \), then at \( \theta = \theta_c \) the resistivity \( \rho \) is minimum in the magnetic fields satisfying the condition (19). In sufficiently strong magnetic fields the last term in (23) is proportional to \( H^{-2} \) and thus may be omitted.

In the weak coupling approximation in the layer plane, which is widely used in various calculations (see for example [12-14]), when

\[
A_0(p_x, p_y) = p_x^2/2m_1 + p_y^2/2m_2,
\]

(24)

the resistivity in the layer plane is completely independent of the magnetic field. Making use of the relation (22) and the equation (11) one can readily show that all components of the \( \sigma_{ij} \) matrix can be expressed through \( \sigma_0 \) and \( \sigma_{zz} \)

\[
\begin{align*}
\sigma_{ij} &= \begin{pmatrix}
\gamma_2 \sigma_{xy} & \sigma_{xy} & -\sigma_{zz} \gamma_2 (1 + \gamma_1 \gamma_2)^{-1} \tan \theta \\
-\sigma_{xy} & \sigma_0 - \gamma_2^{-1} \sigma_{xy} & \sigma_{zz} (1 + \gamma_1 \gamma_2)^{-1} \tan \theta \\
\sigma_{zz} \gamma_2 (1 + \gamma_1 \gamma_2)^{-1} \tan \theta & \sigma_{zz} (1 + \gamma_1 \gamma_2)^{-1} \tan \theta & \sigma_{zz}
\end{pmatrix};
\end{align*}
\]  

(25)
\[ \sigma_{xy} + \sigma_{zz} \gamma_2 \left( \frac{\tan \theta}{1 + \gamma_1 \gamma_2} \right)^2 = \frac{\sigma_0 \gamma_2}{1 + \gamma_1 \gamma_2}; \quad \gamma_i = \frac{c m_i}{e H \tau \cos \theta}; \quad i = 1, 2. \]  

(26)

Rather simple calculations result in following expressions of the resistivity, \( \rho \), and the Hall field \( E_{\text{Hall}} \):

\[ \rho = \left( m_1 \sin^2 \phi + m_2 \cos^2 \phi \right) / N e^2 \tau; \quad E_{\text{Hall}} = (j \times H) / N e c \]

(27)

If the electric current is normal to layers then even in a rather simplified model of electron energy spectrum of type (24) the resitivity

\[ \rho_{\|} = \sigma^{-1}_{xx} + \sigma^{-1}_{\phi}(\tan \theta)^2 / (1 + \gamma_1 \gamma_2) \]

(28)

depends on the magnetic field intensity and orientation, and the Hall components are

\[ E_x = j H \sin \theta / N e c; \quad E_y = 0. \]

(29)

It should be noted that \( \rho_{\|} \) is only weakly affected by the shape of the \( A_0(p_x, p_y) \) function and the formula (28), which is valid for any magnetic field intensity, describes correctly at least qualitatively the behavior of magnetoresistance. The fact that \( \rho \) is independent of magnetic field as indicated by the formula (27) is a consequence both of the parabolic form of the momentum dependence of \( A_0(p_x p_y) \) and of the assumption that other charge carrier groups can be ignored. Violation of only one of the above condition gives rise to a magnetoresistance, \( \Delta \rho \).

At \( \tan \theta \gg 1 \) when electrons move on closed sharply elongated orbits the main contribution to the integral over \( \varphi \) in the formula (17) comes from narrow vicinities around the steady phase points where \( V_x \approx 0 \). In the case of a weakly corrugated FS there are only two turning points on the orbit. Those are points at \( \varphi = \varphi_1, \varphi_2 \) where \( p_y(\varphi_1) = p_y^{\text{max}} \) and \( p_y(\varphi_2) = p_y^{\text{min}} \). Retaining only terms of order of \( \eta^2 \) one can easily obtain an explicit dependence of \( \sigma_{zz}(\infty) \) on \( \theta \) for arbitrary dispersion law of charge carriers in a quasi-two-dimensional conductor.

