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Excitation of acoustic beams in layered substrates

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Résumé. — On discute dans l'article un mécanisme d'excitation d'ondes acoustiques de volume en faisceaux étroits. L'excitation est produite sur la surface libre ou à l'interface du demi-espace entre la couche et substrat; le matériau dont est constitué la couche a des vitesses caractéristiques plus élevées que celles du substrat. Dans le domaine des épaisseurs de couche supérieures à celles correspondant à la valeur de coupure du mode pseudo-Rayleigh de cette géométrie « rapide sur lent », il existe des modes couplés qui rayonnent dans le substrat pour un domaine étroit d'angle de zénith. Les propriétés de ce faisceau sont étudiées dans deux cas : l'un pour lequel la vitesse de Rayleigh du matériau de la couche est supérieure à la vitesse transversale du substrat mais inférieure à la vitesse longitudinale, dans ce cas seule une onde de cisaillement est rayonnée dans le substrat, dans le second cas la vitesse de Rayleigh du matériau de la couche est supérieure à la vitesse longitudinale du substrat en sorte que, à la fois des ondes de cisaillement et des ondes de compression sont rayonnées. Pour des épaisseurs de couche typiques, les faisceaux rayonnés n'ont que quelques degrés de largeur.

Abstract. — A mechanism for the launching of acoustic bulk waves in narrow beams into a solid is discussed. The excitation is at the free surface or on the interface of a layered half-space for which the material of the layer has higher characteristic velocities than those of the substrate material. For a range of layer thickness greater than that corresponding to the cut-off value for the Rayleigh-like mode of this fast-on-slow geometry, there is a form of coupled mode which radiates down into the substrate in a narrow range of zenith angle. The properties of this beam are studied for two cases, one in which the Rayleigh velocity of the layer material exceeds the transverse velocity of the substrate but not its longitudinal, here only a shear wave is radiated into the substrate. In the second case the Rayleigh velocity of the layer material exceeds the longitudinal velocity of the substrate so that both shear and compressional waves are radiated. For typical layer thickness the beams radiated are but a few degrees in width.

1. Introduction.

When a Rayleigh wave is excited on the interface between a solid and a liquid this wave phase-matches to a compressional wave in the liquid near a specific angle and thus radiates in a relatively narrow beam. Such an arrangement has been used to form an acoustic lens by exciting a circular converging Rayleigh wave on the interface [1]. It is also well known that the higher Rayleigh-like modes for layered half-spaces with slow layers on faster substrates become cut-off or start to radiate when the mode velocity equals the substrate shear velocity [2]. Beyond cut-off these modes radiate into the solid near a phase-matching angle.

It is the purpose here to consider fast layers on slower substrates. In this case the dispersion curve of mode phase-velocity as a function of layer thickness

rises from the Rayleigh velocity of the substrate material at zero layer thickness to a cut-off at the shear velocity of the substrate material [3]. The regime of interest here is for a relative layer thickness beyond the cut-off value and in particular the interest lies with the angular distribution of the radiation of acoustic waves into the substrate in this regime with the intent of using such excitation to produce focussed beams within solids. To date only isotropic material combinations have been considered.

2. Analysis.

The geometry consists of an isotropic layer of thickness d characterized by elastic constants c_{11}^L and c_{44}^L and by a density ρ^L on an isotropic half-space characterized by c_{11} , c_{44} and ρ . As shown in figure 1

Fig. 1. — Coordinate system for the layered half-space. Layer and substrate are isotropic.

the z -axis of the coordinate system points into the substrate and the origin is taken on the interface. It is assumed initially that the excitation is a given stress distribution of time dependence $\exp(-j\omega t)$ applied either on the free surface $z = -d$ or on the interface. Thus if the layer material were infinite in thickness the bulk shear waves therein would have a wave vector $k_T^L = \omega(\rho^L/c_{44}^L)^{1/2}$ and the longitudinal waves $k_L^L = \omega(\rho^L/c_{11}^L)^{1/2}$ while similar expressions apply for the substrate material.

