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Superconducting kernel symmetry for an anisotropic superconductivity near H_{C2} and phase transitions in UPt_3

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Abstract. — The symmetry group of anisotropic Cooper pairing in magnetic field H includes the magnetic translations and rotations around H . On the basis of these operations Kasimir's operator which classifies the superconducting nuclei near H_{C2} is constructed. Such kernel classification is used to find *all* possible reasons of kink in $H_{C2}(T)$ -dependence existence and to interpret the phase transitions in $H - T$ plane in the heavy-fermion superconductor UPt_3 at the assumption of two superconducting states coexistence. The phase transition lines in space nonuniform state $H_{C1} < H < H_{C2}$ is shown to be either the lines of superconducting order parameter parity violation with respect to reflection in a perpendicular to magnetic field plane or the lines of triangular vortex lattice distortion or the lines of Abrikosov's lattice period multiplication. Last case near H_{C1} signifies the phase transition with changing of magnetic vortex quantization.

1. Introduction.

Search of superconductors with anisotropic pairing is an old and intriguing problem. The candidates of such substances are the heavy-fermion materials UPt_3 , $CeCu_2Si_2$ etc. One of the most convincing proofs of nontrivial superconductivity here is a power law of some physical characteristics increasing. Another peculiar property of these substances are explained by virtue of multicomponent order parameter to be a phase transition between different superconducting phases. At first those were observed in UPt_3 after Th doping: $UPt_3 \rightarrow U_{1-x}Th_xPt_3$ where at $x > 0.017$ the superconducting critical temperature splits. Two close superconducting transitions in UPt_3 with critical temperatures T_a and T_b : $T_a \approx 0.5K$, $T_a - T_b \approx 0.05K$ [1-3] also attract considerable attention. Surprising phase diagram of such transitions in magnetic field will be discussed in the paper.

It was found [2,4] that if magnetic field lies in a basal plane $x - y$ of hexagonal close-packed crystal lattice of UPt_3 (point group symmetry $G = D_6$) an experimental upper critical field dependence $H_{C2}(T)$ has a kink at $T = T_m \approx 0.4K$ which is likely to correspond to crossing of the phase transition lines between two superconducting states (See Fig.1). The second order phase transition line which at $H \rightarrow 0$ tends to second critical temperature T_b (line $M - T_b$) is the line of

Fig. 1. — Superconducting phase diagram of UPt_3 for a magnetic field perpendicular to the z -axis (from [2]).

Fig. 2. — Superconducting phase diagram (probable) of UPt_3 for magnetic field which is directed an arbitrary angle to axis z .

heat capacity jump [2]. The line $M - C$ were seen with an ultrasonic attenuation [3-6] and with a torsional oscillator [7].

Recent experiments in the perpendicular to the basal plane magnetic field (H parallel to 6-fold symmetry axis z) as well as the experiments with the field oriented at 45° to z also show the anomalies in the specific heat [8], thermal expansion [8] and ultrasonic velocity [9,10] which correspond to the series of superconducting phase transition lines in $H - T$ plane. However it is still not known yet, whether the kink in $H_{C2}(T)$ -dependence as in figure 1 exists or the phase transition lines behavior as in figure 2 takes place.

Probable interpretation of such phase diagram [11-13] is based on the assumption that two superconducting transition temperatures T_a and T_b are the transitions to the anisotropic super-

conducting states a and b which correspond to the different crystal group G irreducible representations (IR) [a] and [b] [14,15]. That is to say in other words, the electron pairing potential $V(\mathbf{k}, \mathbf{k}')$ has nontrivial dependence on electron momenta $\mathbf{k}$ and $\mathbf{k}'$ [15]:

$$V(\mathbf{k}, \mathbf{k}') = \sum_s U_s(T) \sum_m \Phi_m^s(\mathbf{k}) \Phi_m^s(\mathbf{k}') \quad (1)$$

(here $V(\mathbf{k}, \mathbf{k}')$ is expanded on the IR basic functions $\Phi_m^s(\mathbf{k})$ of the crystal group G ; the index s is the number of IR and the index m specifies the basis functions of the same IR). The critical temperature of the superconducting transition to the state transforming under s – IR is defined by equation: $U_s(T) = 0$ and the critical temperatures T_a, T_b for [a] and [b]IR of G are sufficiently close.