\[
\sigma_{zz}(\infty, \eta) = \frac{4 \pi e^2 \tau \cos^2 \theta}{(2 \pi \hbar)^2 |v'_x(\varphi_1)| \sin \theta} \sum_{n=1}^{\infty} n A_n^2(\varphi_1) \left(1 + \sin \frac{n D_p \tan \theta}{\hbar q} \right) \]

(30)

\[ D_p = p_y^{\text{max}} - p_y^{\text{min}}, \quad v'_x(\varphi) = \frac{\partial v_x(\varphi)}{\partial \varphi} \]

The \( n \)th term reaches its minimum at an angle \( \theta = \theta_n \) which satisfies the condition \( n \tan \theta_n = \tan \theta_c = -2 \pi \hbar q / D_p \left( M - \frac{1}{4} \right) \) where \( M \) is an integer. It means that the asymptotic value \( \sigma_{zz}(\infty) \) is nonzero at any angle \( \theta \). If one assumes that \( A_n \) decreases with growing \( n \) as \( \eta^n \) then the terms with \( n \geq 2 \) in the formula (30) can be omitted.

Thus, investigating the dependence of the resistivity on the orientation of the magnetic field one can determine the shape of the FS section by the plane \( p_H = \text{const} \) and some other details of the electron energy spectrum.

If \( \theta \rightarrow \pi / 2 \) the condition \( \gamma \ll 1 \) cannot be satisfied at any value of the magnetic field, since the effective cyclotron electron mass \( m^* = (2 \pi \cos \theta)^{-1} \oint dp_y / v_x \) in this case diverges. At \( \theta = \pi / 2 \) the resistivity \( \rho \) is a parabolic function of \( H \) because of the carriers drifting along open trajectories.

2. If \( \eta \tan \theta > 1 \) or in the case, when \( A_n \) decreases with growing \( n \) slower than \( \eta^n \), then also at \( \theta \approx 1 \), the FS can contain a self-intersection point at \( \mathbf{p}^0 \), where

\[ V_y(\mathbf{p}^0) \cos \theta - V_z(\mathbf{p}^0) \sin \theta = 0, \quad V_x(\mathbf{p}^0) = 0, \quad \epsilon(\mathbf{p}^0) = \epsilon_F \]

(31)
The rotation period of the charge carriers in a small layer in the neighbourhood of the self intersecting orbit $p_H = p_H^0$ diverges logarithmically when $p_H \to p_H^0$:

$$T = 2\pi \Omega_0^{-1} \ln \left| \frac{p_F}{(p_H - p_H^0)} \right|, \quad p_F = \hbar q$$

and the condition of the strong magnetic field ($\Omega \gg 1$) cannot be satisfied at all $p_H$ values. Contribution of these conduction electrons to the conductivity tensor components does not satisfy the Onsager principle of kinetic coefficient symmetry and can substantially affect the magnetoresistance behaviour in magnetic fields that satisfy the condition $\Omega \gg 1$. Here $\Omega_0$ is a rotation frequency of an electron in magnetic field on an orbit that is removed sufficiently far from the self-intersecting one.

Averaging in the formula (9) for $\sigma_{ij}$ over $p_H$ values that are found on FS in a small layer $\delta_p = p_H - p_H$ resembles averaging the polycrystal resistance over all crystallite orientations when FS is open and electron rotation period on strongly elongated orbits may also take arbitrary great values $[11,15]$.