Defining scalar potentials ϕ^L and ϕ and vector potentials $\mathbf{h}^L$ and $\mathbf{h}$, the displacements in the substrate are given by [4]

$$\mathbf{u} = \nabla\phi + \nabla \times \mathbf{h} \quad \text{with} \quad \nabla \cdot \mathbf{h} = 0 \quad (1)$$

and the potentials satisfy the wave equations

$$\begin{aligned} \nabla^2 \phi &= -k_L^2 \phi \\ \nabla^2 h_i &= -k_T^2 h_i \quad \text{with} \quad i = x, y, z. \end{aligned} \quad (2)$$

There are similar expressions for the layer.

Each of the potentials is Fourier transformed in a given plane z . Thus using upper-case symbols for transformed quantities we have for example

$$\Phi(k_x, k_y) \exp(jk_z z) = \mathcal{F} \{ \phi(x, y, z) \} \quad (3)$$

where in order to satisfy the wave equation

$$k_x^2 + k_y^2 + k_z^2 = k_L^2. \quad (4)$$

Thus each point (k_x, k_y) in k -space in this Fourier optics approach represents as usual a plane wave of amplitude Φ propagating in the direction of the wave vector $\mathbf{k}$ which has components k_x , k_y and k_z . The sign of k_z is discussed in the next paragraph.

Since the substrate is infinite in depth, for each value of (k_x, k_y) we have solutions of the wave equation for the substrate given by

$$\Phi \exp(jk_z z), \quad H_1 \exp(jk_z z) \quad \text{and} \quad H_2 \exp(jk_z z) \quad (5)$$

and to satisfy the divergence equation (1.b),

$$k_z H_z = -(k_1 H_1 + k_2 H_2). \quad (6)$$

Here the direction 1 is parallel to the radial vector to the point being considered in the (k_x, k_y) plane, and direction 2 is perpendicular to it in the same plane. The radial length in this plane is $k_r = (k_x^2 + k_y^2)^{1/2}$. That is for the moment the (x, y) coordinates are rotated to suit the point in k -space. In each case for k_z , the sign of the square root is taken positive, as in $k_{z1} = +(k_L^2 - k_r^2)^{1/2}$ when $k_r < k_L$, propagating waves; and $k_{z1} = +j(k_r^2 - k_L^2)^{1/2}$ when $k_r > k_L$, evanescent waves. Similar definitions apply to the other z -directed components k_{z1} , k_{z1}^L and k_{z1}^L in the substrate and layer respectively.

In the layer, since it is finite in thickness, two solutions must be kept for each of the potentials giving

$$\Phi^+ \exp(jk_{z1}^L z), \quad H_1^+ \exp(jk_{z1}^L z), \quad H_2^+ \exp(jk_{z1}^L z) \quad (7)$$

and

$$\Phi^- \exp(-jk_{z1}^L z), \quad H_1^- \exp(-jk_{z1}^L z), \quad H_2^- \exp(-jk_{z1}^L z). \quad (8)$$

A solution to the problem is then given by any combination of the 9 terms in equations (6), (7) and (8) above which satisfy the boundary conditions.

The nine boundary conditions are the continuity of each displacement component across the interface at each point, i.e. for $z = 0$

$$u_i^L = u_i \quad i = x, y, z; \quad (9)$$

the normal stresses on the free surface $z = -d$ equal to the applied stresses t^d on that surface

$$T_{iz}^L = t_{iz}^d(x, y) \quad (10)$$

and the difference in normal stresses across the interface equal to any applied stress t^0 on that interface

$$T_{iz}^L - T_{iz} = t_{iz}^0(x, y). \quad (11)$$

The boundary conditions are expressed in terms of the potentials, then transformed on the planes $z = 0$ and $z = -d$ and thus become expressions in terms of the transformed potentials of equations (6), (7) and (8). The result is the matrix equation

