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HAL Id: jpa-00245337
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Submitted on 1 Jan 1985

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Excitation of acoustic beams in layered substrates

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(Reçu le 20 juillet 1984, accepté le 15 octobre 1984)

Résumé. — On discute dans l'article un mécanisme d'excitation d'ondes acoustiques de volume en faisceaux étroits. L'excitation est produite sur la surface libre ou à l'interface du demi-espace entre la couche et substrat; le matériau dont est constitué la couche a des vitesses caractéristiques plus élevées que celles du substrat. Dans le domaine des épaisseurs de couche supérieures à celles correspondant à la valeur de coupure du mode pseudo-Rayleigh de cette géométrie « rapide sur lent », il existe des modes couplés qui rayonnent dans le substrat pour un domaine étroit d'angle de zénith. Les propriétés de ce faisceau sont étudiées dans deux cas : l'un pour lequel la vitesse de Rayleigh du matériau de la couche est supérieure à la vitesse transversale du substrat mais inférieure à la vitesse longitudinale, dans ce cas seule une onde de cisaillement est rayonnée dans le substrat, dans le second cas la vitesse de Rayleigh du matériau de la couche est supérieure à la vitesse longitudinale du substrat en sorte que, à la fois des ondes de cisaillement et des ondes de compression sont rayonnées. Pour des épaisseurs de couche typiques, les faisceaux rayonnés n’ont que quelques degrés de largeur.

Abstract. — A mechanism for the launching of acoustic bulk waves in narrow beams into a solid is discussed. The excitation is at the free surface or on the interface of a layered half-space for which the material of the layer has higher characteristic velocities than those of the substrate material. For a range of layer thickness greater than that corresponding to the cut-off value for the Rayleigh-like mode of this fast-on-slow geometry, there is a form of coupled mode which radiates down into the substrate in a narrow range of zenith angle. The properties of this beam are studied for two cases, one in which the Rayleigh velocity of the layer material exceeds the transverse velocity of the substrate but not its longitudinal, here only a shear wave is radiated into the substrate. In the second case the Rayleigh velocity of the layer material exceeds the longitudinal velocity of the substrate so that both shear and compressional waves are radiated. For typical layer thickness the beams radiated are but a few degrees in width.

1. Introduction.

When a Rayleigh wave is excited on the interface between a solid and a liquid this wave phase-matches to a compressional wave in the liquid near a specific angle and thus radiates in a relatively narrow beam. Such an arrangement has been used to form an acoustic lens by exciting a circular converging Rayleigh wave on the interface [1]. It is also well known that the higher Rayleigh-like modes for layered half-spaces with slow layers on faster substrates become cut-off or start to radiate when the mode velocity equals the substrate shear velocity [2]. Beyond cut-off these modes radiate into the solid near a phase-matching angle.

It is the purpose here to consider fast layers on slower substrates. In this case the dispersion curve of mode phase-velocity as a function of layer thickness rises from the Rayleigh velocity of the substrate material at zero layer thickness to a cut-off at the shear velocity of the substrate material [3]. The regime of interest here is for a relative layer thickness beyond the cut-off value and in particular the interest lies with the angular distribution of the radiation of acoustic waves into the substrate in this regime with the intent of using such excitation to produce focussed beams within solids. To date only isotropic material combinations have been considered.

2. Analysis.

The geometry consists of an isotropic layer of thickness \( d \) characterized by elastic constants \( c_{11} \) and \( c_{44} \) and by a density \( \rho \) on an isotropic half-space characterized by \( c_{11}, c_{44} \) and \( \rho \). As shown in figure 1
Fig. 1. — Coordinate system for the layered half-space. Layer and substrate are isotropic. 