A fraction of electron orbits $\delta_p/p_F$ for which $\Omega \ll 1$ exponentially decreases with the rise of $\Omega \gg 1$, as can be seen in the formula (32). Therefore, their contribution $\Delta \sigma_{ij}$ to the conductivity tensor components can be neglected if the magnetic field is sufficiently strong. Nevertheless, there exists a magnetic field range, where $\Delta \sigma_{ij}$ exceeds the contribution of all other electrons to $\sigma_{ij}$. Let us evaluate the contribution of electrons in $\sigma_{ij}$, for which $\Omega \ll 1$, supposing that the Fourier components of the electron velocity $v_j^{(k)}$ in a region of not very large values of $k$ decrease when $k$ grows:

$$v_j^{(k)} = (2\pi)^{-1} \int_0^{2\pi} v_j(\varphi) \exp(-ik\varphi) d\varphi = \left( v_j^2 - \overline{v_j^2} \right)^{1/2} \alpha_j/k^q; \quad q \geq 1; \quad \alpha_j \leq 1. \quad (33)$$

Summation over $k$ at $\Omega \ll 1$ can be replaced by integration and after some relatively simple calculations one gets at $q = 1$ the following expression for $\Delta \sigma_{ij}$

$$\Delta \sigma_{ij} = \sigma_0 \left( c_{ij} \ln \frac{p_F}{\delta p} + D_{ij} \Omega_0 \ln \frac{p_F}{\delta p} \right) \frac{\delta p}{p_F} = \sigma_0 \Omega_0 \tau (c_{ij} + D_{ij} \ln \Omega_0 \tau) \quad (34)$$

Numerical factors $c_{ij} = c_{ji}$ and $D_{ij} = D_{ji}$ are of order of unity, if $i$ and $j$ do not coincide with $z$, and $C_{iz}$ and $D_{iz}$ are proportional to $\eta^2$.

The resistivity at $j \perp H$ in the case of a FS section $p_H = p_H^0$ containing only a single self-intersection point, has a following form

$$\rho = \rho_0 \left\{ 1 + (\Omega \tau)^3 \exp(-\Omega \tau) \right\} / \left\{ 1 + (\Omega \tau)^4 \exp(-2\Omega \tau) \ln^2(\Omega \tau) \right\} \quad (35)$$

It can be easily seen that the magnetoresistance can both grow and decrease with an increase of $H$ when $\Omega \tau < 5$:

$$\rho = \left\{ \begin{array}{ll}
\rho_0 (\Omega \tau)^3 \exp(-\Omega \tau) ; & \text{at } (\Omega \tau)^2 \ln(\Omega \tau) < \exp(\Omega \tau) < (\Omega \tau)^3 \\
\rho_0 & \text{at } (\Omega \tau)^3 < \exp(\Omega \tau)
\end{array} \right. \quad (36)$$

The magnetoresistance of some organic conductors behaves in a similar manner (see for example $[6,10]$).

Anomalous behaviour of the magnetoresistance can also appear at higher magnetic field intensities if the self-intersecting electron orbit is close to the central FS section. On the FS section
$p_H = 0$ there can be only an even number of self-intersection points or none at all (see Fig. 1). If the FS sections $p_H = 0$ and $p_H = p_H^0$ lie close to each other the electron has to pass at least two "dangerous" part of its orbit (in the vicinity of point $O_1$ and $O_2$ in the Fig. 2) where it moves slowly on the orbit in the magnetic field. In that case $\Omega_0\tau$ in the index of power of the exponent in the formula (36) for $\rho$ should be replaced by $\Omega_0\tau/2$, which widens up to $\Omega_0\tau = 10$ the magnetic field range where the resistivity can still be appreciably affected by magnetic field variations

$$
\rho = \begin{cases} 
\rho_0 (\Omega_0\tau)^{-1} \ln^{-2} (\Omega_0\tau) \exp (\Omega_0\tau/2); & \text{at } \exp (\Omega_0\tau/2) < (\Omega_0\tau)^2 \ln (\Omega_0\tau) < (\Omega_0\tau)^3; \\
\rho_0 (\Omega_0\tau)^3 \exp (-\Omega_0\tau/2); & \text{at } (\Omega_0\tau)^2 \ln (\Omega_0\tau) < \exp (\Omega_0\tau/2) < (\Omega_0\tau)^3; \\
\rho_0 & \text{at } (\Omega_0\tau)^3 < \exp (\Omega_0\tau/2). 
\end{cases}
$$

(37)

Fig. 1. — Section $p_H = \text{const.}$ of Fermi Surface corrugated cylinder type in the cases when $p_H = 0$ (AA' and BB') and at $\theta = 0$ (CC').

