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Radial fingering in viscoelastic media, an experimental study

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Résumé. — Nous avons effectué des expériences de digitation viscoélastique en géométrie radiale, dans des suspensions concentrées de particules colloïdales (des argiles), en conditions de tension interfaciale très faible. Le taux de déplacement, la largeur moyenne des branches et la dimension fractale associée à la croissance de la figure de digitation ont été mesurés dans une plage très large de viscosité et de pression d'injection. Le processus de croissance est fractal dans toutes les conditions expérimentales que nous avons étudiées, avec des exposants fractals, D_G , couvrant tout le domaine compris entre 1,3 à faible vitesse d'avancement du front, et 2,0, à très haute vitesse. Le taux de déplacement, δ , défini comme la fraction volumique de pâte déplacée par l'eau lorsque l'eau sort de la pâte, est négligeable à vitesse nulle, et semble tendre vers une valeur asymptotique de ~ 0.25 à grande vitesse. δ est presque entièrement déterminé par l'évolution, en fonction de la vitesse, du taux de branchement. La largeur moyenne des branches n'intervient que faiblement.

Abstract. — We report measurements of two dimensional radial fingering in viscoelastic concentrated suspensions (pastes) of colloidal particles, in small surface tension conditions. The displacement efficiency, the average finger width and the fractal dimension for growth have been measured in a broad range of paste viscosity and driving pressure. The growth process appears to be fractal in all the experimental conditions, with fractal exponents, D_G , covering the whole range from 1.3 at very small tip velocity to 2.0 at very high tip velocity. The displacement efficiency, δ , defined as the volume fraction of paste displaced by water when water flows out of the paste, is almost zero at very low velocity, and seems to tend toward an asymptotic value close to 0.25 at high velocity, δ is almost entirely dominated by the velocity dependence of the tip-splitting cascade. The average finger width has very little influence.

List of symbols.

b	: thickness of rectangular or radial Hele-Shaw cells.	P_i	: injection pressure.
T	: surface tension between the less viscous and the more viscous fluid.	$V(t)$	: volume of water injected in the cell at time t .
μ	: viscosity of the more viscous fluid.	$R(t)$	: average diameter of a radial pattern at time t .
U	: velocity of the interface.	D_G	: fractal exponent for growth, derived from $V(t) \sim R(t)^{D_G}$.
N_{ca}	: capillary number $= \mu U/T$.	V_t	: total volume of water injected in the paste when the finger tips flow out of the paste.
W	: width of rectangular Hele-Shaw cells (channels).	N_b	: number of main branches (fingers) of water leaving the injection hole.
λ	: relative width of the viscous finger with respect to channel width in Hele-Shaw channels.	δ	: displacement efficiency, defined as the ratio of the total volume of injected water (V_t) over the total volume of paste.
ϕ	: Laplacian potential.	l	: finger width.
σ	: shear stress.	$S(R)$	: surface area of a radial pattern of average diameter R .
σ_0	: yield stress.	B	: control parameter used in viscous fingering between immiscible fluids in Hele-Shaw
$\dot{\gamma}$	: shear rate.		
m	: shear-thinning exponent.		
S/L	: solid to liquid ratio (weight by weight) in the clay suspension (paste).		

- channels (Saffman-Taylor flow). $B = T/12(w/b)^2 \mu U$.
- $\bar{U}$: average tip velocity in radial cells, defined as $R(t)/t$ at the time where the water tips flow out of the paste.
- V_0 : volume of paste in the cell.
- R_0 : diameter of the paste « cake », before injecting water.
- $\bar{l}$: average finger width.
- δ_0 : displacement efficiency extrapolated at zero tip velocity.
- D_{G0} : fractal exponent for growth extrapolated at zero tip velocity.

1. Introduction.

The hydrodynamic instability which affects the interface between a low viscosity fluid and a high viscosity fluid when pushing the former into the latter is one of the most actively studied pattern formation problem. The simplest patterns are obtained with immiscible Newtonian fluids (air and oil, or water and oil, for instance) flowing in a Hele-Shaw [1] channel, i.e. between two parallel plates separated by a narrow space, b , small compared with the other dimensions of the cell. Those are the conditions used by Saffman and Taylor in their original work [2].