The z-axis of the coordinate system points into the substrate and the origin is taken on the interface. It is assumed initially that the excitation is a given stress distribution of time dependence \( \exp(-j\omega t) \) applied either on the free surface \( z = -d \) or on the interface. Thus if the layer material were infinite in thickness the bulk shear waves therein would have a wave vector \( k_s^L = \omega(\rho_s^L/c_{44}^L)^{1/2} \) and the longitudinal waves \( k_L^L = \omega(\rho_s^L/c_{11}^L)^{1/2} \) while similar expressions apply for the substrate material.

Defining scalar potentials \( \phi_L \) and \( \phi \) and vector potentials \( \mathbf{h}_L \) and \( \mathbf{h} \), the displacements in the substrate are given by [4]

\[
\mathbf{u} = \nabla \phi + \nabla \times \mathbf{h} \quad \text{with} \quad \nabla \cdot \mathbf{h} = 0
\]

and the potentials satisfy the wave equations

\[
\nabla^2 \phi = -k_s^L \phi
\]

\[
\nabla^2 h_i = -k_L^L h_i \quad \text{with} \quad i = x, y, z.
\]

There are similar expressions for the layer.

Each of the potentials is Fourier transformed in a given plane \( z \). Thus using upper-case symbols for transformed quantities we have for example

\[
\Phi(k_x, k_y) \exp(jk_z z) = \mathcal{F} \{ \phi(x, y, z) \}
\]

where in order to satisfy the wave equation

\[
k_s^Z + k_L^Z + k_L^Z = k_s^L.
\]

Thus each point \( (k_x, k_y) \) in \( k \)-space in this Fourier optics approach represents as usual a plane wave of amplitude \( \Phi \) propagating in the direction of the wave vector \( \mathbf{k} \) which has components \( k_x, k_y \) and \( k_z \).

The sign of \( k^2 \) is discussed in the next paragraph.

Since the substrate is infinite in depth, for each value of \( (k_x, k_y) \) we have two solutions for each of the potentials giving

\[
\Phi^+ \exp(jk_z^L z), \quad H^+_L \exp(jk_z^L z), \quad H^+_2 \exp(jk_z^L z)
\]

and to satisfy the divergence equation (1.b),

\[
k_{st} H_2 = -(k_1 H_1 + k_2 H_2). \quad (6)
\]

Here the direction 1 is parallel to the radial vector to the point being considered in the \( (k_x, k_y) \) plane, and direction 2 is perpendicular to it in the same plane. The radial length in this plane is \( k_{st} = (k_1^2 + k_2^2)^{1/2} \).

That is for the moment the \( (x, y) \) coordinates are rotated to suit the point in \( k \)-space. In each case for \( k_z \), the sign of the square root is taken positive, as in \( k_{st}^L = + (k_1^2 - k_2^2)^{1/2} \) when \( k_1 < k_2 \), propagating waves; and \( k_{st}^L = + j(k_1^2 - k_2^2)^{1/2} \) when \( k_1 > k_2 \), evanescent waves. Similar definitions apply to the other \( z \)-directed components \( k_{st} \), \( k_{st}^L \) and \( k_{st}^L \) in the substrate and layer respectively.

In the layer, since it is finite in thickness, two solutions must be kept for each of the potentials giving

\[
\Phi^+ \exp(jk_{st}^L z), \quad H^+_L \exp(jk_{st}^L z), \quad H^+_2 \exp(jk_{st}^L z)
\]

and

\[
\Phi^- \exp(-jk_{st}^L z), \quad H^-_L \exp(-jk_{st}^L z), \quad H^-_2 \exp(-jk_{st}^L z).
\]

A solution to the problem is then given by any combination of the 9 terms in equations (6), (7) and (8) above which satisfy the boundary conditions.

