Asymmetrical ideal concentrators with polygonal absorbers

F. Bloisi, J. Quartieri, L. Vicari, D. Ruggi

► To cite this version:

F. Bloisi, J. Quartieri, L. Vicari, D. Ruggi. Asymmetrical ideal concentrators with polygonal absorbers. *Revue de Physique Appliquée*, 1985, 20 (12), pp.857-862. 10.1051/rphysap:019850020012085700 . jpa-00245402

HAL Id: jpa-00245402

<https://hal.science/jpa-00245402>

Submitted on 1 Jan 1985

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Classification

Physics Abstracts

42.78 — 42.80 — 86.30R

Asymmetrical ideal concentrators with polygonal absorbers

F. Bloisi, J. Quartieri, L. Vicari

Dipartimento di Fisica Nucleare Struttura della Materia e Fisica Applicata,
Facoltà di Ingegneria, Università di Napoli, Napoli, Italy

and D. Ruggi

A.P.R.E. S.p.A., Roma, Italy

(Reçu le 14 septembre 1984, révisé le 29 juillet 1985, accepté le 2 août 1985)

Résumé. — Récemment nous avons étudié des concentrateurs symétriques avec des absorbeurs polygonaux qui présentent des avantages par rapport aux autres concentrateurs. Dans ce papier nous avons étendu l'utilisation des absorbeurs polygonaux au cas de concentrateurs asymétriques. Nous présentons ici une nouvelle famille d'appareils que l'on peut construire connaissant les paramètres suivants : angle d'acceptation, inclinaison et surface de collection. Nous démontrons aussi que ces systèmes sont idéaux dans le sens qui a été défini pour les concentrateurs asymétriques par Mills and Giutronich.

Abstract. — In a recent paper we introduced symmetrical non-focusing concentrators with polygonal absorbers, showing some advantages which they have with respect to other concentrators. In this paper we extend our study of the use of polygonal absorbers to the case of asymmetrical non-focusing concentrators. We introduce here a new family of devices that can be constructed once given the following parameters of use : acceptance angle, inclination an collecting area. We show that they are ideal in the sense defined by Mills and Giutronich for asymmetrical concentrators.

Nomenclature.

A_1	area of the receiving surface ;
$A_{1\perp}$	useful receiving area (orthogonal projection of A_1) ;
A_2	absorbing area ;
l_0	cross section of the receiving area ;
l_1, l_2, l_n^*	lengths of the sides of the absorber in ACAA or ACPA (cross section) ;
n	number of sides of the absorber in ACPA ;
p	perimeter length of the absorber in ACPA (cross section) ;
R_c	concentration factor for symmetrical or asymmetrical systems ;
R_c^*	concentration factor for symmetrical systems ;
r	radius of the circumference in which the absorber is inscribed (ACPA) ;
α	acceptance angle of the system ;
τ	inclination of the system.

1. Introduction.

In a recent paper [1] we introduced a new family of symmetrical non-focusing concentrators with polygonal absorbers. They are particularly well suited when thermal insulation is obtained inserting the absorber into a vacuum glass tube. In particular the optical losses in the gap between the absorber and the mirror are avoided owing to the geometry of the system.

The direction of the optical axis of a concentrator is usually determined by the position of the light source in order to have the maximum flux trough the surface for its main position. In symmetrical devices the normal to the collecting surface has the same direction of the optical axis. Nevertheless in some cases the collecting surface is constrained by structural or other limits (i.e. it has to be horizontal). The problem can be solved by truncation of symmetrical concentrators [2-4], which results in a reduction of the concentrating

factor, or by means of asymmetrical concentrators [5-8]. This second solution is usually to be preferred [5].

In this paper we study the use of polygonal absorbers in asymmetrical devices. The geometries used present the same advantages of the symmetrical ones while they are well suited to be employed on horizontal planes. Besides that we study in some details the problem of obtaining the maximum allowed area of the absorber, once given the diameter of the glass tube. Due to the ideal nature of the concentrators it is equivalent to find the maximum allowed receiving area.

