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Inverse scattering on electromagnetic measurements in a stratified medium

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Résumé. — Dans le cadre d'un modèle théorique monodimensionnel, nous étudions les possibilités de déduire la permittivité d'un milieu stratifié à partir de mesures du champ électromagnétique rayonné par un émetteur se déplaçant perpendiculairement aux interfaces. Des résultats très surprenants sont démontrés. Nous donnons une méthode simple et très efficace que nous testons numériquement sur des mesures expérimentales simulées.

Abstract. — In the framework of a theoretical one-dimensional model, we investigate the possibility of deducing the permittivity of a stratified medium from measurements of the electromagnetic field radiated by an emitter which moves in a direction perpendicular to the interfaces. Surprising results are demonstrated : a very efficient and simple method is given and numerically checked on simulated experimental data.

1. Introduction.

The determination of the complex permittivity of a stratified medium from measurements of an electromagnetic field is a technique currently used in Optics and Electromagnetics. In Optics, the source is placed in the upper medium (air) and, for instance, measurements of the reflected field for different angles of incidence may permit one to reconstruct a permittivity profile [1]. In geophysics, bore-hole measurements are implemented by using an emitter and receivers moving in the direction perpendicular to the surface, in order to retrieve the complex permittivity of the formation surrounding the bore-hole [2-4]. In this paper, we mainly deal with this second problem. Our first aim is to study the conditions under which one may hope to retrieve the permittivity. To this end, we define a one dimension problem, simpler than the situations encountered in practice, but very close from a theoretical point of view. Thanks to the simplicity of this model, we are able to state some surprising conclusions about

the possibilities of such a technique. Then, we present a very efficient and easily implemented method. Unfortunately, the generalization of this new method to the three dimensional case is not straightforward, at least in the current experimental conditions, but we hope to be able to achieve this generalization in the near future.

2. Direct and inverse problems.

2.1 THE MODEL (FIG. 1). — We consider a stratified medium having a complex permittivity $\varepsilon(z)$ piecewise constant. An emitter E located at $z = \zeta$ generates a field whose complex amplitude F (with time dependence in $e^{-i\omega t}$) satisfies the equation :

$$F'' + k^2(z) F = \delta(z - \zeta) \quad (1)$$

F and dF/dz being continuous at the interfaces, $k^2(z) = k_0^2 \varepsilon(z)$ being a piecewise constant function ($k_0 = 2\pi/\lambda_0$, where λ_0 denotes the wavelength in vacuum), and δ being the Dirac distribution. Of course,

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Fig. 1. — The model : the horizontal full lines represent the interfaces and the horizontal dashed lines represent the location of the emitter (E) and the two receivers (R_1 and R_2).

the function F must satisfy an outgoing wave condition for $z \rightarrow \pm \infty$. In other words, F must decrease or propagate towards $z \rightarrow \infty$ (resp. $z \rightarrow -\infty$) in the upper (resp. lower) medium. Two receivers R_1 and R_2 are located at $z = r_1$ and $z = r_2$, with $r_1 = \zeta + l_1$, $r_2 = \zeta + l_2$, $l_2 > l_1 > 0$. Throughout this paper, we suppose that l_1 and l_2 are fixed, while ζ (and consequently related quantities such as r_1 , r_2 , $(r_1 + r_2)/2$, etc...) is varied.

In practice, the theoretical model we are dealing with can be connected to the 2D model described in [2] in the following way : the field $F(z)$ corresponds to the θ component $E_\theta(z, \rho_0)$ of the electric field radiated at a distance ρ_0 of the Oz axis by a current loop with axis Oz and radius ρ_0 , the receivers R_1 and R_2 being loops of radius ρ_0 placed at distances l_1 and l_2 from the emitter such that :

$$l_2 - l_1 \ll l_1.$$

Under these conditions, the electric field $E_\theta(z, \rho_0)$ on the receivers satisfies an equation close to equation (1).

2.2 THEORY OF THE DIRECT PROBLEM. — Here, we consider the following « direct » problem : $k^2(z)$ being known, calculate $F(z)$.