The nine boundary conditions are the continuity of each displacement component across the interface at each point, i.e. for \( z = 0 \)

\[
u^L_i = u_i \quad i = x, y, z;
\]

denotes the normal stresses on the free surface \( z = -d \) equal to the applied stresses \( t^d \) on that surface

\[
T^L_z = t^d_z(x, y)
\]

and the difference in normal stresses across the interface equal to any applied stress \( t^0 \) on that interface

\[
T^L_z - T_z = t^0(x, y).
\]

The boundary conditions are expressed in terms of the potentials, then transformed on the planes \( z = 0 \) and \( z = -d \) and thus become expressions in terms of the transformed potentials of equations (6), (7) and (8). The result is the matrix equation

\[
\begin{bmatrix}
\Phi^+ \\
H^+_L \\
H^+_2
\end{bmatrix} = \frac{1}{ck_{st}^L} \times
\begin{bmatrix}
A & 0 \\
0 & B
\end{bmatrix}
\begin{bmatrix}
\Phi^- \\
H^-_L \\
H^-_2
\end{bmatrix}
\]

(12)
The matrices are familiar ones for surface-wave propagation \([3, 5]\), and for a given isotropic material combination and value of \(k\) and \(r\) are functions of \(k\), only. The values of \(k\) which make the determinant of \([A]\) equal to zero are the wave numbers of the propagating Rayleigh-like modes for a layer of a given \(d\) on a half-space, while the zeroes of the determinant of \([B]\) give the Love modes. The latter will not be discussed further here.

### 3. Fast-on-slow combinations.

Here we are concerned with the case where the layer material is « faster » than the substrate, i.e. \(k_f > k_r\), with the sagittal part only of equation (12), and in particular with the waves radiated into the substrate, that is \(4J\) and \(H_2\). If the elements of the reciprocal matrix of \([A]\) are designated by \(A_{nn}^{-1}\) then

\[
B = \begin{bmatrix}
    (k_f^2)^2/k_{st}^2 & - (k_f^2)^2/k_{st}^2 & - k_f^2/k_{st} \\
    -2 k_f k_{st} & (k_f^2)^2 - 2 k_f^2 & 2 k_f k_{st}^2 \\
    2 k_r^2 - (k_f^2)^2 & -2 k_r k_{st}^2 & 2 k_r^2 - (k_f^2)^2 \\
    -2 k_r k_{st} e^{-\alpha} & [(k_f^2)^2 - 2 k_f^2] e^{-\alpha} & 2 k_r k_{st} e^\alpha \\
    [2 k_r^2 - (k_f^2)^2] e^{-\alpha} & -2 k_r k_{st} e^{-\alpha} & [2 k_r^2 - (k_f^2)^2] e^\alpha
\end{bmatrix}
\]

where

\[
d' \equiv j k_{st}^2 d \quad d'' = j k_{st}^2 d \quad r = c_{44}/c_{44} \quad (15)
\]

and

\[
\tau_{ij}^0(k_x, k_y) = \mathcal{F} \{ \phi_{ij}^0(x, y) \} \quad i = 1, 2, z.
\]

\[
\tau_{ij}^\alpha(k_x, k_y) = \mathcal{F} \{ \phi_{ij}^\alpha(x, y) \} \quad (16)
\]

The matrices are familiar ones for surface-wave propagation \([3, 5]\), and for a given isotropic material combination and value of \(d\), \([A]\) and \([B]\) are functions of \(k\), only. The values of \(k\) which make the determinant of \([A]\) equal to zero are the wave numbers of the propagating Rayleigh-like modes for a layer of a given \(d\) on a half-space, while the zeroes of the determinant of \([B]\) give the Love modes. The latter will not be discussed further here.

Since radiation into the solid is of interest, the values of \(k\), of concern are in the range where \(k_{st}\) (and perhaps \(k_z\) also) is real, i.e. \(k_r < k_f\). In this range the determinant of \([A]\) is non-zero, that is for the chosen finite value of \(d\) there are no poles of the \(A_{nn}^{-1}(k)\) functions on the real \(k\) axis.

