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On atmospheric optical communications

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Résumé. — La diffraction d'une porteuse puissante qui se propage dans une chaîne de communication non guidée (atmosphère sèche) est plus rigoureusement étudiée. Quatre points pertinents sont examinés i) distribution radiale de la température, ii) paramètre « largeur moyenne du faisceau », iii) facteur de mérite normalisé de distorsion optique, et iv) mise en forme d'un champ optique pour réduire la diffraction créée par les effets réfractifs.

Abstract. — This paper investigates, more rigorously, the diffraction of a powerful optical carrier propagating in an unguided communication channel (dry atmosphere). Four main relevant topics are considered i) the radial temperature distribution, ii) the average beamwidth parameter, iii) a dimensionless optical distortion figure of merit, and iv) the irradiance tailoring technique as a method to reduce the diffraction caused by refractive effects.

1. Introduction.

The diffraction caused by the thermal blooming phenomenon associated with the propagation of a CW laser beam in an open medium constitutes one of the important nonlinear propagation phenomena. When a laser beam propagates in an absorbing medium, such as the atmosphere, heating of the medium takes place. This heating results in refractive index gradients across the laser beam. These gradients act as a distributed lens which diffractions the beam and decreases the peak intensity along the propagation path [1].

The problem of calculating the thermal blooming effect was solved on the basis of geometrical optics and paraxial approximation [2]. Light propagating through an inhomogeneous medium whose refractive index has weak changes with cylindrical symmetry around the propagation path was studied by Majias [3]. The cylindrical TEM$_{00}$ and TEM$_{10}$ laser modes were considered and explicit analytical expressions for the light ray and intensity profiles for short and long time intervals were derived. Jones and McMordie [1] reported that the magnitude of the effects of the thermal blooming could be characterized by four dimensionless parameters or numbers : the Fresnel number, an absorption number, a distortion number, and a slewing number. Sodha et al. [4] deeply investigated the self-distortion of laser beams propagation in dielectrics, plasmas, and semiconductors. Based on the paraxial approximation, they reduced the problem to a nonlinear differential equation in a single characterizing parameter, namely, the « beamwidth parameter ». Wallace et al. [5] evaluated the impact of irradiance tailoring as a method of reducing the influence of the refractive effects caused by the heating of the medium. The process is to create, by irradiance shaping, only correctable phase changes along the propagation path. The practicality of designing lasers of a specific irradiance distribution is, however, largely unanswered. Generally, irradiance tailoring may be achieved by special splitters and filters. El-Badawy and El-Halafawy [6, 7] investigated partially the thermal blooming phenomenon in dry atmosphere.

In the present work, the heating of clear air by powerful Gaussian laser beams is studied. The impact of irradiance tailoring technique is applied to design a suitable electric field distribution to reduce the diffraction caused by the refractive effects and to increase the focal length. Thermal blooming phenomenon is rigorously treated taking into account the coupling between the laser and clear air only at the entrance. The affecting parameters are studied in details over a wide range of variation. The correlations...
of a dimensionless optical distortion figure of merit and the other affecting parameters are also investigated.

Section 2 describes the basic model. Section 3 deals with mathematical formulation and analysis of the problem. The results and discussions are exposed in section 4. Section 5 points out the main conclusions.

\[ I(r, z)/I(r, 0) = \exp \left[ - \sigma_a z - \int_0^z \left( \nabla_i + \frac{\nabla_i F}{F} \right) \int_0^z \left( \nabla_i n/n \right) dz' \right], \]

where \( \nabla_i \) is the transverse gradient and \( z \) is the propagation distance.

The refractive index \( n \) and the pressure \( P \) are related to the medium (air) density \( \rho \) and temperature \( T \), respectively, through Gladstone's law and the perfect gas law as

\[ n = 1 + \frac{2}{3} K \rho_m, \]

and

\[ P = R_a \rho_m T, \]

where \( R_a \) is the universal gas constant and \( K \) is a constant proportional to the medium polarizability. The temperature \( T \) is determined from the energy balance equation:

\[ - \nabla_i k_i \nabla T = \sigma_a I(r, z). \]

Based on data of Kumar [8], it could be assumed that both the thermal conductivity \( k_i \) and the density \( \rho_m \) are linearly related to the normalized temperature \( T/T_0 \) (where \( T_0 \) is the initial temperature) in the range of interest as:

\[ k_i = k_0(1 + \alpha T), \]

and

\[ \rho_m = \rho_0(1 + \beta T), \]

with \( k_0 = 4.64 \times 10^{-3} \text{ W/mK} \), \( \alpha = 4.263 \), \( \rho_0 = 1.95 \text{ kg/m}^3 \), and \( \beta = -0.356 \).