Mills and Giutronich [9] have shown that, in order to be ideal, a linear non refractive concentrator with acceptance angle α (Fig. 1a) must have an entrance to exit aperture ratio R_c :

$$R_c = \frac{A_1}{A_2} = \frac{1}{\sin(\alpha/2) \sin(\Phi' + \alpha/2)} = \frac{2}{\cos \Phi + \cos \Phi'}.$$

Let us define the inclination τ as the angle between the normal to the collecting surface (ZC in Fig. 1a) and the bisector to the acceptance angle (VC in Fig. 1a). We obtain :

$$\Phi = \pi/2 + \tau - \alpha/2$$

$$\Phi' = \pi/2 - \tau - \alpha/2$$

and

$$R_c = \frac{1}{\sin(\alpha/2) \cos \tau}. \quad (1)$$

If the system is symmetrical ($\tau = 0$) we have the usual expression :

$$R_c^* = \frac{1}{\sin(\alpha/2)}.$$

Fig. 1. — Asymmetrical and symmetrical tilted concentrators.

We emphasize that an asymmetrical concentrator has not a better concentration factor than a symmetrical one along the optical axis; infact the useful collecting area, $A_{1\perp}$, is smaller than A_1 :

$$A_{1\perp} = A_1 \cos \tau$$

as it can be seen by figures 1a and 1b. Asymmetrical

concentrators are useful when external conditions do not allow to use symmetrical ones.

2. Asymmetrical concentrators with angular (ACAA) and polygonal (ACPA) absorbers.

With reference to figure 2, for given :

A_1 = collecting area (cross section $\overline{P_1P_2}$)

α = acceptance angle ($\widehat{P_1VP_2}$)

τ = inclination angle ($\widehat{ZCQ_2}$)

Fig. 2. — ACAA construction.

let us construct the reflector (H_1P_1 and H_2P_2) and the absorber (H_1VH_2) in the following way :

— let us consider the point H_1 on P_2V and the point H_2 on P_1V so that

$$\begin{aligned} \overline{VH_1}/\overline{P_1C} &= 1/R_c \\ \overline{VH_2}/\overline{P_2C} &= 1/R_c \end{aligned} \quad (2)$$

— let us consider the parabola with vertex H_1 and focus V . It can be shown [1] that P_1 belongs to this parabola, so we can draw the arc H_1P_1 ;

— similarly we draw the arc H_2P_2 of the parabola with vertex H_2 and focus V .

The asymmetrical concentrator with angular absorber just described (ACAA) is an ideal concentrator. The receiving to absorbing surface ratio is R_c , by construction. It is also easily shown that all rays crossing the receiving surface P_1P_2 within the acceptance angle $\widehat{P_1VP_2}$ reach the absorber either directly or after a reflection on the surfaces H_1P_1 or H_2P_2 . We observe that ACAA can be thought to be composed by two halves of two symmetrical concentrators : the left (right) one (Fig. 2) would have receiving surface $P_1Q_1(P_2Q_2)$, reflector $P_1H_1(P_2H_2)$, absorber $H_1V(H_2V)$ and axis VC .

We can substitute one of the two sides of the absorber (i.e. the largest : H_2V) of an ACAA with a polygonal $VK_1K_2...H_1$ of the same length (Fig. 3).

To keep the concentrator ideal we must add a reflective surface H_2H_1 which is the involute of $K_1K_2...H_1$. By a general property of involutes, every ray crossing K_1H_2 is reflected on the surface $K_1K_2...H_1$; the only difference in the optics of the device is given by an increase of the average number of reflections. Since the receiving to absorbing surface ratio has not been changed also the asymmetrical concentrator with polygonal absorber (ACPA) is an ideal concentrator.

There are many ways to construct the desired polygonal $K_1K_2...H_1$. In order to be of practical interest we want the absorber to be well suited to be insulated by means of a vacuum glass tube. To this aim we impose the following conditions :

- the convex polygon $VK_1K_2...H_1V$ must be inscribable in a circumference;
- for a fixed number of sides the radius of the circumference must be minimum (while the perimeter is fixed by the condition (2)).

Fig. 3. — ACPA construction.

3. Construction of optimal receiver.

Let us examine the absorber cross section in figure 3. The perimeter

$$p = \overline{VH_1} + \overline{VK_1} + \overline{K_1K_2} + \dots + \overline{K_{n-2}H_1} = \overline{VH_1} + \overline{VH_2}$$

and the length of one side

$$l_1 = \overline{VH_1}$$

are fixed by the geometry of the system, i.e. by :

- receiving area (in section $l_0 = \overline{P_1P_2}$);
- tilt angle ($\tau = \widehat{ZCQ_2}$);
- acceptance angle ($\alpha = \widehat{P_1VP_2}$).