Taking the inverse transform of say equation (17) to find the scalar potential \(\phi(x, y, z)\) according to equation (3), i.e.

\[
\phi(x, y, z) = (1/4 \pi^2) \int \int \Phi \exp \left\{ j(k_x x + k_y y + k_{st} z) \right\} dk_x dk_y \quad (19)
\]

can be considered as adding up plane waves, one for each value of \((k_x, k_y)\) weighted by the function \(\Phi(k_x, k_y)\) of equation (17), for all values of \((k_x, k_y)\). Some of the plane waves have real values of \(k_{st}\) or \(k_z\), and thus propagate into the substrate and contribute to the « summation » at depths much larger than the wavelengths concerned and others have imaginary values of \(k_{st}\) or \(k_z\), and are evanescent and contribute only for small values of \(z\). In this isotropic case the quantities \(A_{nn}^{-1}(k)\) in equation (17) and (18) are, for radiating waves, in turn in the nature of weighting factors that are functions of the zenith angle \(\beta = \tan^{-1}(k_x/k_z)\), with \(k_z = k_x(k_z)\) or \(k_{st}(k_z)\) and real.

Before plotting these functions, consider first some prototype examples of excitation. If the excitation is a very long line-source of compressive stress \(T_{zz}\) along the \(y\)-axis on the interface, then \(\tau_{zz}^\alpha\) is equal to a constant along the \(k_{st}\)-axis and zero elsewhere while the other \(\tau\) functions are zero everywhere. Thus for this line source, only waves represented by points along the \(k_{st}\)-axis will contribute in the « summation » of equation (19), the « 2 » axis will be the \(y\)-axis and

\[
\Phi(k_x, 0) = KA_{33}^{-1}(k_x) \quad (20)
\]

\[
H_s(k_x, 0) = KA_{66}^{-1}(k_x) \quad (21)
\]

where \(K\) is a constant.

If the long line-source is translated to \(x = X_0\) then again only points along the \(k_{st}\)-axis contribute and contributions are multiplied by the phase factor
exp\(jk_x\) et \(X_0\), an obvious property of the Fourier transform.

Now if the source is a ring source of compressional stress of radius \(R\) applied on the interface then \(\tau_{zz}\) will be a constant times the zero-order Bessel function and the other \(\tau\) functions will be zero, thus \(\Phi\) and \(H_z\) will be functions of \(k_r\) only

\[
\Phi(k_x, k_y) = KA_{6x}^{-1}(k_x) J_0(k_y R) \quad (22)
\]
\[
H_z(k_x, k_y) = KA_{6z}^{-1}(k_x) J_0(k_y R) \quad (23)
\]

where corresponding to each point \((k_x, k_y)\), \(H_z\) contributes only to \(h(x, y)\) in the local azimuthal direction \((k_x = k_r \cos \alpha; \ k_y = k_r \sin \alpha)\). Note that for large values of \(k_y, R\), that is waves tilted appreciably to the \(z\)-axis,

\[
J_0(k_y R) \approx (2\pi k_y R)^{-1/2} \exp(-j\pi/4) \quad (24)
\]

giving phase shifts reminiscent of those associated with two displacements of the long line source above.

4. Form of transform.

Let us turn now to some examples of the radiation functions \(A_{ij}^{-1}\) for different material combinations. Figure 2 shows \(|A_6^{-1}(k_x)|\), the vector potential factor involved in free-surface compressional excitation in equation (18), for layers of aluminum of different thickness \(d/\lambda_T\) on a copper substrate. For \(d\) less than about 0.4 \(\lambda_T\) the \(A^{-1}\) matrix is singular and the locations of the poles give the dispersion relation of the propagating Rayleigh-like modes for this fast-on-slow combination. With these bound modes \(k_{zt}\) and \(k_{rr}\) are of course imaginary.