$$\begin{bmatrix} A & \vdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \vdots & B \end{bmatrix} \times \begin{bmatrix} \Phi^+ \\ H_2^+ \\ \Phi^- \\ H_2^- \\ \Phi \\ H_2 \\ \dots \\ H_1^+ \\ H_1^- \\ H_1 \end{bmatrix} = \frac{1}{c_{44}^L} \times \begin{bmatrix} 0 \\ 0 \\ \tau_{1z}^0 \\ \tau_{zz}^0 \\ \tau_{1z}^d \\ \tau_{zz}^d \\ \dots \\ 0 \\ \tau_{2z}^0 \\ \tau_{2z}^d \end{bmatrix} \quad (12)$$

in which

$$A = \begin{bmatrix} k_r & -k_{zt}^L & k_r & k_{zt}^L & -k_r & k_{zt} \\ k_{zt}^L & k_r & -k_{zt}^L & k_r & -k_{zt} & -k_r \\ -2k_r k_{zt}^L & (k_T^L)^2 - 2k_r^2 & 2k_r k_{zt}^L & (k_T^L)^2 - 2k_r^2 & 2rk_r k_{zt} & -r(k_T^2 - 2k_r^2) \\ 2k_r^2 - (k_T^L)^2 & -2k_r k_{zt}^L & 2k_r^2 - (k_T^L)^2 & 2k_r k_{zt}^L & r(2k_r^2 - k_T^2) & -2rk_r k_{zt} \\ -2k_r k_{zt}^L e^{-d'} & [(k_T^L)^2 - 2k_r^2] e^{-d''} & 2k_r k_{zt}^L e^{d'} & [(k_T^L)^2 - 2k_r^2] e^{d''} & 0 & 0 \\ [2k_r^2 - (k_T^L)^2] e^{-d'} & -2k_r k_{zt}^L e^{-d''} & [2k_r^2 - (k_T^L)^2] e^{d'} & 2k_r k_{zt}^L e^{d''} & 0 & 0 \end{bmatrix} \quad (13)$$

$$B = \begin{bmatrix} (k_T^L)^2/k_{zt}^L & -(k_T^L)^2/k_{zt}^L & -k_T^2/k_{zt} \\ (k_T^L)^2 & (k_T^L)^2 & -rk_T^2 \\ (k_T^L)^2 e^{-d''} & (k_T^L)^2 e^{d''} & 0 \end{bmatrix} \quad (14)$$

where

$$d' \equiv jk_{zt}^L d \quad d'' \equiv jk_{zt}^L d \quad r = c_{44}/c_{44}^L \quad (15)$$

and

$$\begin{aligned} \tau_{iz}^0(k_x, k_y) &= \mathcal{F} \{ t_{iz}^0(x, y) \} \quad i = 1, 2, z. \\ \tau_{iz}^d(k_x, k_y) &= \mathcal{F} \{ t_{iz}^d(x, y) \} \end{aligned} \quad (16)$$

The matrices are familiar ones for surface-wave propagation [3, 5], and for a given isotropic material combination and value of d , $[A]$ and $[B]$ are functions

of k_r only. The values of k_r which make the determinant of $[A]$ equal to zero are the wave numbers of the propagating Rayleigh-like modes for a layer of a given d on a half-space, while the zeroes of the determinant of $[B]$ give the Love modes. The latter will not be discussed further here.

3. Fast-on-slow combinations.

Here we are concerned with the case where the layer material is « faster » than the substrate i.e. $k_T^L < k_T$, with the sagittal part only of equation (12), and in particular with the waves radiated into the substrate, that is Φ and H_2 . If the elements of the reciprocal matrix of $[A]$ are designated by A_{mn}^{-1} then

$$\Phi(k_x, k_y) = A_{53}^{-1}(k_r) \tau_{1z}^0(k_x, k_y) + A_{54}^{-1}(k_r) \tau_{2z}^0(k_x, k_y) + A_{55}^{-1}(k_r) \tau_{1z}^d(k_x, k_y) + A_{56}^{-1}(k_r) \tau_{2z}^d(k_x, k_y) \quad (17)$$

$$H_2(k_x, k_y) = A_{63}^{-1}(k_r) \tau_{1z}^0(k_x, k_y) + A_{64}^{-1}(k_r) \tau_{2z}^0(k_x, k_y) + A_{65}^{-1}(k_r) \tau_{1z}^d(k_x, k_y) + A_{66}^{-1}(k_r) \tau_{2z}^d(k_x, k_y) \quad (18)$$