Now, we shall recall a classical expression for $F(z)$ [5], which is nothing but the Green function associated with the differential equation (1). We assume that the stratified medium is composed by a finite number $N + 1$ of homogeneous materials, numbered from the bottom to the top. We call s_i ($i \in (1, N)$) the ordinate of the i th interface (between the i th and $(i + 1)$ th

materials) and k_i and ε_i the values of $k(z)$ and $\varepsilon(z)$ in the i th material. We define the two functions $H_p(z)$ and $H_m(z)$ continuous, with continuous first derivative, satisfying the homogeneous wave equation :

$$H''_{p,m} + k^2(z) H_{p,m} = 0, \quad (2)$$

H_p satisfying an outgoing wave condition for $z \rightarrow +\infty$ (upper medium) and H_m the same condition for $z \rightarrow -\infty$ (lower medium). Taking into account the form of the field in $\exp(ik_{N+1} z)$ and $\exp(-ik_1 z)$ in these two extreme media, the outgoing wave condition can be written in the mathematical form :

$$H'_p - ik_{N+1} H_p = 0 \quad \text{in the upper medium} \quad (3)$$

$$H'_m + ik_1 H_m = 0 \quad \text{in the lower medium} \quad (3')$$

So, $H_{p,m}(z)$, which satisfies a linear homogeneous differential equation of the second kind and the boundary condition (3) or (3'), is not unique, two arbitrary solutions being proportional.

Finally, let us define the two functions :

$$G_p(z, \zeta) = \begin{cases} H_p(z) & \text{if } z > \zeta, \\ H_p(\zeta) & \text{if } z \leq \zeta, \end{cases} \quad (4)$$

$$G_m(z, \zeta) = \begin{cases} H_m(\zeta) & \text{if } z \geq \zeta, \\ H_m(z) & \text{if } z < \zeta. \end{cases} \quad (5)$$

and the wronskian :

$$W = H_m H'_p - H_p H'_m,$$

which is independent of z (this result may be established by multiplying Eq. (2) by $H_{m,p}$ and by subtracting the two equations so obtained, the final result, i.e. $H''_p H_m - H''_m H_p = 0$, showing that $W' = 0$).

An elementary calculation shows that :

$$F(z) = G_p(z, \zeta) G_m(z, \zeta) / W, \quad (6)$$

and it is to be noticed that the value of $F(z)$ so obtained does not depend on the determination of H_p and H_m which have been chosen.

Finally, the function $F(z)$, which depends on the parameter ζ , can be calculated from the two functions $H_p(z)$ and $H_m(z)$ independent of ζ . This property considerably simplifies the numerical application.

2.3 NUMERICAL SOLUTION OF THE DIRECT PROBLEM. — This problem reduces to the calculation of the function $H_p(z)$ and $H_m(z)$ defined in section 2.2.

For the sake of simplicity, we only describe the computation of $H_p(z)$. In the j th material, the field can be written in the form :

$$H_p(z) = a_j^+ \exp(+ik_j z) + a_j^- \exp(-ik_j z), \quad (7)$$

a_j^+ and a_j^- being complex numbers.

In the following, we shall describe the field by the

values $V^+(z)$ and $V^-(z)$ of each term in the right hand side of (7). Thus, in the j th material :

$$H_p(z) = V^+(z) + V^-(z) \quad (7)$$

with

$$V^\pm(z) = a_j^\pm \exp(\pm ik_j z),$$

and $V(z)$ will denote the vector $(V^+(z), V^-(z))$.

It must be noticed that in the j th material, $V(z)$ and $V(z')$ are linked by the relation :

$$V(z') = Q(z', z, k_j) V(z), \quad (8)$$

Q being a 2×2 matrix given by :

$$Q(z', z, k) = \begin{vmatrix} \exp ik(z' - z) & 0 \\ 0 & \exp -ik(z' - z) \end{vmatrix} \quad (9)$$

The unknowns in our numerical implementation are the vectors A_n and B_n respectively equal to the limit of $V(z)$ above and below the n th interface (Fig. 2) :

$$A_n = V(s_{n+}), \quad (10)$$

$$B_n = V(s_{n-}). \quad (10')$$

From (8), it turns out that, in the n th material, A_{n-1} can be deduced from B_n by the relation (Fig. 2) :

$$A_{n-1} = M_n B_n, \quad (11)$$

with

$$M_n = Q(s_{n-1}, s_n, k_n).$$

Furthermore, a straightforward calculation shows that the continuity of F and dF/dz entails the matrix relation (Fig. 2) :

$$B_n = T_n A_n, \quad (12)$$

Fig. 2. — Determination of the function $H_p(z)$ when $N = 3$.