It is seen that for layer thickness of the order of \(\lambda_T\) there is a peak in the \(A_6^{-1}\) curve which becomes narrower and higher with increasing thickness, asymptoting to the pole at \(k_x = k_R\) corresponding to the Rayleigh wave on the free surface of an infinitely thick layer. The values of \(k_{zt}\) are real in this range and the peak corresponds to a weighting factor or radiation pattern for shear waves which can be expressed as a function of the zenith angle \(\beta = \tan^{-1} k_y/k_{rr}\) Values of \(\beta\) are shown along the top scale. These curves can be interpreted as a \(k\)-space representation of the generation of a Rayleigh-wave by a line source on the free surface which couples more-or-less strongly to the shear waves in the substrate concentrated near the phase-matching zenith angle \(\tan^{-1} k_y/k_{rr}\).

Before looking at these waves in real space several features of such curves should be noted. The phase of \(A_6^{-1}\) for the aluminum-on-copper case with \(d = \lambda_T\) is shown in figure 3 and has the 180° phase change over the width of the amplitude maximum as is characteristic of simple damped resonant systems. The graphs of \(|A_6^{-1}|\), the scalar potential factor in equation (17), have the same general shape as figure 2 in the range shown, however the corresponding substrate propagation factors \(k_{zt}\) for longitudinal waves are imaginary for this material combination near the maxima \(k_z \approx k_n\). Such longitudinal disturbances in the substrate are necessary to satisfy the boundary conditions but do not propagate into the substrate.

With the aluminum-on-copper combinations the radiation maxima of figure 2 do not occur if the excitation is on the interface, that is \(A_6^{-1}\) in equation (18) is calculated.

It is interesting to consider the case of a layer material fast enough that its Rayleigh velocity exceeds the longitudinal as well as the shear velocity of the substrate, i.e. \(k_R < k_L\). In this case there is the possibility of coupling from the Rayleigh wave to both longitudinal and shear waves propagating in the substrate. Figure 4 shows the transforms \(H_y\) and \(\Phi\) for a line source on the free surface \((A_6^{-1}\) and \(A_5^{-1}\) respectively) for a layer of beryllium 1 \(\lambda_T\) in thickness on a PZT substrate. Here again there are strong maxima but at a value of \(k_x\) larger than \(k_R^0\) also in contrast to the case of figure 2 here the maximum of \(\Phi\) can occur in a range where \(k_{zt}\) as well as \(k_{rr}\) is real, and we would expect propagation into the substrate of longitudinal as well as shear waves. While the maxima of \(H_y\) and \(\Phi\)
Fig. 4. — Transforms of vector and scalar potentials for a Be layer on a PZT substrate. Line excitation on the free surface. $d = \lambda_T$.

occur at the same value of $k_x$, the angle of maximum radiation for the two polarizations will be quite different because the corresponding values of $k_z$ are different.

The dependence of $H_y$ on layer thickness for the Be layer on PZT and free-surface excitation is shown in figure 5. As the layer becomes thicker the locations of the maxima asymptote to the Rayleigh wave number of the layer material from larger values and the amplitude of the maxima decrease. As the layer becomes thinner the maxima increase and move to higher $k$-values, for $d < 0.4 \lambda_T$ the maximum of $\Phi$ lies to the right of $k_\beta = k_T$ where $k_T$ is imaginary and the corresponding longitudinal waves do not propagate into the substrate. For $d$ less than about 0.1 $\lambda_T$, the maximum occurs at $k_z > k_T$ so that the corresponding shear wave is also cut off, and for still smaller layer thickness there arise the poles corresponding to the propagating Rayleigh-like layer modes. For this material combination the maxima of figure 5 still occur if the line source excitation is moved to the interface.

Fig. 5. — Transform of vector potential for various thickness of Be layer on PZT substrate. Scale of the broken curves four times that of solid curves.