Since radiation into the solid is of interest, the values of k_r of concern are in the range where k_{zt} (and perhaps k_z also) is real, i.e. $k_r < k_T$. In this range the determinant of $[A]$ is non-zero, that is for the chosen finite value of d there are no poles of the $A_{mn}^{-1}(k_r)$ functions on the real k_r axis.

Taking the inverse transform of say equation (17) to find the scalar potential $\varphi(x, y, z)$ according to equation (3), i.e.

$$\varphi(x, y, z) = (1/4\pi^2) \iint \Phi \exp \{ j(k_x x + k_y y + k_{zt} z) \} dk_x dk_y \quad (19)$$

can be considered as adding up plane waves, one for each value of (k_x, k_y) weighted by the function $\Phi(k_x, k_y)$ of equation (17), for all values of (k_x, k_y) . Some of the plane waves have real values of k_{zt} or k_z and thus propagate into the substrate and contribute to the « summation » at depths much larger than the wavelengths concerned and others have imaginary values of k_{zt} or k_z and are evanescent and contribute only for

small values of z . In this isotropic case the quantities $A_{mn}^{-1}(k_r)$ in equation (17) and (18) are, for radiating waves, in turn in the nature of weighting factors that are functions of the zenith angle $\beta = \tan^{-1}(k_r/k_z)$, with $k_z = k_{zt}(k_r)$ or $k_z(k_r)$ and real.

Before plotting these functions, consider first some prototype examples of excitation. If the excitation is a very long line-source of compressive stress T_{zz} along the y -axis on the interface, then τ_{zz}^0 is equal to a constant along the k_x -axis and zero elsewhere while the other τ functions are zero everywhere. Thus for this line source, only waves represented by points along the k_x -axis will contribute in the « summation » of equation (19), the « 2 » axis will be the y -axis and

$$\Phi(k_x, 0) = KA_{54}^{-1}(k_x) \quad (20)$$

$$H_y(k_x, 0) = KA_{64}^{-1}(k_x) \quad (21)$$

where K is a constant.

If the long line-source is translated to $x = X_0$ then again only points along the k_x -axis contribute and contributions are multiplied by the phase factor

Fig. 4. — Transforms of vector and scalar potentials for a Be layer on a PZT substrate. Line excitation on the free surface. $d = \lambda_T$.

occur at the same value of k_x , the angle of maximum radiation for the two polarizations will be quite different because the corresponding values of k_z are different.

The dependence of H_y on layer thickness for the Be layer on PZT and free-surface excitation is shown in figure 5. As the layer becomes thicker the locations of the maxima asymptote to the Rayleigh wave number of the layer material from larger values and the amplitude of the maxima decrease. As the layer becomes thinner the maxima increase and move to higher k -values, for $d < 0.4 \lambda_T$ the maximum of Φ lies to the right of $k_x = k_L$ where k_{zi} is imaginary and the corresponding longitudinal waves do not propagate into the substrate. For d less than about $0.1 \lambda_T$, the maximum occurs at $k_x > k_T$ so that the corresponding shear wave is also cut off, and for still smaller layer thickness there arise the poles corresponding to the propagating Rayleigh-like layer modes. For this material combination the maxima of figure 5 still occur if the line source excitation is moved to the interface.

Fig. 5. — Transform of vector potential for various thickness of Be layer on PZT substrate. Scale of the broken curves four times that of solid curves.