After Kroll and Watson [9], both the absorption coefficient \( \sigma_a \) and the polarizability \( K \) appear to be inversely proportional to the square of the laser frequency \( \omega \) through the relations:

\[ \sigma_a = \sigma_0(\omega_0/\omega)^2 P, \]

and

\[ K = K_0(\omega_0/\omega)^2, \]

where \( P \) is the pressure in standard atmospheric units. For dry air, \( \sigma_0 = 3 \times 10^{-4} \text{ m}^{-1} \) [10], \( K_0 = 2.181 \times 10^{-4} \) [2, 11], and \( \omega_0 = 1.7777 \times 10^{14} \text{ rad/s} \).

Following our previous work [6], the spatial profile \( F(r, 0) \), defined as \( I(r, 0) = I_0 F(r, 0) \), is tailored under the following forms.

2.1 TEM_{00} mode. — i) Gaussian distributions.

\[ F(r, 0) = \exp(\pm (r/r_0)^2). \]

The negative sign is for the Gaussian distributions and the positive sign for the inverse distributions. The index \( i \) equals 1, 2 and 3 for the normal, flat, and superflat distributions, respectively.

ii) Parabolic distributions

\[ F(r, 0) = 1 + \alpha_i(r/R)^2. \]

where \( R \) is the beam radius, and \( \alpha_i \) is a parameter in the range (−1, 10). The index \( i \) runs from 0 to 10.

iii) Biquadratic distributions

\[ F(r, 0) = 1 + \alpha_i(r/R)^2 - (r/R)^4. \]

These spatial profiles have a maximum value \( f_m \) at a fixed radial position \( r_m \) where \( r_m = R/\sqrt{2} \) and \( f_m = 1 + \alpha_i/4 \).

2.2 TEM_{10} mode. — Following Majias [3], the distributions are tailored as

\[ F(r, 0) = (1 - m(r/R)^2)^2 \exp(-m(r/R)^2). \]

The parameter \( m \) controls the radial position \( r_2 \) at which \( F(r, 0) = 0 \). Thus \( r_2 = R/m \).

2.3 TE_{20} mode.

\[ F(r, 0) = A(r/R)^2m(1 - (r/R)^2), \]

where \( A \) is designed so that the maximum value of \( F(r, 0) \) occurring at \( r_m \) is always equal to unity where \( r_m = R/\sqrt{m(1 + m)} \).
The above distributions have the same maximum intensity wherever it is.

The dimensionless optical distortion figure of merit, DODFM, introduced by the authors in reference [6] proved to be a good measuring tool to evaluate the impact of irradiance tailoring as a method to reduce the diffraction caused by the refractive effects. This figure is designed under the form

\[ F_m = \frac{Q}{\Delta T \langle \partial n/\partial T \rangle H}, \]  

(14)

where \( Q \) is the power transferred per unit length from the hot propagation path to the surrounding cold blanket, \( \Delta T \) is the maximum radial temperature difference, \( \langle \partial n/\partial T \rangle \) is the average temperature rate of change of the refractive index, and \( H \) is the total absorbed power per unit length. From the optical communication point of view, one must have high \( Q \) and small \( \partial^2 T / \partial \rho^2 \) and \( H \). The figure of merit \( F_m \) designed in this manner makes a good propagation process (of less diffraction) be characterized by a large value of \( F_m \).