Saffman-Taylor flow is characterized by a transient in which the originally plane interface is destabilized by a few modes [3] with wavelengths scaling as $\sim b(T/\mu U)^{1/2}$, in which T is the interfacial tension, U the velocity of the interface and μ the viscosity of the more viscous fluid (or, more generally, the viscosity difference between the high viscosity and the low viscosity fluid) [4]. However, the most characteristic feature is the long-time steady-state pattern which, when the capillary number, $N_{ca} = \mu U/T$, is not too high, is one smooth finger of relative size λ (with respect to the channel width, w), which moves steadily through the channel. λ goes continuously from ~ 1 (full width) at very small velocities to ~ 0.5 (one half of the channel width) at large velocities [1, 5]. Although the mechanism leading to the selection of a finger size is not entirely clear, there is no doubt that capillarity and film draining have an important stabilizing effect [6].

In fact, Saffman-Taylor fingers are extremely robust patterns. One has to go to very high values of the capillary number (which measures the ratio of viscous forces over surface tension) before disturbances appear. The most obvious disturbance is tip-splitting. It was already evidenced by Saffman and Taylor themselves [2] and recently by several others, either in rectangular [6-8] or in radial [9, 10] Hele-Shaw cells. However, before tip-splitting — which is a symmetric disturbance — occurs, other simpler asymmetric or symmetric modes might appear, as

predicted by Bensimon *et al.* [11], and observed experimentally by Tabeling *et al.* [6].

A totally different pattern structure is obtained when the stabilizing effects controlling Saffman-Taylor flow are minimized. Nittmann *et al.* [8, 12, 13] showed that by using (i) a pair of fluids with negligible interfacial tension and (ii) a viscous fluid with non-Newtonian (shear-thinning) properties (water into aqueous polymer solutions or aqueous latex suspensions), extensive tip-splitting occurs, leading to highly ramified patterns with a fractal structure. The close similarity of fractal viscous fingering (VF) patterns with other fractal patterns obtained by diffusion limited aggregation (DLA) [14] or dielectric breakdown (DB) [15] was very soon pointed out by several authors [8, 16, 17]. This similarity can be at least intuitively understood since VF, DLA and DB are all governed by a potential field obeying a Laplace equation, $\nabla^2 \phi = 0$, with equivalent boundary conditions. The potential is (minus) the pressure in VF, the particle concentration in DLA, and the electric potential in DB. Fractal VF, DLA and DB patterns therefore belong to the general class of Laplacian fractals [18].

Although the main morphological features of fractal VF patterns have been reported (in fact, almost exclusively in those cases where resemblance with DLA is the most striking) [8, 11, 12, 19] several important data are still lacking before a comprehensive picture, comparable to what has been achieved for classical Saffman-Taylor flow, could be constructed. In particular, the relationships between the fractal dimension, the pattern homogeneity, the pattern surface, the « finger » shape and profile on the one hand, and the flow parameters as well as the visco-elastic properties of the viscous fluid on the other hand, have still to be determined.

The purpose of this paper is to establish some of these relationships experimentally. We used water as the low viscosity fluid and concentrated aqueous suspensions (pastes) of colloidal clay particles as the high viscosity fluids. Since the water/clay paste is in fact a water/water interface, the interfacial tension is essentially zero and fractal patterns are easily obtained, as we showed recently [19]. Aqueous clay pastes are viscoelastic media with a threshold for flow (yield stress) which may be very high. The existence of a threshold for flow is probably an additional factor favourable to the development of ramification since it prevents the disturbances from vanishing as the front (and the stress) moves forward.

This paper will be devoted to a qualitative description of the patterns and of the flows, and to a semi-quantitative correlation of the pattern structure with the flow parameters, emphasizing the fractal aspects of the process. In a subsequent paper [20] we will concentrate on the correlation between the viscoelas-

tic properties of the pastes and the finger characteristics.