2. General method.

The problem of such superconducting phase transition is rather complicated since magnetic field penetration in a sample at $H_{C1} < H < H_{C2}$ breaks crystal symmetry group and leads to the space nonuniform state in which superconducting order parameter $\Delta_{\alpha\beta}(\mathbf{k}, R) = \langle \Psi_\alpha(r) \Psi_\beta(r') \rangle$ depends not only on the position on Fermi-surface $\mathbf{k}$ (here the Fourier transformation on $r - r' \rightarrow \mathbf{k}$ being made) but on the Cooper pair center coordinate $R = (r + r')/2$. So the phases classification in a space nonuniform region isn't the classification in terms of IR of crystal group symmetry G ; it may be the reason of the a and b phases mixing in a magnetic field and the kink existing in $H_{C2}(T)$ -dependence is not obviously in advance.

Now we will briefly outline the general way of the following speculations. The $H_{C2}(T)$ calculation problem is the problem of eigenfunction and eigenvalue calculation for the order parameter integral equation in magnetic field [16-19]:

$$\Delta_{\alpha\beta}(\mathbf{k}, R) = \hat{\mathcal{L}}(H, T) \Delta_{\alpha\beta}(\mathbf{k}, R) \quad (2)$$

$\hat{\mathcal{L}}$ acts on $\mathbf{k}, R$ variables and it has an anisotropy of the pairing potential (1).

The Ginsburg-Landau differential equations [20] are the particular case of equation (2) when $T \rightarrow T_c$.

There exist many eigenvalues of such operator $\hat{\mathcal{L}}$ and correspondingly many candidates – "eigenfields" – $H_i(T)$ for the upper critical field $H_{C2}(T)$, however, only the phase with greatest value of $H_i(T)$ does occur: $H_{C2}(T) = \max\{H_i(T)\}_i$. If at some temperature $T = T_m$ two dependencies $H_1(T), H_2(T)$ cross and $H_1(T) > H_2(T)$ when $T < T_m$ and $H_2(T) > H_1(T)$ when $T > T_m$ the kink in $H_{C2}(T) = \{H_1(T), T < T_m; H_2(T), T > T_m\}$ dependence must be observed at $T = T_m$.

Such an exotic eigenvalue behavior hardly takes place in usual superconductors but it may be stimulated by two close superconducting transitions T_a and T_b in anisotropic states if the upper critical field H_b of the phase b at $T > T_m$ is greater than the upper critical field H_a of the phase a. Really, due to the interaction between phase a and phase b the kink in $H_{C2}(T)$ may disappear. It will be so if eigenvalues of operator $\hat{\mathcal{L}}$ corresponding to H_a and H_b have the same "quantum numbers" i.e. if they belong to the same IR's of operator $\hat{\mathcal{L}}$ group symmetry (which is not coincides with crystal group G).

To interpret the phase transitions in the field $H \leq H_{C2}(T)$ let's assume that $H_{C2}(T) = \max\{H_i(T)\}_i = H_1(T)$ and there exist some other "eigenfield" $H_2(T)$ which is slightly lower than $H_1(T)$ (e.g. in vicinity of $H_1(T) - H_2(T)$ intersection). Below $H_{C2}(T) = H_1(T)$ the linear approximation (2) is not valid and to find the order parameter structure it needs to solve nonlinear

Gor'kov's like + Maxwell's equations. $\Delta_1(\mathbf{k}, R)$ is known to form Abrikosov triangular kernel lattice structure. Due to the closeness $H_2(T)$ to $H_{C2}(T)$ the phase $\Delta_2(\mathbf{k}, R)$ will be also appear. The $\Delta_2(\mathbf{k}, R)$ will be created by virtue of second order phase transition at renormalized value $H_2(T)$ if the nuclear $\Delta_2(\mathbf{k}, R)$ appearance give rises to $\Delta_1(\mathbf{k}, R)$ -lattice symmetry violation. In other case $\Delta_2(\mathbf{k}, R)$ will create together with phase $\Delta_1(\mathbf{k}, R)$ at $H_2(T)$.

So, we will discuss the superconducting kernel symmetry classification near $H_{C2}(T)$ and we will find the quantum numbers which enumerate the kernels. We will consider kernel structure in a crystal like UPt_3 with point group symmetry D_6 and all possible ways of different representations [a] and [b] mixing in a nonuniform superconducting state near $H_{C2}(T)$.