3. Analysis.

3.1 Temperature Distribution. — Assuming that the coupling between laser and air is only at the entrance of the medium, the heat equation is written as

\[ -\nabla.k_0 \nabla T = \sigma_a I_0 F(r, z). \]  

(15)

Taking equation (5) into consideration, substituting \( T_n = T/T_o, \rho = r/R \) (for Gaussian distributions \( R \) is the 1/e radius), and \( F(r, z) = F(r, 0) \exp(-\sigma_a z) \), and assuming as a solution for equation (15)

\[ T_n(\rho, z) = T_0 \exp(-\sigma_a z), \]  

(16)

equation (15) reduces to

\[ (1 + \alpha T_n) \left( T_n + \frac{1}{\rho} T_n + R^2 \sigma_a^2 T_n \right) + \right. \]

\[ \left. + \alpha(T_n^2 + \sigma_a^2 R^2 T_n^2) = -AF(\rho, 0), \right. \]  

(17)

where \( A \) is the dimensionless quantity \( \sigma_a R^2 I_0/k_0 T_o \). The solution of equation (17) is assumed as a power series:

\[ T_n = \sum_{i=0} b_{2i} \rho^{2i}. \]

Such a solution satisfies the boundary condition

\[ \frac{\partial T}{\partial \rho} = 0 \quad \text{at} \quad \rho = 0. \]  

(18)

At the interface between the hot propagation path and the cold surrounding air, the following boundary condition is applied:

\[ -k_n \left. \frac{\partial T}{\partial r} \right|_{r=R} = h(T - T_0) \left|_{r=R} \right., \]  

(19)

where \( h \) is the air-air heat transfer coefficient [12]. Using equation (13) in (19) gives

\[ \sum_{i=0} b_{2i} \rho^{2i}(1 + i\delta) = 1, \]  

(20)

where \( \delta = 4/N_u \rho_w \) with \( N_u \) the Nusselt No. (for air \( N_u = 2000 R \) [12]) and \( \rho_w \) the normalized radial position at \( r = R \). The solution of (20) yields \( b_0 \) as a function of both the medium properties and the laser characteristics.

3.2 Thermal Blooming Analysis. — The solution of (1) is assumed under a simple closed form for initially Gaussian laser beam,

\[ l(r, z) = \frac{I_0}{f^2(r, z)} \exp(-r^2/(r^2 + f^2(r, z))) \times \]

\[ \exp(-\sigma_a z), \]  

(21)

where \( f(r, z) \) is a single characterizing parameter known as the « beamwidth parameter »; it characterizes the phenomenon of thermal blooming both quantitatively and qualitatively. At \( z = 0, f(r, 0) = 1 \). Using (21), (1) yields

\[ \frac{1}{f^2} \exp \left[ -\rho^2 \left( \frac{1}{f^2} - 1 \right) \right] = \]

\[ = \exp \left[ -\frac{1}{n} \int_0^z (V_i + g(f)) \times \int_0^z V_i n d\alpha d\alpha' \right], \]  

(22)

where

\[ g(f) = 2 \rho^2 V_i f - 2 \rho f/r_o - 2 \rho^2 V_i f \]  

(23)

Assuming that the transverse refractive index gradient, \( V_i n \), to be constant along the propagation path, substituting \( F_n = 1/f \) and taking the logarithmic differentiation w.r.t. \( z \)-coordinate, one obtains:

\[ \nabla_z F_n (1 - \rho^2 F_n) = -\frac{z}{2n} [F_n \nabla_z^2 n + g(F_n) \nabla_z n], \]  

(24)

where

\[ g(F_n) = 2(1 - \rho^2 F_n^2) \nabla_z F_n/F_n - 2 \rho F_n^2/r_o. \]

Using equation (3), where \( P \) is constant, with equation (2) yields

\[ \nabla_z n = -K \rho_m \nabla_z T/T, \]  

(25)

\[ \nabla_z^2 n = -K \rho_m (\nabla_z T/T - 2(\nabla_z T/T)^2). \]  

(26)

Substituting equations (25) and (26) into (24), \( F_n \) is obtained using a computational technique and assuming decoupling between the laser beam and the propagation path.
4. Results and discussion.

The problem is numerically solved with high accuracy and low negligible truncation error in the series solution. Monotonic relationships are found among the set of dependent variables \{ T, n, f_a, DODFM \} and the set of independent variables \{ E_0, R, \lambda, F(0, 0) \}.

The variations of the increase in the radial temperature, \((T - T_0)\), with the normalized radial position, \(\rho\), at different values of the electric field, \(E_0\), and the assumed sets of parameters are shown in figure 1. It is clear that the radial temperature, \(T\), increases with the increase in \(E_0\) at any radial position. This is expected since the absorbed power is proportional to the quantity \(E_0^2\).