2. Materials and methods.

Crude ground Wyoming bentonite from NL Industries was used to prepare the clay pastes. The clay was dispersed in distilled water by shaking the mixture. The solid to water ratio (S/L) in the pastes ranged from 0.05 to 0.10. Just before injecting the pastes into the Hele-Shaw cell, the pastes were stirred for five minutes. Particular care was taken to control the rheological history (shaking, stirring and rest periods) of the pastes. This was indeed found to be absolutely necessary to obtain reproducible results.

All the experiments reported here were performed with a horizontal radial cell (diameter : 0.5 m) in order to avoid the wall effect evidenced by Nittmann *et al.* [8] in rectangular cells. Injection of dyed water was performed at constant pressure, through a hole (diameter : 1 mm) at the centre of the bottom glass plate. The water was stored in a measuring glass vessel. In the low pressure experiments, the injection pressure was kept constant by moving the reservoir upwards in order to keep the water level at the same height above the cell during the whole experiment. In that way, the volume of injected water, $V(t)$, was continuously monitored. This was no longer possible in the high pressure experiments, in which the pressure was kept constant by pressurizing the water reservoir. Photographs of the patterns were taken at regular intervals in some experiments. The average diameter of the patterns, $R(t)$, (i.e. the diameter of the circle passing through the most remote finger tips) was measured on the photographs.

The rheological properties of the clay pastes were measured with a Couette viscosimeter. The shear stress (σ) vs. shear rate ($\dot{\gamma}$) curves are shown in figure 1. The shear-thinning properties of the pastes are obvious. Also shown are the extrapolated yield stresses (σ_0) of the pastes. This rheological behaviour is satisfyingly accounted for by a generalized Casson equation :

$$(\sigma - \sigma_0) \sim \dot{\gamma}^m. \quad (1)$$

The shear-thinning exponent, m , is a decreasing function of paste concentration, whereas σ_0 is a steeply increasing function of the concentration (Table I). This expresses the development of the elastic properties.

3. Pattern morphology.

We describe here the morphological tendencies as a function of paste concentration and injection pressure, at constant cell spacing, b . We will focus on simple properties or parameters such as pattern homogeneity, pattern surface, and fractal dimension.

Fig. 1. — Shear stress (σ) vs. shear rate ($\dot{\gamma}$) curves for three clay pastes used in this study. S/L is the solid to liquid, i.e. the clay to water ratio (weight by weight) in the paste. The arrows indicate the yield stress.

Table I. — Rheological parameters of the clay pastes.

S/L	0.05	0.06	0.07	0.08	0.09	0.10
σ_0 (Pa)	1.6	3	8.2	30	82	137
m	0.60	0.55	0.51	0.48	0.45	0.42

S/L is the clay to water ratio (weight by weight) in the paste.

σ_0 is the yield stress.

m is the shear-thinning exponent used in equation (1) : $(\sigma - \sigma_0) \sim \dot{\gamma}^m$.

σ is the shear stress.

$\dot{\gamma}$ is the shear rate.

However, it will be clear to the reader that there is much more information in the patterns than is extracted by those simple features.

We will distinguish, somewhat arbitrarily, three regimes characterized by the low, medium or high value of the following couple of parameters : the viscosity of the clay paste (the term « viscosity » in used here to designate all the viscoelastic properties of the paste which increase upon increasing the solid to liquid ratio, S/L , in the paste) and the injection pressure (P_i). Actually, it turns out that each of these regimes corresponds to a morphology class.