From (2) we obtain

$$\begin{aligned} p &= l_0/R_c \\ l_1 &= \lambda p \end{aligned} \tag{3}$$

where R_c is the concentration ratio (1) for an ideal system and

$$\lambda = \frac{\overline{P_1C}}{\overline{P_1P_2}} = \frac{1 - \operatorname{tg}(\alpha/2) \operatorname{tg} \tau}{2} . \tag{4}$$

So our problem can be expressed as follows :
Let us examine all convex n -side polygons inscribable in a circumference, which satisfy the following conditions :

- the perimeter length is p ;
- the length of one side ($\overline{VH_1}$) is l_1 ;
- the angle ($\widehat{H_1VH_2}$) between this side and the one adjacent is α .

We will select the polygon (or polygons) which can be inscribed in the smallest circumference and we will find the radius of such circumference.

We will examine here the general case of n -side polygon, observing that for $n \rightarrow \infty$ we must substitute the polygonal $K_1K_2...K_{n-2}H_1$ with the arc of circumference K_1H_1 (Fig. 4).

Fig. 4. — Several absorber types.

A polygon with perimeter length p , inscribable in a circumference of radius r can be represented by a point in the plane (p, r) even if the correspondence is not one to one. Sometime, to deal with adimensional quantities, we will refer all lengths to l_1 , using the plane $(p/l_1, 2r/l_1)$. The polygons satisfying the previous conditions are represented by a set of points in the plane (p, r) . If, for each radius value, we construct the specified polygons having the minimum or the maximum perimeter length, we can define two functions : $p_{n\min}(r)$ and $p_{n\max}(r)$. The polygons of interest cannot fall outside the region of plane delimited by these functions (dashed area in Figs. 5 and 6).

Fig. 5. — Allowed ranges for the polygon perimeters. $\alpha \leq 90^\circ$. Fig. 6. — Allowed ranges for the polygon perimeters. $\alpha > 90^\circ$.

The boundary of this area, which, for a fixed value of p , has the smallest value of r , defines the desired function

$$r = r_{n \min}(p).$$

For the construction of the polygon, the side l_1 must not be greater than the diameter of the circumference, and the side l_2 must be internal or at least tangent to the circumference. Such conditions give

the following restrictions on the admissible values of r :

$$2r/l_1 \in \begin{cases} [1, \infty[& \text{for } \alpha \in [0, \pi/2] \\ [1/\sin \alpha, \infty[& \text{for } \alpha \in]\pi/2, \pi] \end{cases} \quad (5)$$

if the point K_1 belongs to the longer of the two arcs VH_1 (type I polygon); or

$$2r/l_1 \in [1, 1/\sin \alpha] \quad \text{for } \alpha \in [0, \pi/2] \quad (6)$$

if the point K_1 belongs to the shorter one (type II polygon); in this case the construction is impossible if $\alpha \in]\pi/2, \pi]$.

In both cases, for a fixed value of the radius, the perimeter length of the inscribed polygon can be varied only changing the position of points $K_2 K_3 \dots K_{n-2}$ on the circumference (which is obviously impossible if $n = 3$). As it can be easily verified, the minimum perimeter length is obtained when $K_2 \dots K_{n-2}$ coincide with one of the points H_1 or K_1 ; while the maximum perimeter length is obtained when the points $K_2 \dots K_{n-2}$ divide the arc $H_1 K_1$ into $n-2$ equal parts. So if $p_n^I(r)$ and $p_n^{II}(r)$ are the perimeters of such polygons, respectively of the I and of the II type, we have

$$p_{n \min}(r) = \min \{ p_n^I(r), p_n^{II}(r) \}$$

$$p_{n \max}(r) = \max \{ p_n^I(r), p_n^{II}(r) \}.$$

To evaluate p_n^I and p_n^{II} we observe that l_1 is given while l_2 is independent on n :

$$l_2 = l_1 \cos \alpha \pm \sin \alpha \sqrt{4r^2 - l_1^2} \quad (7)$$

where the minus sign appears when K_1 belongs to the shortest of the two arcs VH_1 (type II polygon). Moreover, the sum of the lengths of the remaining sides is

$$l_n^* = 2r\Gamma_n(\alpha)$$

where

$$\Gamma_n(\alpha) = \begin{cases} (n-2) \sin \frac{\alpha}{n-2} & \text{for } n = 1, 2, \dots \\ \alpha & \text{for } n \rightarrow \infty \end{cases}$$

so that we can write (including the case $n \rightarrow \infty$) :

$$p_n(r) = l_1 [1 + \cos \alpha + \Gamma_n(\alpha) 2r/l_1 \pm \sin \alpha \sqrt{(2r/l_1)^2 - 1}]. \quad (8)$$