The increase in \(E_0\) yields an increase in the radial gradient of the refractive index, \(\nabla n\), at any radial position and results in an increase in the beam cross section and consequently a decrease in the power along the propagation path. This is depicted in figures 2 and 3, where the average beamwidth parameter \(f_a\) increases with the increase of \(E_0\). \(f_a\) is defined as:

\[
f_a = \frac{1}{R} \int_0^R f(r, z) \, dr,
\]

where \(f(r, z)\) is the beamwidth parameter.

\[
\text{Fig. 1. — Variation of}(T - T_0)\text{with }\rho\text{ for different values of }E_0\text{ and the assumed set of parameters.}
\]

\[
\text{Fig. 2. — Variation of }\nabla n\text{ with }\rho\text{ for different values of }E_0\text{ and the assumed set of parameters.}
\]

\[
\text{Fig. 3. — Variation of }f_a\text{ with }z\text{ for different values of }E_0\text{ and the assumed set of parameters.}
\]
The variation of the dimensionless optical distortion figure of merit, DODFM, with $E_0$ for different values of $R$ and the assumed set of other parameters is displayed in figure 4, where the DODFM decreases with the increase in $E_0$.

The axial normalized temperature (on-axis), $b_o$, may be approximately obtained by solving

$$b_o + b_2 \rho_o^2(1 + \delta) \approx 1,$$

or

$$b_o - \rho_o^2(1 + \delta)[Aa_o + R^2 \sigma^2(b_o + 2 ab_o^2)]/4(1 + ab_o) = 1,$$  \hspace{1cm} (28)

taking into consideration that :

$$Aa_o \gg R^2 \sigma^2(b_o + 2 ab_o^2). \hspace{1cm} (29)$$

Equation (28) yields

$$b_o = 1 - \frac{1}{\alpha} + 2[Aa_o \rho_o^2(1 + \delta) + 4]/(\alpha - 1). \hspace{1cm} (30)$$

It is clear that $b_o$ is nearly linearly related to the dimensionless quantity $A = \sigma_o R^2 I_0/k_0 T_o$. Thus it could be concluded that $b_o$ and consequently $T$ is proportional to $E_0^2$. The radial gradient of the temperature distribution is always negative (the heat flux is always outside the hot path). This in turn yields a positive radial gradient for the refractive index which can be derived from equations (2) and (3) as :

$$\nabla_r n = \frac{2}{3} K \frac{b_m}{T} \nabla_r T.$$  \hspace{1cm} (31)

The variations of the increase of the radial temperature, $(T - T_o)$, with the normalized radial position, $\rho$, at different values of the beam radius, $R$, and the assumed set of other parameters are displayed in figure 5. This figure reveals that, at any radial position, both $T$ and $|\nabla_r T|$ increase with the increase of $R$ which is true on the basis of equation (30). As the beam radius increases, the radial gradient of the refractive index $\nabla_r T$ increases also as it is clear in figure 6. The average beamwidth parameter $f_\alpha$ decreases with the increase of $R$ at any propagation distance as shown in figure 7. From (24) one can say that $\nabla_r f$ is proportional to the inverse of $R^2$. Thus, along the propagation path, $f$ increases as $R$ decreases. In spite of the increase of $f$, the beam radius at any propagation distance ($z > 0$) decreases with the decrease of the initial radius (at $z = 0$).
The variation of the DODFM with the beam radius, \( R \), for different values of \( E_0 \) and the assumed set of other parameters is displayed in figure 8; from which it is clear that the DODFM decreases with the increase of \( R \).

The variations of the increase in the radial temperature, \( (T - T_0) \), with \( \rho \) at different angular frequencies \( \omega \) and the assumed set of other parameters are shown in figure 9.

Based on equation (7) and figure 9, it is revealed that the interaction process between the optical carrier and the dry atmosphere is weakened as \( \omega \) increases. Thus, both the temperature \( T \) and its radial gradient \( \nabla_r T \) decrease, at any radial position, with the increase in the laser frequency. Consequently, the radial gradient of the refractive index \( \nabla_r n \) decreases as \( \omega \) increases as shown in figure 10.