3.1 LOW VISCOSITY-INJECTION PRESSURE REGIME.

— Figures 2 to 5 form a typical set of patterns obtained at increasing injection pressure in a rela-

tively fluid paste ($S/L = 0.05$) in a thin cell ($b = 0.2$ mm). The pattern represents the part of the clay paste which has been displaced by the water injected at the centre of the cell. Although we have no quantitative measure of the thickness of the clay film which is drained behind, there is no doubt that it is much smaller than the cell thickness, b . Indeed, the transparency of the regions where water has penetrated is such that one can look through those

$e=0.2\text{mm}$ $S/L=0.05$ $P_i=100\text{ Pa}$

Fig. 2. — Pattern obtained by injecting water in a clay paste, with a clay to water ratio in the paste of 0.05 and an injection pressure of 0.1 kPa. The cell spacing is 0.2 mm.

$e=0.2\text{mm}$ $S/L=0.05$ $P_i=700\text{ Pa}$

Fig. 4. — Pattern obtained by injecting water in a clay paste with a clay to water ratio in the paste of 0.05 and an injection pressure of 0.7 kPa. The cell spacing is 0.2 mm.

$e=0.2\text{mm}$ $S/L=0.05$ $P_i=300\text{ Pa}$

Fig. 3. — Pattern obtained by injecting water in a clay paste with a clay to water ratio in the paste of 0.05 and an injection pressure of 0.3 kPa. The cell spacing is 0.2 mm.

$e=0.2\text{mm}$ $S/L=0.05$ $P_i=900\text{ Pa}$

Fig. 5. — Pattern obtained by injecting water in a clay paste with a clay to water ratio in the paste of 0.05 and an injection pressure of 0.9 kPa. The cell spacing is 0.2 mm.

regions at a distance of 0.5 meter (the depth of our set-up) without any noticeable light scattering. At such a distance, a film of clay a few microns thick would already prevent a clear vision.

Another important point is that the interface between the water and the clay paste is sharp. There is no mixing of the fluids nor diffusion of the dye (which is anionic, in order to avoid adsorption on the clay, which is itself a cation exchanger) during the time of an experiment.

Despite the small viscosity contrast and the extremely small front velocity in some cases (down to 10^{-4} m s^{-1}), the patterns are all extensively branched. However, their homogeneity varies considerably. Generally speaking, the width of the branches is larger at the centre of the patterns than at the periphery. This tendency decreases as the injection pressure and the tip velocity increases, but it is very noticeable in the low pressure experiments, where the centre of the pattern is in some cases a compact spot, which means that water has completely displaced the clay paste in that spot (Fig. 3 is a clear example). This is parallel to the time-dependence of the injected volume (which is equivalent to the time-dependence of the pattern area), as shown in figure 6. In the very low pressure experiment, one can separate two different flow regimes. The first one corresponds to compact displacement. The second one, which settles in only after the first one has almost stopped, corresponds to the growth of branches. As the pressure increases, the separation between the two regimes vanishes and one single regime is observed when branching starts from the centre (beyond $\sim 700 \text{ Pa}$).

Fig. 6. — Examples of flow curves in clay pastes, with a clay to water ratio in the paste of 0.05. V is the volume of water injected at time t . P_i is the injection pressure. The end point of each curve is the point where the water flows out of the cell.

Daccord *et al.* [12] showed that the growth of fractal VF patterns in polymer solutions occurs only by consecutive splitting of the leading tips. No growth occurs in the shielded interior regions. This was found to be also the case here, even in very inhomogeneous patterns. Figure 7 shows four stages in the growth of the pattern obtained at $P_i = 1.5 \text{ kPa}$. One can see that going from one stage to the next one merely involves the addition mass at the periphery.

The eventually fractal structure of the injection patterns was tested by plotting the volume of water injected at time t , $V(t)$, vs. the size of the pattern at that time, $R(t)$, in double Log plot. We call the exponent derived from such plots the *fractal exponent for growth*, D_G . Surprisingly, reasonably good

Fig. 7. — Growth sequence of the pattern obtained at $P_i = 1.5 \text{ kPa}$ in a clay paste with clay to water ratio of 0.05.

Fig. 8. — Volume of water injected vs. diameter of the pattern in double Log plot. The clay to water ratio is 0.06 and 0.05 in the upper and in the lower row, respectively.