3. UPt_3 , some simplifications.

To avoid overcomplicated notations let's consider the concrete realistic case of hexagonal crystal symmetry of UPt_3 . For the simplicity let's assume that the d -wave superconducting pairing does occur. It means that the superconducting order parameter $\Delta_{\alpha\beta}(\mathbf{k}, R) = i\sigma_y \Delta(\mathbf{k}, R)$ expands over the spherical harmonics $Y_{2,\pm 2} = (k_x \pm ik_y)^2$, $Y_{2,\pm 1} = (k_x \pm ik_y)k_z$, $Y_{2,0} = 2k_z^2 - k_x^2 - k_y^2$.

Hexagonal crystal field breaks the 5-fold d -representation degeneration of isotropic group SO_3 to two 2-dimensional representations E_1, E_2 : $Y_{2,\pm 1} \rightarrow E_1$, $Y_{2,\pm 2} \rightarrow E_2$ and one 1-dimensional representation A_1 : $Y_{2,0} \rightarrow A_1$ of group D_6 (other IR: A_2, B_1, B_2 of D_6 do not correspond to d -wave pairing). It corresponds to d -wave critical temperature splitting into three critical temperatures: $T_{E_1}, T_{E_2}, T_{A_1}$. Two of these temperatures are believed [11-13] to be the phase transition temperatures T_a and T_b .

The hexagonal crystal field influence first of all distorts isotropic group symmetry SO_3 to the group symmetry of cylinder D_∞ . It is this distortion that leads to d -wave critical temperature splitting. The effect is proportional to the square of critical field. It breaks the rotational symmetry D_∞ around axis z to the hexagonal group D_6 to less extent (proportional to the forth order terms of crystal field).

So, for the simplicity we will take into account only the first, more strong crystal field effect, - only $SO_3 \rightarrow D_\infty$ distortion takes place. We will consider two magnetic field directions with respect to axis z : $\mathbf{H} \parallel z$ and $\mathbf{H} \perp z$. If $\mathbf{H} \parallel z$ calculations for D_∞ coincide with those for isotropic group SO_3 since after magnetic field introduction the residual symmetry groups are the same. If $\mathbf{H} \perp z$, magnetic field is directed along 2-fold crystal symmetry axis and the upper critical field influence on the kernel symmetry classification is essential.

The generalization of our further method for an arbitrary wave functions of all IR is quite straightforward but it requires more sophisticated formulas.

4. Phase classification when crystal field influence is not essential ($UPt_3, \mathbf{H} \parallel z$).

At first we will discuss phase classification near $H_{C2}(T)$ for $\mathbf{H} \parallel z$.

As the integral operator $\hat{L}$ is a linear one, the eigenfunctions realize the basis of IR for the operator $\hat{L}$ group symmetry and the problem analysis may be carried out in a model independent terms of such symmetry classification for the whole temperature region without solving the appropriate equation (2).

The eigenfunctions of integral equation (2) are the basic functions of irreducible representations of the operator $\hat{L}$ group symmetry. This group includes Cooper pair magnetic translations on an arbitrary vector $\mathbf{a}$:

$$\hat{T}_{\mathbf{a}} \Delta(\mathbf{k}, R) = \Delta(\mathbf{k}, R + \mathbf{a}) \exp(i \frac{2e}{c} \mathbf{a} \mathbf{A}(R)) \quad (\hbar = 1)$$

and the rotation $\hat{L}_\phi$ on an angle ϕ with respect to magnetic field direction $\mathbf{H}$. The infinitesimal generators of such operations are:

$$\begin{aligned}\hat{t}_x &= -i\partial_x - e/cHy, \quad \hat{t}_y = -i\partial_y + e/cHx \\ \hat{l} &= \hat{l}_k + \hat{l}_R\end{aligned}\quad (3)$$

Here $\hat{l}_k$ and $\hat{l}_R$ are projections of angle momentum operators on the magnetic field direction in the $\mathbf{k}$ and R spaces; we choose the vector-potential in a Coulomb gauge $\mathbf{A} = [\mathbf{H} \times \mathbf{R}]/2$.

So, the operators $\hat{t}_\pm = (\hat{t}_x \pm i\hat{t}_y)/\sqrt{2}$, $\hat{l}$ define the integral operator $\hat{\mathcal{L}}$ eigenfunction classification and the eigenvalue degeneration due to noncommutativity of magnetic translations $\hat{t}_+$, $\hat{t}_-$.