the transmission matrix T_n being given by :

$$T_n = \begin{vmatrix} t_n^+ & t_n^- \\ t_n^- & t_n^+ \end{vmatrix} \quad (13)$$

with

$$t_n^+ = (k_n + k_{n+1})/2 k_n, \quad (14)$$

$$t_n^- = (k_n - k_{n+1})/2 k_n. \quad (14')$$

It is worth noting that the inverse of T_n is obtained very easily by replacing k_n by k_{n+1} and conversely.

Equation (12) allows us to generalize equation (8) and to state the following fundamental lemma :

Lemma : The vector $V(z)$ at any ordinate may be known as soon as its value $V(z_0)$ for an arbitrary ordinate z_0 is given. The matrix relation between $V(z)$ and $V(z_0)$ may be obtained by using (8) or (11) inside each material separating the points z and z_0 , and equation (12) at each interface between these points.

Finally, it must be remarked that $A_{N+1}^- = 0$, due to the outgoing wave condition on H_p . Now, if the coefficient A_{N+1}^+ is given, we know the vector $A_{N+1} = (A_{N+1}^+, 0)$ therefore the lemma shows that $V(z)$ can be deduced linearly from A_{N+1} . Since A_{N+1} only depends on a multiplicative constants A_{N+1}^+ , $V(z)$ and $H_p(z)$ depend linearly on A_{N+1}^+ as well and we know that the value of $F(z)$ given by (6) is independent of this multiplicative constant which may be chosen to be equal to unity. Figure 2 outlines the determination of the A_j and B_j from A_{N+1} .

2.4 DEFINITION OF THE INVERSE PROBLEM. — Let S and η be the quantities defined by :

$$\eta = \frac{r_1 + r_2}{2} = \zeta + \frac{l_1 + l_2}{2}$$

$$S = \frac{F(r_2)}{F(r_1)}.$$

So $S(\eta)$ contains both the ratio of the amplitude and the phase difference of the measurements implemented by the two receivers R_1 and R_2 .

The inverse problem we study is the following : knowing $S(\eta)$ for a set of values η_i of η in arithmetic progression, find the piecewise constant function $k^2(z)$. Of course, the choice of this particular inverse problem is inspired by the situation in actual experimental borehole measurements.

3. Some properties of the solutions of the direct problem.

3.1 THE INVARIANCE PROPERTY. — Let us state the following property called « invariance property » :

If the locations of R_1 and R_2 are fixed, the signal $S(\eta)$ only depends on the permittivity profile of the stratified medium above the first receiver R_1 . As a consequence, neither the permittivity $\epsilon(z)$ below R_1 , nor the location of the source (provided it remains below R_1) has any influence on the signal.

To demonstrate this surprising property, we first use equation (6) to express the signal $S(\eta)$. Since $r_2 > r_1 > \zeta$, it turns out that :

$$S(\eta) = H_p(r_2)/H_p(r_1). \quad (15)$$

Obviously, $S(\eta)$ does not depend on the location of the emitter E. Furthermore, the lemma shows that $V(z)$ and thus the signal S , can be deduced from $A_{N+1} = (1, 0)$ using the linear relations (12) and (8) in the region located between S_N and z (Fig. 2). Since these linear relations only require the knowledge of $\varepsilon(z)$ between z and ∞ , the permittivity for $z < r_1$ has no influence on $S(\eta)$.

3.2 THE EQUIVALENCE PROPERTY. — Let $\tilde{\varepsilon}(z)$ be a fictitious permittivity profile, and $\tilde{S}(\eta)$ the corresponding signal which would be measured in the stratified medium of permittivity $\tilde{\varepsilon}(z)$. The equivalence property can be stated in the following manner :

For any value z_0 , there exists a fictitious stratified structure of permittivity $\tilde{\varepsilon}(z)$ constant for $z > z_0$ and « equivalent » to the actual stratified structure, i.e. having the following properties :

$$\tilde{\varepsilon}(z) = \varepsilon(z) \quad \text{for } z < z_0, \quad (16)$$

$$\tilde{\varepsilon}(z) = \varepsilon_f \quad \text{for } z > z_0, \quad (16')$$