The injection pressures (P_i) and the fractal exponents for growth (D_G) are indicated on the graph.

linear plots were obtained over about one decade in pattern size, in spite of the inhomogeneity of the patterns (Fig. 8). Although a large uncertainty affects each fractal exponent derived from those $V(t) \sim R(t)^{D_G}$ relationships, one can distinguish two major tendencies: (i) all D_G 's are lower than 1.7, the fractal dimension of regular DLA aggregates and of homogeneous, fractal, and radial VF patterns [13, 19]; (ii) as the injection pressure and the tip velocity increases, D_G also increases.

Another simple parameter was measured: the total volume injected in the paste, at the point where the water flows out of the paste, V_t . V_t is equivalent to the maximum surface area of the pattern. For a given cell thickness, V_t is determined by several other parameters: (i) the number of main branches leaving the injection hole, N_b ; (ii) the branching cascade of each of those main branches; (iii) the average width of the branches. N_b is a remarkably constant parameter: $N_b = 5$ or 6, not only in the patterns that we discuss here, but also in all the radial patterns that we will consider later on in the paper. We also found that in the low viscosity contrast-low injection regime, V_t is an increasing linear function of P_i (this is no longer true in the high

viscosity contrast-high injection pressure conditions, as we shall see).

Basically, the same type of results is obtained upon injecting water into slightly more viscous pastes in which the clay/water ratio is 0.06. However, a general tendency towards homogeneity is noticeable, as well in the patterns (Figs. 9-10) as in the flow curves (Fig. 11). The total injected volume is still increasing linearly with injection pressure, P_i , but a tendency to saturation is already noticeable at the highest P_i that we used (2.5 kPa). The fractal exponent for growth, D_G , increases steadily from ~ 1.4 to ~ 1.7 as P_i increases from 0.9 to 2.2 kPa (Fig. 8).

3.2 MEDIUM VISCOSITY-INJECTION PRESSURE REGIME. — Figure 12 exemplifies the phenomenon which is characteristic of this regime: the « detachment » of the water fingers from the glass walls. For instance, in figure 12, one can see that, in a large portion of the pattern near the centre, the « regular » fingers in which the water occupies the whole space between the glass walls (except for the microscopic film drained behind), are surrounded by more diffuse regions, in which the water finger is

Fig. 9. — Pattern obtained by injecting water in a clay paste with a clay to water ratio of 0.06. The injection pressure is 1.3 kPa. The cell spacing is 0.2 mm.

Fig. 10. — Pattern obtained by injecting water in a clay paste with a clay to water ratio in the paste of 0.06. The injection pressure is 2.5 kPa. The cell spacing is 0.2 mm.

midway in the gap of the cell, between two thick clay paste films. Interestingly, the regular fingers in the pattern form a very homogeneous (as far as finger width is concerned) backbone. Detached fingering has been observed in a range of S/L ratios between 0.07 and 0.10, and a range of injection pressure between ~ 1 and ~ 15 kPa. It is different from the seepage phenomenon observed by Daccord *et al.* [12], in which the water finger flows between the upper plate and the mass of the viscous fluid.

Inspite of the simultaneous growth of two types of fingers, the flow curves, $V(t) = f(t)$ (Fig. 13), are still smooth, without any inflection point. On the opposite, the $V(t) = f[R(t)]$ Log-Log plots show

Fig. 11. — Flow curves in clay pastes, with a clay to water ratio in the paste of 0.06. V is the volume of water injected at time t . P_i is the injection pressure.

Fig. 12. — Pattern obtained by injecting water into a clay paste, with a clay to water ratio in the paste of 0.07. The injection pressure is 6 kPa. The cell spacing is 0.2 mm.

Fig. 13. — Flow curves in clay pastes, with a clay to water ratio in the paste of 0.07. V is the volume of water injected at time t . P_i is the injection pressure.

either an important upward bending or a break. The fractal exponents in the second part of the curves have physically meaningless values (> 2), suggesting that the break corresponds to a point where detached fingering starts near the centre of the pattern without significant growth on the periphery. Examination of the patterns shows that this is indeed the case.