Notice that

$$\sin \alpha \leq \Gamma_n(\alpha) \leq \alpha$$

and the minimum value is for the triangle ($n = 3$).

To study the function $p_n(r)$ (Figs. 5 and 6) we derive it with respect to r obtaining

$$\frac{\partial p_n}{\partial r} = 2\Gamma_n(\alpha) \pm \sin \alpha \left[1 - \left(\frac{l_1}{2r} \right)^2 \right]^{-1/2}.$$

So for type I polygons (plus sign) the function p_n does uniformly increase with r , while for type II polygons (minus sign) p_n does uniformly decrease with r in the respective domains (5) or (6). The ranges of variation of p are

$$p_n^I(r)/l_1 \in \begin{cases} [1 + \cos \alpha + \Gamma_n(\alpha), \infty[& \text{for } \alpha \in [0, \pi/2] \\ [1 + \Gamma_n(\alpha)/\sin \alpha, \infty[& \text{for } \alpha \in]\pi/2, \pi] \end{cases} \quad (9)$$

for a type I polygon, and

$$p_n^II(r)/l_1 \in [1 + \Gamma_n(\alpha)/\sin \alpha, 1 + \cos \alpha + \Gamma_n(\alpha)]$$

for

$$\alpha \in [0, \pi/2] \quad (10)$$

for a type II polygon.

To evaluate, for a fixed value of the perimeter, the radius of the smallest circumference in which a polygon satisfying the previously specified conditions can be inscribed, we must consider several cases :

Case a1

$$\alpha \in [0, \pi/2] \\ p/l_1 \in [1 + \cos \alpha + \Gamma_n(\alpha), \infty[. \quad (11)$$

In this case $r_{n\min}(p)$ is the inverse function of $p_n^I(r)$:

$$r_{n\min}(p) = \begin{cases} \frac{l_1}{2} \frac{\psi \Gamma_n(\alpha) - \sin \alpha \sqrt{\psi^2 + \sin^2 \alpha - \Gamma_n(\alpha)}}{\Gamma_n^2(\alpha) - \sin^2 \alpha} & \text{for } n > 3 \\ \frac{l_1}{2} \frac{\psi^2 + \sin^2 \alpha}{2 \psi \sin \alpha} & \text{for } n = 3 \end{cases} \quad (12)$$

with

$$\psi = p/l_1 - 1 - \cos \alpha.$$

Case a2

$$\alpha \in [0, \pi/2]$$

$$p/l_1 \in [1 + \cos \alpha + \sin \alpha, 1 + \cos \alpha + \Gamma_n(\alpha)].$$

In this case it is simply :

$$r_{n\min}(p) = l_1/2.$$

Case a3

$$\alpha \in [0, \pi/2]$$

$$p/l_1 \in [2, 1 + \cos \alpha + \sin \alpha].$$

In this case $r_{n\min}(p)$ is the inverse function of $p_n^II(r)$, and is given by the second of expressions (12).

Case b1

$$\alpha \in]\pi/2, \pi]$$

$$p/l_1 \in [1 + \Gamma_n(\alpha)/\sin \alpha, \infty[.$$

In this case $r_{n\min}(p)$ is the inverse function of $p_n^I(r)$, and is given by the expression (12).

Case b2

$$\alpha \in]\pi/2, \pi]$$

$$p/l_1 \in [2, 1 + \Gamma_n(\alpha)/\sin \alpha].$$

In this case :

$$r_{n\min}(p) = \frac{l_1}{2} \frac{1}{\sin \alpha}.$$

We make some remarks on the shape of the polygons we obtain in different cases.