To find such level classification let's note that due to the obvious commutation rules:

$$[\hat{t}_+, \hat{t}_-] = 2\frac{e}{c}H, \quad [\hat{t}_\pm, \hat{l}] = [\hat{t}_\pm, \hat{l}_R] = \mp \hat{t}_\pm \quad (4)$$

Operator

$$\hat{\mathcal{J}} = \hat{l} + \hat{t}_+ \hat{t}_- / (2\frac{e}{c}H) \quad (5)$$

which commutes with integral operator $\hat{\mathcal{L}}$ commutes also with the system symmetry operators $\hat{t}_+$, $\hat{l}$. Such operator (the Kasimir's operator) enumerates the levels of the operator $\hat{\mathcal{L}}$ i.e. the operator $\hat{\mathcal{L}}$ eigenfunctions $\Delta(\mathbf{k}, R)$ are the eigenfunctions of the operator $\hat{\mathcal{J}}$, different levels of the operator $\hat{\mathcal{L}}$ corresponding to different eigenvalues of the operator $\hat{\mathcal{J}}$.

Now we will find the explicit type of eigenfunctions $\Delta(\mathbf{k}, R)$ as combination of spherical harmonics $Y_{s,m}(\mathbf{k})$: ($\hat{l}_k Y_{s,m}(\mathbf{k}) = m Y_{s,m}(\mathbf{k})$) and corresponding to the n -Landau level wave functions $f_n(R)$. Introducing destruction and creation operators of the Landau levels: $\hat{a}^+$, $\hat{a}$ $\hat{a}^+ \hat{a} f_n = (n + \frac{1}{2}) f_n$)

$$\hat{a}^+ = \hat{a}_x + i\hat{a}_y, \quad \hat{a} = \hat{a}_x - i\hat{a}_y; \quad \hat{a}_x = -i\partial_x + e/cHy, \quad \hat{a}_y = -i\partial_y - e/cHx \quad (6)$$

and focusing on the identity: $\hat{a}^+ \hat{a} = \hat{t}^+ \hat{t} / (2\frac{e}{c}H) + \hat{l}_R$ we will present the operator $\hat{\mathcal{J}}$ as:

$$\hat{\mathcal{J}} = \hat{l}_k + \hat{a}^+ \hat{a} \quad (7)$$

The eigenfunctions $\Delta(\mathbf{k}, R)$ of the integral operator $\hat{\mathcal{L}}$ are the eigenfunctions of the operator $\hat{\mathcal{J}}$:

$$\Delta_N^\pm(\mathbf{k}, R) = \sum_{N=m+n} f_n(R) \Psi_m^\pm(\mathbf{k}) \quad (8)$$

where

$$\Psi_m^\pm(\mathbf{k}) = \sum_s \alpha_s Y_{s,m}^\pm(\mathbf{k})$$

The index N is a characteristic quantum number of the functions $\Delta(\mathbf{k}, R)$; the quantum number - sign $\pm$ - corresponds to the discrete system symmetry under reflection in $x - y$ plane which is perpendicular to the magnetic field $\mathbf{H}$ direction. This sign is the $\Delta(\mathbf{k}, R)$ function parity under this transformation. (Naturally, all functions $\Delta(\mathbf{k}, R)$ have the same parity under $\mathbf{k}$ -space inversion; it corresponds to a singlet - triplet pairing classification).

Coefficients $\alpha_s = \alpha_s(H, T)$ may be found only by solving an appropriate integral equation like (2) for the particular potential $V(\mathbf{k}, \mathbf{k}')$.

Using (8) we write all possible types of the functions $\Delta(\mathbf{k}, R)$ in the case of d -wave pairing ($V(\mathbf{k}, \mathbf{k}') \sim (\mathbf{k}\mathbf{k}')^2$), when in (8) only the $Y_{2,m}$ functions are present. These functions are:

$$\Delta_N^+(\mathbf{k}, R) = \alpha_1^N (k_x + ik_y)^2 f_N(R) + \alpha_3^N (2k_z^2 - k_x^2 - k_y^2) f_{N+2}(R) + \alpha_5^N (k_x - ik_y)^2 f_{N+4}(R)$$

$$\Delta_N^-(\mathbf{k}, R) = \alpha_2^N (k_x + ik_y) k_z f_N(R) + \alpha_4^N (k_x - ik_y) k_z f_{N+2}(R) \quad (9)$$