ε_f being a well chosen complex number,

$$\tilde{S}(\eta) = S(\eta) \quad \text{for } r_2 < z_0. \quad (16'')$$

The demonstration of this property is a direct consequence of the lemma : if $\tilde{V}(z)$ denotes the vector $V(z)$ of the fictitious stratified structure, it suffices to choose ε_f in order to have $\tilde{V}(z_0) = V(z_0)$. This will entail that $\tilde{V}(z) = V(z)$ for $z < z_0$ (for $z > z_0$, the permittivity of the two structures are not identical and thus the equality non longer holds). If the point z_0 is located in the n th material of the actual structure, the vector $\tilde{V}(z_0)$ will be deduced from the vector $\tilde{A}_n$ of the fictitious structure by a relation similar to (12) :

$$\tilde{V}(z_0) = \tilde{B}_n = \tilde{T}_n \tilde{A}_n, \quad (17)$$

$\tilde{t}_n^+$ and $\tilde{t}_n^-$ being obtained from (14) and (14') replacing k_{n+1} by $k_f = k_0 \varepsilon_f$. Because of the outgoing wave condition, $\tilde{A}_n = (\tilde{A}_n^+, 0)$ and the question which arises is the following : is it possible to find $\tilde{A}_n^+$ and k_f in such a way that $\tilde{V}(z_0)$ takes a given value $V(z_0)$? A straightforward calculation using equation (17) shows that it suffices to choose k_f such that :

$$k_f = k_n(V^+(z_0) - V^-(z_0))/(V^+(z_0) + V^-(z_0)). \quad (18)$$

3.3 CONSEQUENCES OF THE INVARIANCE AND EQUIVALENCE PROPERTIES ON THE INVERSE PROBLEM. — Let us suppose that a set of measurements of $S(\eta)$ has been implemented, $z_{\min}$ been the smallest value of r_1 and $z_{\max}$ the greatest value of r_2 during the measurement. A

direct consequence of the two properties established in the preceding section is that these measurements would be unchanged if the part of the structure located below $z_{\min}$ were modified in an arbitrary manner, or if the part of the structure located above $z_{\max}$ were replaced by an homogeneous medium having a well chosen permittivity ε_f . So, it is obvious that *the retrieval of $\varepsilon(z)$ outside the interval $(z_{\min}, z_{\max})$ is quite impossible.*

This surprising property has a fundamental importance for the solution of the inverse problem.

4. The approximate method of the « homogeneous material » (HM).

4.1 PRINCIPLE OF THE METHOD. — This method lies on assumptions similar to those of the method described in [2]. If the structure is assumed to be homogeneous from $-\infty$ to $+\infty$, which $k(z) = k_1$, the function $H_p(z)$ reduces to an exponential function :

$$H_p(z) = \alpha \exp(ik_1 z),$$

thus

$$S(\eta) = \exp(ik_1(l_2 - l_1))$$

and it turns out that k_1 can be deduced from one measurement :

$$k_1 = -i \text{Log}(S(\eta))/(l_2 - l_1). \quad (19)$$

So, for each measurement of $S(\eta)$, the above assumption enables one to associate a value $\varepsilon(\eta) = k_1^2/k^2$ using (19). It is worth noting that the determination of k_1 from (19) is not unique, unless the distance $l_2 - l_1$ between the two receivers is chosen in such a way that :

$$(l_2 - l_1) \text{Re}(k_1) < 2\pi. \quad (20)$$

Under these circumstances, $\text{Log}(S(\eta))$ will be chosen in order to have an imaginary part less than 2π .

4.2 NUMERICAL RESULTS. — We consider in figure 3 a stratified structure whose permittivity is given by a dashed line. We have computed the corresponding value of $S(\eta)$ using the numerical method described in paragraph 2, then the approximate value of $\varepsilon(\eta)$ given by (19) (full line in Fig. 3). The main conclusion is that this approximate method is able to give accurate value of $\varepsilon(z)$, except at the vicinity of an interface. More precisely, the lemma and elementary considerations lead us to state the following rule of thumb.