In cases a1) and b1) :

— points $K_2 \dots K_{n-2}$ are equidistributed on the arc $H_1 K_1$ so that the polygon is uniquely defined;

— for some values of the parameters p , l_1 , α and n the polygon can be inscribed in a half-circumference. This is true for example if

$$\alpha \in [0, \pi/2]$$

$$2r/l_1 \geq 1/\cos \alpha$$

or equivalently

$$\alpha \in [0, \pi/2]$$

$$p/l_1 \geq \frac{1 + \cos \alpha + \Gamma_n(\alpha)}{\cos \alpha} \quad (13)$$

In cases a2) and b2) :

— points $K_2 \dots K_{n-2}$ are not equidistributed over the arc $H_1 K_1$ so that the polygon is not uniquely defined;

— side VH_1 is a diameter of the circumference so that the polygon can be inscribed in a half-circumference.

In case a3) :

— in this range of values of p the polygon is always a triangle (points $K_2 \dots K_{n-2}$ coincide with point H_1 or point K_1);

— point K_1 belongs to the shortest arc VH_1 so that the radius increases as the perimeter decreases;

— for the same reason the polygon can be inscribed in a half-circumference.

4. Conclusions.

For the sake of completeness the ACPA have been studied for every possible value of the parameters. Nevertheless we observe that for the applications to the use of solar energy for heating purposes the devices of greater interest are the ones that cannot be inscribed in half a circumference, otherwise there exists a smaller circle which contains the polygon, even if not all of the vertices belong to the circumference. On the other hand, for the polygons satisfying the conditions (11) and (13) it does not exist a tube in which they can be inserted of diameter less than the computed one. The corresponding limit values on the tilt angle τ as a function of α can be obtained by using equations (3) and (4). These limits, that are slightly affected by the choice for n , are shown in figure 7 for $n = 4$.

Fig. 7. — Range of most interest for ACPA practical applications.

In the usual applications it is useful to set $\alpha \approx 60^\circ$. In this case, the allowed range for τ is $20^\circ \leq \tau \leq 45^\circ$, corresponding to a wide range of latitudes [10].

Figure 8 shows the dependence of the radius on the number of sides, for $\alpha = 60^\circ$. As it can be seen there is a sharp reduction of the radius from $n = 3$ to $n = 4$ while for $n > 4$ there are only small further reductions. Since increasing the number of the sides increases the manufacturing difficulties, $n = 4$ appears to be a good choice.

A particularly interesting situation is when $VK_1 = VH_1$ (Fig. 3) and the absorber is symmetric. For instance, for $\alpha = 60^\circ$ and $\tau = 30^\circ$ the quadrangular absorber is symmetric and is just the same as introduced for symmetrical devices. This implies the possibility of making asymmetrical devices constructing asymmetrical mirrors but using the same absorbers of symmetrical ones.

Fig. 8. — Radius to perimeter ratio versus number of sides.

References

- [1] BLOISI, F., RUGGI, D. and VICARI, L., Ideal concentrators with polygonal absorbers, *Revue Phys. Appl.*, submitted for publication.
- [2] BRASLAVSKAYA, N. V. and BARANOV, V. K., Truncated « focones » and « foclines », *Gelioteknika* 13 (1977), 25-30.
- [3] MCINTIRE, V. R., Truncation of nonimaging cusp concentrators, *Solar Energy* 23 (1979) 351-355.
- [4] JONES JR., R. E., Circular arc approximation of truncated CPC collectors, *Solar Energy* 25 (1980) 139-147.
- [5] MILLS, D. R. and GIUTRONICH, J. E., Assymetrical non-imaging cylindrical solar concentrators, *Solar Energy* 20 (1978) 45-55.
- [6] MILLS, D. R., The place of extreme assymetrical non-focusing concentrators in solar energy utilization-technical note, *Solar Energy* 21 (1978) 431-434.
- [7] MILLS, D. R. and GIUTRONICH, J. E., New ideal concentrators for distant radiation sources, *Solar Energy* 23 (1979) 85-87.
- [8] RABL, A., Comparison of solar concentrators, *Solar Energy* 18 (1976) 93-111.
- [9] MILLS, D. R. and GIUTRONICH, J. E., Symmetrical and asymmetrical ideal cylindrical radiation transformers and concentrators, *J. Opt. Soc. Am.* 69 (1979) 325-328.
- [10] MILLER, C. W., Collections times for through-type concentrators having arbitrary orientation, *Solar Energy* 20 (1978) 399-404.