Fig. 2.

3. Quasi-two-dimensional electron energy spectrum favours an inclusion of a great number of charge carriers into the formation of quantum oscillatory effects and therefore results in a con-
siderable increase of the amplitude of Shubnikov-de Haas and de Haas-van Alphen oscillations, in comparison with isotropic conductors. In metals the Shubnikov-de Haas effect is mainly manifested by a dependence of the thermoelectric power on the inverse value of the magnetic field strength and Shubnikov-de Haas oscillations of the magnetoresistance were experimentally established only in semimetals of the bismuth type. The specific feature of FS of layered conductors results not only in an increase of the quantum oscillation amplitude but in their peculiar angular dependence as well, especially in the angle range, where self-intersecting electron orbits are certain to be present.

In the quasiclassical approximation the energy levels of charge carriers in the magnetic field with closed orbits in the momentum space can be determined using the Lifshits-Onsager relation

\[ S(\epsilon, p_H) = 2\pi \hbar e H(n + \gamma(n))/c; \]
\[ n = 0, 1, 2, 3, \ldots, 0 < \gamma(n) \leq 1; \] (38)

where \( S \) - is an area of a section of the isoenergetical surface by the plane \( p_H = \text{const} \). We neglect here spin splitting of conduction electron energy levels, that gives rise to Pauli paramagnetism.

Quantization of the charge carrier energy \( \epsilon_n(p_H) \) reduces the appearance of singularities in the density of states of the charge carriers

\[ \nu(\epsilon) = 2H|e|(2\pi\hbar)^{-2}/c \cdot \sum_{n=0}^{\infty} \int dp_H \delta(\epsilon - \epsilon_n(p_H)) \sim \sum_n (\partial \epsilon_n(p_H)/\partial p_H)^{-1} \bigg|_{\epsilon_n(p_H)=\epsilon} \] (39)

which appear periodically with a step proportional to \( 1/H \), and that is the cause of the quantum oscillation effects [16,17]. Since at \( n = \text{const} \) we have \( \partial S/\partial p_H + \partial S/\partial \epsilon \cdot \partial \epsilon/\partial p_H = 0 \) it follows that the singularities in the density of states \( \nu(\epsilon) \) occur for charge carriers with \( \partial S/\partial p_H = 0 \) or carriers with \( \partial S/\partial \epsilon = \infty \); those are conduction electrons with an extremal section \( S_\epsilon = S(p_e) \) of FS, or electrons whose orbits contain a self-intersection point \( p^0 \).

Far from the section \( p_H = p^0_H \) electron energy spectrum in the quasiclassical approximation \( (n \gg 1) \) is a sequence of equidistant levels with an interval \( \hbar \Omega_0 \) between two neighbouring states, and with \( \gamma = 1/2 \). However in the neighbourhood of \( p^0_H \) a rearrangement of electron orbits and changes of their connectivity at \( p_H \to p^0_H \) lead to a considerable complication of the energy spectrum and oscillatory dependence of \( \gamma \) upon \( H \) [18]. Azbel, who has calculated the electron energy spectrum near the saddle-point \( p^0 \) on FS, indicated that their contribution into the quantum oscillation effect is \( \epsilon_F/\hbar \Omega_0 \) smaller than that of electrons with the extremal section area FS [19].

Using quantum kinetics equations [17] or Kubo's method [20] it can be easily shown that amplitudes of the Shubnikov-de Haas oscillations of magnetoresistance, formed by the above said electrons, are related in a similar manner. We don't present here details of those calculations and limit ourselves by remarks of general nature.