The determination of $\varepsilon(z)$ from the method of the homogeneous medium gives accurate results as soon as the two following conditions are satisfied :

— R_1 and R_2 are in the same material,

— The distance d between R_2 and the closest interface above it is greater than $d_0 \simeq 2/\text{Im}(k_i)$, k_i being the wave number in the material. This explains the failure of the method in the second material from the left.

Fig. 3. — Numerical computation of $\varepsilon(z)$ from the HM method with $\lambda = 12$, $l_1 = 0.635$, $l_2 = 1.27$, $d = 0.3175$. ----- Actual value of $\text{Re}(\varepsilon(z))$ and $\text{Im}(\varepsilon(z))$; ——— corresponding results obtained from the HM method.

In conclusion, this method is very simple to implement but can provide erroneous conclusions since the presence of « horns » in the vicinity of the interfaces could be interpreted as the consequence of the presence of fictitious layers which, in fact, do not exist. This phenomenon becomes greater as the resistivity in the material increases.

5. A new method : the locally homogeneous material (LHM) method.

5.1 PRINCIPLE OF THE METHOD. — From equations (7) and (15), we deduce that $S(\eta)$ can be expressed in the form :

$$S(\eta) = \frac{a_j^+ \exp(ik_j r_2) + a_j^- \exp(-ik_j r_2)}{a_j^+ \exp(ik_j r_1) + a_j^- \exp(-ik_j r_1)}$$

provided both R_1 and R_2 are in the j th material.

In order to simplify the above expression, we define :

$$\gamma = \exp(ik_j a), \quad (21)$$

where a denotes the distance $r_2 - r_1$ between the two receivers, and :

$$R = a_j^- / a_j^+.$$

After straightforward calculations, we get :

$$S(\eta) = \frac{\gamma + R \exp(-2ik_j r_1)/\gamma}{1 + R \exp(-2ik_j r_1)}. \quad (22)$$

From the above equation, it turns out that the signal $S(\eta)$ which is a complex number, depends on two unknown complex numbers k_j (or γ) and R , provided the two receivers are in the same material. Of course, it is quite impossible to deduce these two unknowns from one measurement. On the other hand, it can be conjectured that two successive measurements, for two different values of η , will provide enough informations to retrieve R and k_j , provided these two measurements are such that the receivers remain in the same material. Under these conditions, denoting by S_1 and S_2 the signals $S(\eta)$ and $S(\eta + d)$ obtained successively, we obtain the two equations :

$$S_1 = \frac{\gamma + R \exp(-2ik_j r_1)/\gamma}{1 + R \exp(-2ik_j r_1)}, \quad (23)$$

$$S_2 = \frac{\gamma + R \exp(-2ik_j r_1)/\gamma \delta}{1 + R \exp(-2ik_j r_1)/\gamma \delta}, \quad (24)$$

where

$$\delta = \exp(2ik_j d). \quad (25)$$

Now, the problem is the following : S_1 and S_2 being known, find k_j (γ and δ depending on the only variable k_j). With this aim, we first eliminate $R \exp(-2ik_j r_1)$ between (23) and (24) and, after elementary calculations, we obtain the fundamental equation :

$$\delta = \frac{\gamma^2 S_2 - \gamma(S_1 S_2 + 1) + S_1}{\gamma^2 S_1 - \gamma(S_1 S_2 + 1) + S_2}. \quad (26)$$

This equation contains the two unknown complex numbers γ and δ which are, in fact, linked to the only unknown k_j (see equations (21) and (25)), and thus verify the equation :

$$\delta = \gamma^{2d/a}. \quad (27)$$

Thus we have to solve the system of two non linear equations (26) and (27) with two unknowns γ and δ . These two unknowns must be less than unity in modulus since k_j has a positive imaginary part in (21) and (25), due to the losses in the material.

Before solving this system of equations, it is to be noticed that the determination of γ and δ does not imply a unique determination of k_j . In order to force the uniqueness, we must assume, for instance, that the phase of $\exp(ik_j a) = \gamma$ is positive and less than 2π , which implies that :

$$a \text{Re}(k_j) < 2\pi. \quad (28)$$

It is very important to notice that this condition implies the uniqueness of the determination of δ

from γ in (27). Indeed, this condition can be written in the form :

$$\text{Log } \delta = (2 d/a) \text{Log } \gamma.$$

If the phase of γ is positive and less than 2π , $\text{Log } \gamma$ is unique, thus δ is unique. So, (28) enables us to write (26) and (27) in the form of a non linear equation :

$$\gamma^{2d/a} = \frac{\gamma^2 S_2 - \gamma(S_1 S_2 + 1) + S_1}{\gamma^2 S_1 - \gamma(S_1 S_2 + 1) + S_2}, \quad (29)$$

where the phase of $\gamma^{2d/a}$ is obtained by multiplying by $2 d/a$ the phase of γ (between 0 and 2π).