5. Beam formation.

The propagation of the waves into the substrate can be considered by multiplying the transformed potentials of equations (17) and (18) by the appropriate propa-

gating factor, equation (5), and taking the inverse Fourier transform, that is, at a depth z

$$\varphi(x, y, z) = \mathcal{F}^{-1} \{ \Phi(k_x, k_y) \exp(jk_{zi} z) \}. \quad (25)$$

The development of the radiating beam for a simple line source excitation is illustrated in figures 6 and 7. For the line source, Φ and H are given by equations (20) and (21) and thus the transforms of figure 4 were multiplied point-by-point by the propagating factor and an inverse Fast Fourier Transform was performed. Figure 6 shows the scalar and vector potentials along a scan of x at a distance $z = 25 \lambda_T$ into the substrate for the case of Be on PZT, $d = \lambda_T$, and a compressive line source on the free surface. At this depth the beams are well developed in that they are assuming angular symmetry about what will become the angle of maximum radiation for the far-field [5, 6] but the translation due to the mechanism of generation, *via* a Rayleigh mode, is quite evident. Marked on the diagram are the intercepts on this scan at $z = 25 \lambda_T$ of radii from the origin at the two angles corresponding to the common maximum in figure 4, ($\beta_m = 44^\circ$ for longitudinal, $\beta_m = 24^\circ$ for shear). The phase across the main beam in each scan is that of a plane wave propagating at these respective angles β_m , not that of cylindrical or plane waves emanating directly from the line source.

Fig. 6. — Beam profiles for the scalar and vector potentials at $z = 25 \lambda_T$ for the case of figure 4, Be on PZT, $d = \lambda_T$. Points marked R_L and R_ϕ give intersection with this plane of line from origin at the angle of the maxima in figure 4.

The progressive stages of the development of the radiating beam are illustrated in figure 7 which shows profiles of the shear wave $h_y(x)$ at different depths for the same geometry as in figure 6. Close to the interface the profile is somewhat exponential in shape as would be associated with the decay of the radiating Rayleigh

Fig. 7. — Development of vector potential beam for the case of figure 4. Lines A and B mark the 3dB beam width. Here line A is also a line from the origin at the angle of the maximum in figure 4.

mode. At large distances the beam becomes more symmetrical and has a full 3 dB width of some 4.2° . As the beam expands the inner 3 dB line (line A) remains

more or less co-linear with the « Rayleigh line », i.e. a radial line at zenith angle $\beta_m = 24^\circ$. The longitudinal beam for this geometry has the same character as in figure 6 but the Rayleigh line is at $\beta_m = 44^\circ$ and the full 3 dB beam width at large z is 10° . Since $\Phi(k_x)$ and $H_y(k_x)$ have approximately the same shape, the beam profiles of $|\Phi|$ and $|h_y|$ near the interface $z \approx 0$ are also of approximately the same shape, but due to the longer wavelength of the longitudinal waves not only is the zenith angle of the beam greater than for the shear, because of the phase matching, but so is the beam width because of the smaller relative radiating aperture.

For the more-typical of the fast-on-slow cases which is shown in figure 2, aluminum-on-copper, only the beam for the shear mode h_y propagates but the progressive development of the beam is similar to that of figure 7 however here the Rayleigh angle is 54° and for $d = 1 \lambda_T$ the full 3 dB beam width is 4.3° .

If the line source is circular rather than linear as considered above, each segment of the circle will produce maximum radiation near the zenith angle β_m thus a focal spot will be produced in the solid on the axis of the circle and at a distance from the interface determined by the radius. Since β_m depends on the layer thickness in wavelengths, the focal length can be changed by varying the frequency of excitation.

Similar arguments regarding the focussed radiation apply to the case of higher Rayleigh-like modes of slow-on-fast layered geometries operated in the cut-off region of these modes, but in such cases it is difficult to excite but one mode.

In summary it has been noted that shear waves can be excited easily in a narrow beam radiating into the substrate by using a fast-on-slow layer combination operating beyond cut-off of the Rayleigh mode. For very fast layers longitudinal as well as shear beams can be excited.

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