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Magnetic field profiling for low temperature $H\downarrow$ confinement

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Résumé. — L'étude et la réalisation d'un système de matériaux ferromagnétiques permettant de réaliser une compression sur l'hydrogène polarisé sont présentées. Ce système permettrait d'approcher les densités critiques de condensation de Bose-Einstein à basse température.

Abstract. — The study and the realization of a ferromagnetic system allowing to perform a magnetic compression of polarized hydrogen are presented. This system would allow to approach the critical Bose-Einstein condensation density at low temperature.

In the course of setting up an experiment on spin-polarized hydrogen ($H\downarrow$) at Grenoble, which would make use of larger refrigeration capacities and magnetic fields than those described by the other groups presently working in the field, it became apparent that if lower temperatures were reached such temperatures would present certain problems that could be tuned with an advantage for achieving the onset of Bose-Einstein condensation. Instead of a mechanical compression, we propose a profile of the magnetic field that would compress the $H\downarrow$ gas to critical densities.

It has been suggested that magnetic field inhomogeneity may be used to confine and study stabilized hydrogen [1]. For a solenoid where the axial component of the magnetic field B_{oz} may be expanded in a quadratic form of the z coordinate (z parallel to the solenoid axis), $B_{oz} = B_0 \left(1 - \left(\frac{z}{z_m} \right)^2 \right)$ the hydrogen density profile has been calculated [1, 2]. We suggest here a quite different mean to high density $H\downarrow$, utilizing a movable system of ferromagnetic materials, saturated in the magnetic field B_0 of a solenoid. This system creates in addition to B_0 a magnetic field gradient ΔB much larger than the B_0 field inhomogeneity. ΔB is of the order of a few tenths of a Tesla and the B_0 inhomogeneity typically does not exceed $10^{-3} \text{ T cm}^{-1}$. The small dimension of the volume in which the field gradient will be operating precludes the use of superconducting coils to create the field gradient. Moreover, the use of ferromagnets of different

magnetization densities gives more possibilities to obtain variable magnetic field profiles.

The dimensions of the experimental cell where the hydrogen magnetic compression is to take place are determined by the magnetic field volume available. A compression scheme in a flat cylinder of height $h = 1 \text{ mm}$ and inner diameter $2R_0 = 30 \text{ mm}$ is presented here.

The experiment principle is to create an additional magnetic field of axial symmetry showing a step intensity profile as shown on figure 1. Such a profile may be approached by two cylindrical ferromagnetic systems movable one with respect to the other. The first system S_1 is a cylindrical ring of outer radius R_1 and inner radius R_2 . The second one is a cylinder of radius R_2 , $R_2^2 \ll R_1^2$ (Fig. 2).

1. Compression scheme.

In a first stage the inner ferromagnetic system S_2 is far (2 cm) from the experimental cell, the field intensity profile is the one shown on figure 1a. Let us denote by

Fig. 1. — Schematic step profile of the additional magnetic field; a) in presence of S_1 , b) in presence of S_1 and S_2 . $2R_0$ is the experimental cell diameter.

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Fig. 2. — Ferromagnetic systems S_1 and S_2 creating the additional field in the experimental cell C.

$F\left(F \sim \frac{R_0^2}{R_1^2}\right)$ the fraction of the cell volume where the

S_1 field B_1 is acting, region I. In this region and in the outermost region the hydrogen atomic densities are xn and n respectively. As long as the hydrogen is far from being degenerate one has $x = \exp \frac{\mu_e B_1}{kT}$, μ_e is the electronic magnetic moment ($\mu_e = 9.2845 \times 10^{-24} \text{ J} \cdot \text{T}^{-1}$), k the Boltzmann constant. Owing to the presence of S_1 the hydrogen density at the outer surface of the cell decreases. If n_0 is the hydrogen cell filling density in the absence of S_1 then $\frac{n}{n_0} = \frac{F}{F+x-1}$.

This depletion of density decreases the surface relaxation rate from the hyperfine state $|b\rangle$ to $|a\rangle$ [3], as the surface relaxation is highly anisotropic and is maximum along the cell surfaces parallel to the field [4]. For a chosen geometry $2R_0 = 30 \text{ mm}$, $2R_1 = 22 \text{ mm}$, $2R_2 = 6 \text{ mm}$ and with $B_1 = 0.1 \text{ T}$ the depletion factor amounts to 0.29 and 0.08 for $T = 150 \text{ mK}$ and 100 mK respectively.

Table I. — Values of $x = \exp \frac{\mu_e B_1}{kT}$ for different values of additional field B_1 at 150 and 100 mK.