Let's point out some particular cases of (9).

$$\Delta_{-4}^+(\mathbf{k}, R) = \alpha_5^{-4} (k_x - ik_y)^2 f_0(R) \quad (9a)$$

$$\Delta_{-2}^+(\mathbf{k}, R) = \alpha_3^{-2} (2k_z^2 - k_x^2 - k_y^2) f_0(R) + \alpha_5^{-2} (k_x - ik_y)^2 f_2(R) \quad (9b)$$

$$\begin{aligned} \Delta_0^+(\mathbf{k}, R) &= \alpha_1^0 (k_x + ik_y)^2 f_0(R) + \\ &+ \alpha_3^0 (2k_z^2 - k_x^2 - k_y^2) f_2(R) + \alpha_5^0 (k_x - ik_y)^2 f_4(R) \end{aligned} \quad (9c)$$

$$\Delta_{-2}^-(\mathbf{k}, R) = \alpha_4^{-2} (k_x - ik_y) k_z f_0(R) \quad (9d)$$

$$\Delta_0^-(\mathbf{k}, R) = \alpha_2^0 (k_x + ik_y) k_z f_0(R) + \alpha_4^0 (k_x - ik_y) k_z f_2(R) \quad (9e)$$

Formulas (9) show possible ways of different representations of group SO_3 mixing near H_{C2} . In general, magnetic field breaks types pairing classification over the irreducible representations of the crystal group G and such mixing will always exist if the mixing phases have (i) the same parity under reflection in the plane $x - y$ and (ii) the same parity under the $\mathbf{k}$ -space inversion (singlet or triplet).

An account of D_∞ symmetry breach to D_6 reduces the set of integral quantum number classification of $\Delta_N(\mathbf{k}, R)$ to the group of residua over mod 6. As a result, it leads to the Landau levels with the same numbers over mod 6 mixing. Such changes in classification do not substantially violate the above considerations since the wave functions $\Delta_a(\mathbf{k}, R)$ and $\Delta_b(\mathbf{k}, R)$ have different quantum numbers over mod 6.

5. Phase classification in crystal field (UPt_3 , case $H \perp z$).

As the group rotation symmetry D_6 when $H \perp z$ is broken to the C_2 let's discuss the general case of the nontrivial pairing in magnetic field which is directed along n -fold symmetry axis C_n . In accordance with (4) we write the operator of c_n rotation on an angle $\frac{2\pi}{n}$ which commutes with Hamiltonian as $e^{i\frac{2\pi}{n}(\hat{i}_k + \hat{i}_R)}$. The superconductivity coherence length ξ is much greater than the lattice period and the Hamiltonian commutes with magnetic translation generators. The operator $\hat{J}$ which commutes with $\hat{i}_\pm$ as well as with $e^{i\frac{2\pi}{n}(\hat{i}_k + \hat{i}_R)}$ has the form:

$$\hat{J} = e^{i\frac{2\pi}{n}(\hat{i}_+ + \hat{i}_- + \hat{i}_R)} e^{i\frac{2\pi}{n}\hat{j}_k} = e^{i\frac{2\pi}{n}(\hat{i}_k + a^\dagger a)} \quad (10)$$

If $n = 2$ the quantum number set is the group of residua over mod 2 which in turn coincides with the kernel classification over the parity ($\pm$) with respect to $y - z$ plane reflection.

The only reason for kink existence may be the competition between Δ^+ and Δ^- pairing. The lines $M - T_b$ and $C - T_b$ (See Fig.1) are the lines of second order phase transitions with parity violation and the kernel lattice has one-axis distortion in z -direction due to crystal field influence.

6. Discussion.

The main question which is to be solved is the nature of superconducting critical temperature splitting in UPt_3 when $H = 0$. If the effect leads to the two superconducting phase coexistence which belong to the different crystal group IR it may be the reason of the kink in $H_{C2}(T)$ dependence. The kink will be exist if the superconducting phases at both sides of it have different quantum numbers.

Now we will discuss the possible phase transitions diagram in $H - T$ plane at $H \leq H_{C2}(T)$ when magnetic field is parallel to 6-fold symmetry axis z . We assume that kink at $H \parallel z$ does really exist and it is due to the different quantum numbers of phases $\Delta_a(k, R)$ and $\Delta_b(k, R)$ near $H_{C2}(T)$.