5. Beam formation.

The propagation of the waves into the substrate can be considered by multiplying the transformed potentials of equations (17) and (18) by the appropriate propagating factor, equation (5), and taking the inverse Fourier transform, that is, at a depth $z$

$$\varphi(x, y, z) = \mathcal{F}^{-1} \{ \Phi(k_x, k_y) \exp(jk_zz) \}.$$  (25)

The development of the radiating beam for a simple line source excitation is illustrated in figures 6 and 7. For the line source, $\Phi$ and $H$ are given by equations (20) and (21) and thus the transforms of figure 4 were multiplied point-by-point by the propagating factor and an inverse Fast Fourier Transform was performed. Figure 6 shows the scalar and vector potentials along a scan of $x$ at a distance $z = 25 \lambda_T$ into the substrate for the case of Be on PZT, $d = \lambda_T$, and a compressive line source on the free surface. At this depth the beams are well developed in that they are assuming angular symmetry about what will become the angle of maximum radiation for the far-field [5, 6] but the translation due to the mechanism of generation, via a Rayleigh mode, is quite evident. Marked on the diagram are the intercepts on this scan at $z = 25 \lambda_T$ of radii from the origin at the two angles corresponding to the common maximum in figure 4, ($\beta_m = 44^\circ$ for longitudinal, $\beta_m = 24^\circ$ for shear). The phase across the main beam in each scan is that of a plane wave propagating at these respective angles $\beta_m$, not that of cylindrical or plane waves emanating directly from the line source.

Fig. 6. — Beam profiles for the scalar and vector potentials at $z = 25 \lambda_T$ for the case of figure 4, Be on PZT, $d = \lambda_T$. Points marked $R_L$ and $R_S$ give intersection with this plane of line from origin at the angle of the maxima in figure 4.

The progressive stages of the development of the radiating beam are illustrated in figure 7 which shows profiles of the shear wave $h_\beta(x)$ at different depths for the same geometry as in figure 6. Close to the interface the profile is somewhat exponential in shape as would be associated with the decay of the radiating Rayleigh
mode. At large distances the beam becomes more symmetrical and has a full 3 dB width of some 4.2°. As the beam expands the inner 3 dB line (line A) remains more or less co-linear with the « Rayleigh line », i.e. a radial line at zenith angle $\beta_m = 24^\circ$. The longitudinal beam for this geometry has the same character as in figure 6 but the Rayleigh line is at $\beta_m = 44^\circ$ and the full 3 dB beam width at large $z$ is 10°. Since $\Phi(k_x)$ and $H_y(k_x)$ have approximately the same shape, the beam profiles of $|\Phi|$ and $|h_y|$ near the interface $z \approx 0$ are also of approximately the same shape, but due to the longer wavelength of the longitudinal waves not only is the zenith angle of the beam greater than for the shear, because of the phase matching, but so is the beam width because of the smaller relative radiating aperture.

For the more-typical of the fast-on-slow cases which is shown in figure 2, aluminum-on-copper, only the beam for the shear mode $h_y$ propagates but the progressive development of the beam is similar to that of figure 7 however here the Rayleigh angle is 54° and for $d = 1 \lambda_T$ the full 3 dB beam width is 4.3°.

If the line source is circular rather than linear as considered above, each segment of the circle will produce maximum radiation near the zenith angle $\beta_m$ thus a focal spot will be produced in the solid on the axis of the circle and at a distance from the interface determined by the radius. Since $\beta_m$ depends on the layer thickness in wavelengths, the focal length can be changed by varying the frequency of excitation.

Similar arguments regarding the focussed radiation apply to the case of higher Rayleigh-like modes of slow-on-fast layered geometries operated in the cut-off region of these modes, but in such cases it is difficult to excite but one mode.

In summary it has been noted that shear waves can be excited easily in a narrow beam radiating into the substrate by using a fast-on-slow layer combination operating beyond cut-off of the Rayleigh mode. For very fast layers longitudinal as well as shear beams can be excited.

Acknowledgments.

The authors wish to acknowledge the support of the Natural Sciences and Engineering Research Council of Canada and the programming help of Mr. M. Esonu.

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