$B_1 \text{ (T)}$	0.5	0.4	0.3	0.2	0.1
$T \text{ (mK)}$					
150	9.3	6.0	3.8	2.4	1.6
100	28.9	14.7	7.5	3.8	2.0

In a second step the central ferromagnetic system S_2 is moved inside S_1 to the cell surface. An additional field B_2 is thus created in the central region II. As the temperature decreases in the experimental cell and as the hydrogen density increases in region II, the hydrogen gas becomes closer to degeneracy. Let us denote by N_1 and N_2 respectively the number of hydrogen atoms in regions I and II. By considering the polarized hydrogen as an ideal Bose gas it comes :

$$N_1 + N_2 = \frac{zx_0 x_1}{1 - zx_0 x_1} + \frac{V_1}{\lambda_T^3} g_{3/2}(zx_0 x_1) + \frac{zx_0 x_2}{1 - zx_0 x_2} + \frac{V_2}{\lambda_T^3} g_{3/2}(zx_0 x_2) \quad (1)$$

$z = \exp \beta\mu$ is the fugacity ; $\beta = 1/kT$; λ_T is the thermal de Broglie wave length ; $x_i = \exp \mu_e B_i$, $i = 0, 1, 2$; $g_{3/2}$ is the usually defined integral [5]. A numerical solution of equation (1) allows the calculation of the initial hydrogen density needed in the cell in the absence of S_1 and S_2 in order to reach the Bose-Einstein condensation after compression with the S_1 and S_2 systems (Table II). For sake of comparison, the density required in the absence of magnetic compression ($B_2 = B_1$) is also quoted.

2. Ferromagnetic systems S_1 and S_2 .

Alloys of nickel, iron and copper of different composition are used to make the ferromagnetic systems S_1 and S_2 . We require these systems to generate B_1 and B_2 magnetic fields with the best step profile (Fig. 1). The problem consists in determining the superficial magnetic distribution $\sigma(\mathbf{r})$ which creates a given magnetic field $B(\mathbf{p})$; $\mathbf{r}$ and $\mathbf{p}$ are the source and probe vectors respectively. The problem is somewhat simplified here as we do not have to consider demagnetization field reacting on the ferromagnets. Furthermore, one takes advantage of the axial symmetry to restrict the calculation to a one dimension problem by integrating over the angular variable (Fig. 3). Thus,

Table II. — Values of the initial polarized hydrogen atomic density (cm^{-3}) required to obtain the Bose-Einstein condensation in presence of the additional field ($B_2 - B_1$) at different temperatures.

$T \text{ (mK)}$	60	80	100
$B_2 - B_1 \text{ (T)}$			
0.5	2.5×10^{17}	2.8×10^{17}	4.8×10^{17}
0.3	2.7×10^{17}	3.7×10^{17}	8.4×10^{17}
0.0	7.2×10^{18}	1.1×10^{19}	1.5×10^{19}

Fig. 3.

$\sigma(r)$ is given by a Fredholm's first type equation :

$$\frac{\mu_0 h}{\pi} \int_0^\infty \sigma(r) K(r, \rho) r dr = B(\rho), \quad (2)$$

$$K(r, \rho) = \frac{E\left(\frac{\pi}{2}/\alpha\right)}{((r + \rho)^2 + h^2)^{1/2} ((r - \rho)^2 + h^2)}$$

where $\sin^2 \alpha = \frac{4 \rho r}{(r + \rho)^2 + h^2}$; $E\left(\frac{\pi}{2}/\alpha\right)$ is the second complete elliptic integral [6].

Discretization of r and ρ in equation (2) leads to a linear equation. Although regularization methods [7] are used for solving equation (2), only oscillatory in r , physically non-acceptable solutions are obtained.

A more practical approach has been used to simulate a step magnetic field profile. The S_1 and S_2 systems are considered as made of nested rings of different thickness e and of different magnetization density J . The field profile given by such a system is calculated and optimized in order to simulate a step profile. One solution is with outer system consisting of three pieces ($J_1 = 10.11 \times 10^5 \text{ Am}^{-1}$, $e_1 = 2 \text{ mm}$; $J_2 = 4.80 \times 10^5 \text{ Am}^{-1}$, $e_2 = 4 \text{ mm}$; $J_3 = 8.12 \times 10^5 \text{ Am}^{-1}$, $e_3 = 2 \text{ mm}$), and the S_2 system of two pieces ($J_4 = 10.27 \times 10^5 \text{ Am}^{-1}$, $e_4 = 1 \text{ mm}$; $J_5 = 6.13 \times 10^5 \text{ Am}^{-1}$, $e_5 = 2 \text{ mm}$). Such a system has been constructed and the experimental field profile measured with a Hall effect gaussmeter is shown on figure 4. The S_1 and S_2 systems were tested in a $B_0 = 1.7 \text{ T}$ magnetic field. The active area of the gaussmeter (0.78 mm^2) was moved in a plane perpendicular to the S_1 and S_2 symmetry axes, at a distance of 2.5 mm to their surface

Fig. 4. — Experimentally measured magnetic field profile (x) obtained with the S_1 and S_2 ferromagnetic systems. The calculated field is shown for sake of comparison.