It is not our purpose to study equation (29) in detail and to show the uniqueness of the solution whose modulus is less than unity. We have verified this property in the cases where $d/a = 1, 0.5, 0.25$ where (29) becomes an algebraic equation. For instance, when $d/a = 0.5$ (29) becomes :

$$\gamma^3 S_1 - \gamma^2(S_1 S_2 + 1) + \gamma S_2 - \gamma^2 S_2 + \gamma(S_1 S_2 + 1) - S_1 = 0,$$

where $\gamma = 1$ is a solution whose modulus is equal to 1 and must be rejected. Eliminating this solution leads to our equation of the second order :

$$\gamma^2 - \left(S_2 + \frac{S_2}{S_1} - 1 + \frac{1}{S_1} \right) + 1 = 0. \quad (30)$$

The product of the two solutions of (30) is equal to 1, and this entails the uniqueness of the solution whose modulus is less than 1.

When the solution of (29) is known for a given value of d/a , it can be known for close values. It suffices to take the Logarithmic derivative of (29) to deduce the variation $\delta\gamma$ from the variation of d/a .

A general means for solving (29) would be to search for the root of this equation in the complex plane using an iterative process (Newton method for instance). A first estimate of the solution would be obtained by considering first the known solution in the case where (29) can be solved analytically ($d/a = 0.5$ for instance), then by modifying slowly d/a in order to follow the solution of the transcendental equation. We have not implemented this general method.

Another means to find γ in the general case is to consider two successive equations (29) with signals S_1, S_2, S_3 , assuming that these three measurements have been performed in the same material. Eliminating $\gamma^{2d/a}$ between these two equations leads to the algebraic equations :

$$\begin{aligned} \frac{\gamma^2 S_3 - \gamma(S_2 S_3 + 1) + S_2}{\gamma^2 S_2 - \gamma(S_2 S_3 + 1) + S_3} &= \\ &= \frac{\gamma^2 S_2 - \gamma(S_1 S_2 + 1) + S_1}{\gamma^2 S_1 - \gamma(S_1 S_2 + 1) + S_2}. \end{aligned}$$

Fig. 4. — The same as figure 3, but with the LHM method and using equation (29).

Fig. 5. — The same as figure 3, but with the LHM method and using equation (30).

Eliminating the solutions $\gamma = \pm 1$ in the above equation leads to the equation of the second order :

$$\gamma^2 + c\gamma + 1 = 0, \quad (31)$$

with

$$c = (2 S_1 S_2 S_3 - S_2^2(S_1 + S_3) - 2 S_2 + S_1 + S_3)/(S_2^2 - S_1 S_3).$$

The solution is the root of (31) whose modulus is less than unity (the product of the two roots is equal to unity).

5.2 NUMERICAL RESULTS. — Figure 4 shows the numerical result obtained using equation (29) for $d/a = 0.5$. Our theoretical predictions are fully confirmed : the permittivity is rigorously given when the two receivers are in the same material. The superiority of this new method is all the more obvious since in the region where the two receivers are not in the same material, the curve of the predicted permittivity in general does not oscillate. It is not so for the curve

in figure 5 obtained from equation (31), which exhibits large horns. At present, we are unable to understand the superiority of equation (29) on equation (31) from a numerical point of view. We conjecture that the origine of this surprising phenomenon may be found in the variational properties of the two formulae.

6. Conclusion.

Our simple model has allowed us to state important conclusions. The LHM method is much more precise and powerful than the HM method which assumes the material to be homogeneous. Moreover, we can conjecture from the rules of invariance and equivalence that the determination of the permittivity of a stratified structure outside the domain of measurement in the two-dimension problem is very difficult, even though the simple rules of the one dimensional problem cannot be generalized to this more difficult problem. A generalization of the LHM method to the two-dimension problem is not straightforward. This is one of the aims of our future work.

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