Two cases are possible.

I) The kernel $\Delta_a(k, R)$ at $T < T_m$ and $\Delta_b(k, R)T > T_m$ have different parity with respect to $x - y$ plane reflection. It is possible if: $[a] = E_1$, $[b] = E_2$ or A_1 .

The parity classification is valid in a nonlinear region. So, the lines $\Delta_a \rightarrow \Delta_a + \Delta_b$ and $\Delta_b \rightarrow \Delta_a + \Delta_b$ are the second order transition lines with parity violation.

II) $\Delta_a(k, R)$ and $\Delta_b(k, R)$ have different quantum numbers N , e.g. $[a] = E_2$, $[b] = A_1$: $\Delta_a(k, R) = \text{phase}(9c)$, $N = 0$ for $T > T_m$ and $\Delta_b(k, R) = \text{phase}(9c)$, $N = -2$ for $T < T_m$. The numerical calculations like [13,20] show that such anzatz is rather realistic.

The phase transition lines interpretation is less trivial than in the case I. The consequential theory of phase transitions in a nonlinear region near the point M will be given elsewhere [21]. Here we will briefly formulate only the results of such consideration. The order parameter amplitude $\rho(R) = \langle \Delta^*(k, R)\Delta(k, R) \rangle_k$ for the phase $\Delta_a(k, R)$ as well as for the phase $\Delta_b(k, R)$ has the triangular lattice structure with a single flux quantum $\Phi = \pi\hbar c/e$ for one lattice cell. The possible scenarios of phase transitions is as follows:

IIa) The lines $\Delta_a \rightarrow \Delta_a + \Delta_b$ and $\Delta_b \rightarrow \Delta_a + \Delta_b$ correspond to the second order phase transition lines of rhombic distortion of triangular lattices. So, in a region $\Delta_a + \Delta_b$ $\rho(R)$ has a rhombic lattice structure.

IIb) These lines is the second order phase transitions with Abrikosov lattice period multiplication in two or even three times. Such multiplication may lead to the two or three flux quanta vortices near H_{C1} . The numerical confirmation of doubly quantized vortices stability was given in [22].

The phase transitions with symmetry breach which is the combination of cases I, IIa, IIb are also possible. What phase transition does really take place depends on the numerical factors in nonlinear equations which define the superconducting order parameter in a mixed state. Some model examples of these possibilities will be published in [21].

Let now magnetic field is slightly deflected from the symmetry axis z . As the point group symmetry D_∞ is broken the quantum number classification by (7) is no more valid and eigenfunctions of equation (2) which corresponded to the different quantum numbers must be mixed. This leads to the kink smearing as it is shown in figure 2, the more being the field deflection the more being such smearing. When H will sweep all the angles from $H \parallel z$ to $H \perp z$ the kink may appear again at $H \perp z$ if the order parameter $\Delta(k, R)$ behavior satisfies to the conditions which was described in section 5.

The probable $H - T$ phase diagram when H is directed at an arbitrary angle to z is shown in figure 2. The lower line of physical properties peculiarities may be the phase (first or second

order) transition line or the line of smear heat capacity jump which is the memory of second order phase transition when $H \parallel z$ or when $H \perp z$. The concert type of this line also depends on the numerical factors in nonlinear superconducting equation [21].

For the sake of completeness let's point out one more opportunity of kink and phase transitions existence: the concurrence between singlet and triplet pairing. The appearance of it does not depend on the magnetic field direction.

Unfortunately, the available experimental data is not perfect to really examine the above conclusions. The most interesting question which come up is the $H - T$ phase diagram appearance if $H \parallel z$ and its behavior when H deviates from z to basal plane. What is the order of phase transitions in mixed state at different magnetic field orientations? Does the kink in $H_{C2}(T)$ at any orientation exist, or phase diagram has the shape as in figure 2?

To clarify such problems the more carefully investigation of heat capacity jump in phase transition lines is necessary. It is worth note that usually the phase mixing effects which smear the heat capacity jump does not appear in Ginsburg-Landau approach in the gradient expansion of equation (2) near T_c . To obtain them it needs to take into account the quartic on gradients terms. So, the experimental temperature solution which would allow to observe the described smearing effects and to make a choose between phase diagrams in figure 1 and figure 2 is estimated to be $(1 - T_a/T_m)^2 T_m \sim 0.02 T_m$.

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