(Fig. 2). The magnetic field gradient along the z axis, $\partial B_z / \partial z$ is less than $3 \times 10^{-3} \text{ T cm}^{-1}$. The small discrepancy between the calculated and experimental magnetic field profiles (Fig. 4) may be accounted for by the uncertainty in the location of the Hall probe with respect to the magnet surface. The 10 % difference between the calculated and experimental profiles corresponds approximately to a 0.3 mm uncertainty in the distance between the Hall probe and the magnet surface.

3. Density profile.

The knowledge of the experimental magnetic field profile in the cell allows one to calculate the H_↓ density as a function of the radial coordinate. For a cell containing N atoms, for densities less than the critical density :

$$n(\rho) = \frac{1}{\lambda_T^3} g_{3/2}(zx(\rho)) \quad (3)$$

$$N = 2 \pi \int_0^{R_0} \rho n(\rho) d\rho \quad (4)$$

where $x(\rho) = \exp \beta \mu B(\rho)$. Equations (3) and (4) may be solved numerically for different temperatures and initial density in the experimental cell, numerical values are given on table III.

Table III. — Calculated values obtained from the experimental field shown on figure 4. n_0 initial density ; n_{max} maximum density ; n_c critical density for Bose-Einstein condensation.

$T \text{ (mK)}$	$n_0 \text{ (} 10^{17} \text{ cm}^{-3} \text{)}$	$n_{\text{max}} \text{ (} 10^{17} \text{ cm}^{-3} \text{)}$	$n_c \text{ (} 10^{17} \text{ cm}^{-3} \text{)}$
70	3.4	30	88
60	3.0	36	71
50	2.6	53	56

The figures shown on table III clearly indicate that at low temperature the local density of polarized hydrogen can be significantly increased by means of the described magnetic compression. The n_0 values quoted in table III correspond to realistic initial densities. The « magnetic bottle technique » [8] has yielded densities up to $3 \times 10^{17} \text{ cm}^{-3}$ at 200 mK and may be capable of achieving somewhat larger values (9). With the ferromagnetic system described here the critical densities would be obtained around 50 mK.

The reasonable agreement between the calculated and experimentally measured magnetic field (Fig. 4) allows us to consider the setting up of similar systems of different chemical composition and which would

produce better magnetic field profiles for increasing $H_{\downarrow}$ density at low temperature.

We present below the results of the calculation performed on two systems made of materials of higher magnetization density as the ones previously used. Magnetization density values up to 30 Am^{-1} can be obtained using Holmium [10]. The high magnetic anisotropy of Holmium (11) is considerably reduced at low temperature and for applied magnetic fields higher than 11 T [12]. Such a situation which corresponds to our working conditions allows to consider the use of Holmium powder instead of single crystal. The geometry and the magnetization values ($J \text{ Am}^{-1} \times 10^{-5}$) are shown in the following diagram (solution A, solution B).

J_7	J_6	J_5	J_4	J_3	J_2	J_1	
0.81	26.46	11.34	7.98	5.88	4.0	11.3	(B)
17.98	1.19	30.80	7.98	5.88	4.0	11.3	(A)
1	2	3	5	7	9	11	$r \text{ (mm)}$

Table IV. — Calculated values obtained from the calculated field shown on figure 5. n_0 initial density; n_{max} maximum density; n_c critical density for Bose-Einstein condensation.

$T \text{ (mK)}$	$n_0 (10^{17} \text{ cm}^{-3})$		$n_{\text{max}} (10^{19} \text{ cm}^{-3})$	$n_c (10^{19} \text{ cm}^{-3})$
	(A)	(B)	(A, B)	
100	5.5	3.0	1.4	1.6
	5.8	3.2	1.5	
80	3.3	1.7	0.9	1.1
	3.6	1.8	1.0	
75	2.9	1.5	0.8	1.0
	3.2	1.6	0.9	

Solution A corresponds to a flat profile at the cell centre, B to a steeper profile (Fig. 5) which would create a $H_{\downarrow}$ concentration over a smaller volume. The $H_{\downarrow}$ density values which could be reached with such a profile are shown on table IV.

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Fig. 5. — Calculated magnetic field profiles (A) and (B). →

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