At any propagation distance, the average beamwidth parameter \( f_{a} \) increases with the increase of the laser frequency \( \omega \) as shown in figure 11. At any fixed set of parameters \( \{ E_0, R, T_0, P \} \), the DODFM increases with the increase in the laser frequency \( \omega \) as depicted in figure 12.

The variations of the DODFM with different spatial profiles for the TEM_{oo} mode at a fixed assumed set of parameters \( \{ R, \lambda, E_0, T_0, P \} \) are illustrated in figures 13 to 16. For the Gaussian distribution, figure 13, the DODFM increases with the decrease
Fig. 9. — Variation of \((T - T_0)\) with \(\rho\) for different values of \(\omega\) and the assumed set of parameters.

Fig. 10. — Variation of \(\nabla_r n\) with \(\rho\) for different values of \(\omega\) and the assumed set of parameters.

Fig. 11. — Variation of \(f_a\) with \(z\) for different values of \(\omega\) and the assumed set of parameters.

Fig. 12. — Variation of DODFM with \(\omega\) for different values of \(R\) and the assumed set of parameters.
in the radial gradient of $F(r, 0)$. This is also valid for the quadratic and the biquadratic distributions, respectively, shown in figures 14 and 15 and in figure 16. The effect of the TEM$_{20}$ mode is shown in figure 17. As $m$ increases the DODFM increases also, i.e., as the peak of the distribution moves towards the cold region ($r > R$), $\nabla_r F(r, 0)$ decreases and the DODFM increases. The variation of the DODFM with different spatial profiles of the TEM$_{10}$ mode is shown in figure 18; from which it is clear that as $m$ increases (i.e., the radial position of the zero value moves towards the centre of the beam), the DODFM increases also. Moreover, as $m$ increases $\nabla_r F(r, 0)$ decreases. Figures 12 to 18 reveal that there is some correlation between the DODFM and $\nabla_r F(r, 0)$.

5. Conclusions.

In the present investigation the diffraction of an optical carrier propagating in an unguided communication channel (dry atmosphere) is rigorously analysed taking into account the coupling between the optical carrier and the medium only at the entrance of the medium. The density and thermal conductivity of the medium are both considered to be temperature-dependent quantities. Both the absorption coefficient and the polarizability of the medium are considered inversely-proportional to the square of the angular
Four main relevant topics are deeply treated, namely, 1) the radial temperature distribution, $T(r, 0)$, 2) the average beamwidth parameter, $f_a$, 3) the dimensionless optical distortion figure of merit, DODFM, and 4) the irradiance tailoring technique as a method to reduce the diffraction caused by the refractive effects; since one of the fundamental limits on the performance of the optical channel is set by diffraction.

In general, $T(r, 0)$, $f_a$, and DODFM are all functions of the optical electric field $E_0$, optical beam radius $R$, and optical frequency $\omega$ at constant pressure.

The main conclusions of the present study are:

i) Both $T(r, 0)$ and $|\nabla_r T(r, 0)|$ increase with the increase of $E_0$ and/or $R$.

ii) Both $T(r, 0)$ and $|\nabla_r T(r, 0)|$ decrease as $\omega$ increases.

iii) $|\nabla_r n|$ increases with the increase of $E_0$ and/or $R$ while it decreases with the increase of $\omega$.

iv) The variation in the beam radius, or the diffraction, along the propagation path due to the refractive effects increases with the increase of $E_0$ and/or $R$ and decreases with the increase of $\omega$.

v) The DODFM can be used as a measure for the diffraction caused by the refractive index variations; as at any set of parameters, the DODFM is positively correlated with $\nabla_r n$. 
vi) The DODFM increases with the decrease of $E_0$ and/or $R$ and increases with the increase of $\omega$.

vii) The DODFM increases with the decrease of radial gradient of the spatial profile $F(\rho, 0)$ whatever the TEM mode is.

viii) From the optical communication point of view, the optical carrier must possess a) low electric field, b) small radius, c) high frequency, i.e., small power and high frequency, and d) spatial profile $F(\rho, 0)$ of small radial gradient ($V, F(\rho, 0) < 0$).

Two points shall be published elsewhere i) the critical optical power i.e., the smallest optical electric field and/or the smallest beam radius, and ii) the coupling between the optical carrier and the medium along over the propagation path.

References