A major difference with respect to the previous regime is that the total injected volume, V_t , is no longer an increasing linear function of P_i . A clear tendency to saturation shows up, at a level of $V_t \approx 7 \times 10^{-3} L$. This saturation value corresponds to a displacement efficiency of ~ 0.25 (the displacement efficiency, δ , being defined as the ratio of the total volume of water injected vs. the total volume of paste in the cell).

3.3 HIGH VISCOSITY-HIGH INJECTION PRESSURE REGIME. — This is the regime in which the most homogeneous patterns were obtained. No seepage nor detached fingering occurs. Figure 14 is an example taken at the lower limit of this regime, in a paste at $S/L = 0.07$ and $P_i = 20$ kPa. One can see that detached fingering has been avoided by using a rather high pressure, but the finger width is not yet totally homogeneous. The fingers are still somewhat wider near the centre of the pattern than at the periphery. Another example, deeper into the regime that we consider ($S/L = 0.10$ and $P_i = 100$ kPa), is

Fig. 14. — One of the main branches of a pattern obtained by injecting water in a clay paste, with a clay to water ratio in the paste of 0.07. The injection pressure is 20 kPa and the cell spacing is 0.2 mm.

shown in figure 15. The average finger width is now constant.

One can also see that the general aspect of the pattern is different. It looks much more prickling in the more concentrated pastes. This is due to an important modification of the local curvature of the fingers, from convex to concave. We will discuss this in detail in a subsequent paper [20].

Because of high velocity of the fingers in this regime, we were unable to record the flow curves, which prevents us from calculating the fractal exponent for growth. The fractal dimension, D , was therefore calculated directly on the patterns.

In the ideal case where the pattern is a perfectly homogeneous (constant finger width, l) self-similar radial array starting with N_b main branches, the surface of the pattern in a box of size R is given by :

$$S(R) = \alpha N_b \left(\frac{R}{l} \right)^D l^2 \quad (2)$$

where α is a dimensionless shape factor. The number

Fig. 15. — One of the main branches of a pattern obtained by injecting water in a clay paste, with a clay to water ratio in the paste of 0.10. The injection pressure is 100 kPa and the cell spacing is 0.2 mm.

of branches intersected by the box walls, $N(R)$, is

$$N(R) = D\alpha N_b(R/l)^{D-1} \quad (3)$$

since

$$dS(R) = l \cdot N(R) dR \quad (4)$$

in an homogeneous pattern. Hence, one can calculate D either by the « mass in the box » method (Eq. (2)) [8], or by the « branching cascade » method, first used by Niemayer *et al.* for dielectric breakdown patterns [15]. Using this method, we obtained $D = 1.78$ and 1.58 for the patterns shown in figures 14 and 15, respectively.

Several experiments were performed at very high injection pressure (up to 400 kPa) in concentrated pastes ($S/L = 0.10$), in a cell reinforced with steel bars in order to avoid excessive deformations. In this regime, injection has an explosive character. A typical pattern is shown in figure 16. The finger width is homogeneous. The fingers are narrow, but the pattern is very densely branched. The fractal dimension, measured by the branching cascade method on the patterns, is 2.0, within experimental error.

4. Control parameters.

The first step towards a rationalisation of classical Saffman-Taylor flow was the hypothesis that a single

Fig. 16. — Pattern obtained by injecting water in a clay paste, with a clay to water ratio in the paste of 0.10. The injection pressure is 120 kPa, and the cell spacing is 0.2 mm. The steel bars reinforcing the cell are visible on the photograph.

parameter might control the finger size and shape. In the original Saffman-Taylor paper [2], this control parameter was proposed to be the capillary number $N_{Ca} = \mu U/T$. Later, the aspect ratio of the cell, w/b , was considered (w is the width of the Hele-Shaw channel) in order to take full account of the pressure drop across the curved interface. This led to a more general dimensionless control parameter, $1/B = 12(w/b)^2 \mu U/T$ [21]. Finally, in order to resolve the residual discrepancy between experimental results and numerical studies [22], Tabeling and Libchaber [5] introduced a renormalized parameter, $1/B^*$, which integrates the influence of the film drained behind on the pressure drop across the interface [23]. With this new parameter, a « universal » λ vs. $1/B^*$ curve was obtained. λ decreases from 1 at $1/B^* = 0$, to about 0.5 at very large $1/B^*$. Whether there is a real asymptotic limit or not is not yet clear [5, 6].