Azbel's theory of calculation of the electron energy spectrum near the saddle-points on FS can be readily generalized on an arbitrary number of self-intersection points of the same classical orbit in the momentum space.

The energy spectrum of the charge carrier, whose orbit contains two "dangerous" parts situated closely to the saddle-points \( O_1 \) and \( O_2 \) (see Fig. 2), is determined from condition of matching of
Schrodinger equation solutions at the points $O_1$ and $O_2$, which has the following form:

$$
\cos \left( \frac{c(S_1 + S_2 + S_3)}{2eHh} + \varphi(k_1) - \varphi(k_2) \right) + q_2 \cos \left( \frac{c(S_1 + S_2 + S_3)}{2eHh} + \varphi(k_1) \right) =
$$

$$
= -q_1 \cos \left( \frac{c(S_1 - S_2 - S_3)}{2eHh} + \varphi(k_2) \right) - q_1 q_2 \cos \frac{c(S_1 - S_2 + S_3)}{2eHh};
$$

$$
\varphi(k_j) = 2k_j \ln \frac{k_j}{e} + i \ln \left( \Gamma \left( \frac{1}{4} + ik_j \right) \Gamma \left( \frac{1}{4} - ik_j \right) \right) - \arg\tan (\tan \pi k_j);
$$

(40)

$$
q_j = (2 \cos h (2\pi k_j))^{-1/2} \exp (-\pi k_j); \quad k_j = c |m_j m'_j|^{1/2} (\varepsilon - \varepsilon_{0j} (p_H))/2eHh; \quad j = 1, 2.
$$

Here $m_1$ and $m'_2$ are principal values of the effective mass tensor in the saddle-points $O_1$ and $O_2$, $\varepsilon_{0j} (p_H)$ is an energy on $p_H$ along a coordinate line, passing the points $O_1$ and $O_2$ in the direction parallel to the $p_H$ axis.

Calculating the oscillating terms of thermodynamical and kinetic characteristics one has to take into account that $\partial S_2/\partial p_H = 0$, but $\partial S_1/\partial p_H \neq 0$ and $\partial S_3/\partial p_H \neq 0$. If the sections $p_H = p_e$ and $p_H = p^0_H$ are close to each other but nevertheless separated by an interval $\Delta p_H = |p_e - p^0_H| \gg p_F (h\Omega_0/\varepsilon_F)^{1/2}$, the charge carriers in those sections neighbourhood make independent contributions to the quantum oscillation effect. Following Azbel, one can readily show that contribution of electrons with the spectrum satisfying the condition (40), in the quantum oscillation Shubnikov-de Haas effect, as in the case of a single “dangerous” section, is $\varepsilon_F/h\Omega_0$ times less than the contribution of electrons with the extremal FS section at infinite mean free time $\tau$ of charge carrier. As $\Delta p_H$ declines the Dingle factor begins to play a substantial part, decreasing $\exp (1/\Omega \tau)$ times the amplitude of oscillations that are due to electrons with $S_e$, and according to the formula (31) the oscillation amplitudes in that case are multiplied by a small factor $|\Delta p/p_F|^{1/\mbox{Dileo}}$.

If $\Delta p \leq p_F (h\Omega_0/\varepsilon_F)^{1/2}$ a substantial rearrangement of energy levels of charge carriers at the extremal FS section takes place and at $p^0_H \rightarrow p_e$ their contribution to quantum oscillation effect becomes considerably smaller. Variations of the magnetic field orientation may reduce $\Delta p$ to an arbitrary small quantity and a sharp fall of the Shubnikov-de Haas oscillation amplitude at variations of $\theta$ seems to be due to appearance of self-intersections points on the extremal FS section. Knowing the angle $\theta$ at which the quantum oscillation amplitude abruptly decreases one can determine $\eta$, i.e. the extent of corrugation of the Fermi surface.

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References