As a first step towards an equivalent « universal » representation of fractal viscous fingering, we tried to plot the displacement efficiency, δ , vs. injection pressure. δ is a dimensionless number (fraction of paste displaced by water) which is equivalent to λ , the relative finger width in Saffman-Taylor flow,

when the finger is smooth and long. Although a general trend, starting with an increasing linear region and ending with a saturation value of $\delta \approx 0.25$ (see Sect. 3) was observed, the data showed considerable scatter.

A better result was obtained by plotting δ vs. the average front velocity, $\bar{U}$, simply calculated from the time for water to flow out of the paste (i.e. from the end points in the flow curves of Figs. 6, 11, 13). This is shown in figure 17. Other attempts to reduce further the scatter of data points were unsuccessful. For instance, using the product of $\bar{U}$ with some viscoelastic parameter (yield stress or apparent viscosity) yield a much worse result.

Fig. 17. — Plot of displacement efficiency, δ , vs. average tip velocity, $\bar{U}$, for all the experiments in which the total injected volume could be measured. The clay to water ratios (S/L) in the pastes are denoted on the figure.

The first point to emphasize is that δ shows a trend which is *opposite* to that of λ in Saffman-Taylor flow. This major difference is not an artifact due to the use of a radial cell instead of a rectangular cell. We observed the same trend in rectangular cells [20]. On the other hand, Patterson [9] has shown that radial fingering with Newtonian fluids having a large interfacial tension is only slightly different from linear Saffman-Taylor fingering.

Let us now analyse the mechanism leading to the quasi-linear increase of δ in the low velocity regime ($\bar{U} < 2 \times 10^{-3} \text{ m s}^{-1}$). We defined δ as the ratio V_t/V_0 , where $V_0 = \pi b R_0^2/4$ is the total volume of paste in a « cake » of diameter R_0 . From equation (2), one easily obtains :

$$\delta = V_t/V_0 \approx (4 \alpha / \pi) N_b (R_0/l)^{D-2}. \quad (5)$$

The increase of δ is *not* coming from an increase of the number of main branches, N_b . As we already

pointed out, $N_b = 5$ or 6 (most often 5) in the whole low velocity regime.

Nor does it come from an important increase of the average finger width, $\bar{l}$, as far as we can estimate. We were unable to measure accurately the average finger width. This would require a digitization of the patterns with a resolution of at least 1000×1000 . Nevertheless, a crude estimate was obtained by averaging the intersection of circles of diameter $R_0/2$ with the patterns for which a photograph, taken at the point where the water fingers reach the boundary of the paste, was available. $\bar{l}$ is plotted vs., the average velocity, U , in figure 18. No strong tendency is obvious in this cloud of data points, although a general and weak *decreasing* trend can be detected. Other attempts to correlate $\bar{l}$ with P_i or P_i/σ_0 did not give better results. In any case, the rather clear evolution of δ (Fig. 17) can hardly be explained in terms of a parallel behaviour of $\bar{l}$ (Fig. 18).

In fact, the whole behaviour of δ between $\bar{U} \approx 0$ and $\bar{U} \approx 8 \times 10^{-2} \text{ m s}^{-1}$ can be almost quantitatively accounted for by the variations of the fractal exponent for growth, D_G . As shown in figure 19, D_G undergoes an important increase, from ~ 1.3 to ~ 1.75 , in the velocity range where a $\sim$ seven fold increase of δ is observed ($0 < \bar{U} < 2 \times 10^{-3} \text{ m s}^{-1}$), and then levels off, just like δ . Quantitatively, the relationship between δ and D_G can be analysed using the following simple scaling law, derived from equation (5), at constant N_b and l :

$$\delta/\delta_0 = (R_0/l)^{D_G - D_{G0}} \quad (6)$$

Fig. 18. — Average finger (or branch) width, $\bar{l}$, vs. average tip velocity for all the experiments in which the time was recorded and a photograph was taken at the point where the water fingers were on the verge of flowing out of the cell. The dashed line is the average of all values ($\bar{l} = 2.85 \times 10^{-3} \text{ m}$). The arrow on the right is the average width at very high velocity ($\bar{U} \approx 10^{-1} \text{ m s}^{-1}$). The full line is (perhaps) the general tendency.

Fig. 19. — Fractal exponent for growth, D_G , vs. average tip velocity, $\bar{U}$, for all the experiments in which the time was recorded and enough photograph (at least five) were taken to have a safe estimate of D_G . The value indicated by the triangular symbol has been measured directly on a pattern. Fractal dimensions of ~ 2 have been measured at even higher velocities ($\bar{U} \approx 10^{-1} \text{ m s}^{-1}$). The dotted curve has been calculated according to equation (6), using the δ vs. $\bar{U}$ curve of figure 17 (see text).

δ_0 and D_{G0} are the displacement efficiency and the fractal dimension for growth extrapolated at zero tip velocity. $\delta_0 = 0.03$ (Fig. 17) and $D_{G0} = 1.3$ (Fig. 19). On the other hand, the average of all the l values in figure 18 is $2.84 \times 10^{-3} \text{ m}$, yielding an average $R_0/l = 150$. From this, and using equation (6), one can predict the $D_G = f(\bar{U})$ curve from the $\delta = f(\bar{U})$ curve, or *vice versa*. The agreement is good. For instance, at $\bar{U} = 1.5 \times 10^{-3} \text{ m s}^{-1}$, $\delta \approx 0.215$, and one predicts $D_G \approx 1.68$. The actual value is ~ 1.66 . The $D_G = f(\bar{U})$ curve, calculated from the $\delta = f(\bar{U})$ curve, is shown in figure 19 as a dotted line. We have certainly not yet reached a strict agreement, but the uncertainty on the experimental data is such that one can hardly hope better than that.

5. Conclusions.

The main conclusion of this work is that radial fingering in viscoelastic shear-thinning fluids, in the absence of important interfacial tension, is intrinsically a fractal growth process. This fractal behaviour, which is associated with fractal growth exponents in the whole range 1.3-2.0, is not restricted to a narrow set of conditions which lead to DLA type patterns. It appears in a broad range of experimental conditions and can be associated with a broad range of morphologies. This is akin to the conclusion of a recent work of Rauseo *et al.* [10], in which « vestiges » (or precursory signs) of fractal behaviour were detected even in the case of Newtonian fluids with an important interfacial tension.

On a more detailed level, we showed that the displacement efficiency is almost totally dominated by the branching or tip-splitting cascade of the fingers. The width of the fingers itself has very little influence. If anything, the finger width decreases when the displacement efficiency increases. Furthermore, the branching cascade, estimated by the fractal exponent for growth, appears to be a simple function of the tip velocity.

This points to at least two directions for further research : (i) understanding the relative insensitivity of the finger width to the flow velocity, or, more generally, relating the finger width to the viscoelastic properties of the more viscous medium and to the driving pressure. De Gennes [24] has recently addressed this point and we will report more experimental data on it very soon [20] ; (ii) reaching a finer description of the morphology of the final patterns and of the disturbances which affect the branches in the growing zone. We merely used the fractal exponent for growth but, obviously, there is much more information in a pattern than what can be extracted by a single parameter. One could for instance use the stretching of the growing zone and the distribution of distances between neighbouring branches in this growing zone, as considered by Pietronero *et al.* [25] or, even more generally, the $f(\alpha)$ function introduced by Halsey *et al.* [26].

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