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The building as a thermodynamic system. Physical model and experimental test

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Résumé. — Dans cet article nous présentons le modèle électrique d'un édifice considéré comme un système thermodynamique complexe. Le modèle est caractérisé par une méthode pour résoudre l'équation de la chaleur de Fourier, permettant une rapide et précise détermination des températures existantes dans des structures massives. Les prévisions obtenues par notre modèle sont comparées avec les résultats expérimentaux obtenus pour un édifice situé à Naples.

L'accord entre résultats théoriques et expérimentaux peut être considéré satisfaisant et il montre l'efficacité du modèle, dont l'extrême rapidité d'exécution se révèle très utile dans plusieurs champs d'application.

Abstract. — We present and discuss a model to simulate, by using an electric network, a complex thermodynamic system, as a building. The model is characterized by a new technique to solve the Fourier equation; this technique is able to obtain the temperature of a massive structure with short computer time. The results of the simulation has been compared with the measured behaviour of a building located in Naples.

Introduction

Let us consider a thermodynamic system which exchanges heat with more than one source. The calculation of the system performances in non-steady conditions, is usually quite difficult. Nevertheless, very often, and in various fields of application, the solution of this problem comes out to be very useful.

In this paper we analyse the thermal behaviour of a building, during winter season; a system which exchanges heat with an internal and an external ambience and, in addition, receives energy from some internal sources.

In different situations, we may be interested in long term (typically a month) or short-term (typically an hour) time resolution. In the first case a suitable approximation can be obtained by using a stationary method of solution; in the second case a dynamical method of solution is unavoidable.

The stationary technique of solution has been widely discussed in other papers (for instance see Ref. 1). Here we want to present a dynamical method of evaluation and its experimental test.

The model we use divides the whole system into

blocks, thermally connected among themselves. A block is defined as a part of the system at a uniform temperature. In this way, the system performances can be calculated by using its analogy with an electrical network.

The model has been used to forecast the thermal behaviour of a real building. The comparison between the results of the calculation and the experimental measurements performed on the building, shows that the model is able to predict the behaviour of the system within an approximation adequate to applications. The slight discrepancies come out to be better related to the unforeseen habits of the users than to inaccuracy of the method.

This paper is organized as follows: in section 1 we define the thermodynamic system under study and discuss the model we use to simulate its behaviour; in section 2 we present the numerical technique we use to solve the Fourier equation of heat flow in solids; in section 3 we discuss the validation method using the experimental results obtained in an apartments building under test for a heating season.

In the conclusions we summarize the results of the model and present some comments on its use.

1. The model.

At a first level of approximation, the system under study can be divided into the following sub-systems (see Fig. 1) :

- i) the external ambiencies ;
- ii) the internal ambiencies ;
- iii) the building frontier ;
- iv) the users (UA) ;
- v) the heating plant (HP) ;
- vi) the automatic control system of the heating plant (AC).

In figure 1 the arrows indicate the direction of the heat flow among the sub-systems. The external ambiency acts as a heat sink whose temperature T_{ext} is modulated. T_{ext} represents one of the time dependent boundary conditions of the building frontier.

Thermal exchanges through the building frontier take place mainly through the following mechanism :

- i) conduction through windows and walls ;
- ii) solar radiation through windows and any other transparent components ;
- iii) air changes and infiltrations.

The heating plant is driven by an automatic control system which controls the heat power delivered to the building (ranging from 0 to a maximum value) according to the needs determined by the values of the external and internal temperatures.

The possible actions of the users can be summarized as follows :

- i) opening the windows ;
- ii) operating internal sources (gas, light, etc.) ;
- iii) metabolic heat production.

In principle, it would appear that the precision of the calculation improves indefinitely by increasing the number of blocks. However, even disregarding the problem of computer time, the precision with which we know the boundary parameters puts limits to the maximum useful number of blocks. To clarify this point we give, in the following, more detailed criteria to choose a convenient number of blocks.

Fig. 1. — Block scheme of the thermodynamic system.

To define a block we must consider a part of a system which appears, as seen by other elements of the system, as a whole. This means that the temperature of each block must be uniform : in other words, the differences of temperature inside the block must be lower than the expected errors. We can also say that the transfer time T_b of a thermal information throughout a block must be much less than the characteristic time T_s of the system. On the other hand, it is useless to choose the blocks so small that T_b becomes smaller than the time resolution with which the boundary temperatures are known.

Usually, the time constant of the whole system is $\cong 40\text{--}60$ hours, while the external data (solar radiation and air temperature) are known on a time base ΔT of the order of one hour. Therefore ΔT can be considered as the transfer time to define the block size.

On this basis we can subdivide our system into blocks as listed in table I.

Each block is simulated with an electrical equivalent circuit, as listed in the last column of table I. Although the skins are listed, for simplicity, among the blocks, the simulation is, in this case, a particular one. In fact the external masonry walls cannot be simulated by a single resistor-capacitor since, usually, the transfer time is, for them, of many hours. The technique used for these components is particularly important and therefore it will be discussed in the next section.

Once the electrical network describing the system is drawn, it is possible to compute analytically the response of the system once the boundary conditions are known.

Table I. — List of the blocks and the electrical equivalents used in the model.

sun	power generator
external air	power generator
burner	power generator
internal sources	power generator
windows	resistor
internal air	resistor and capacitor
internal structures	resistor and capacitor
heating plant	resistor and capacitor
external walls	set of resistors and capacitors

2. The dynamical thermal behaviour of a wall.

As said in the previous section, the electrical resistor-capacitor circuit is able to simulate only part of a wall. Therefore, to represent a wall, we must use the circuit depicted in figure 2.

This scheme corresponds to the subdivision of the wall into $N - 1$ zones separated by N nodes. If $T(j, \Delta t)$ is the temperature of the j -th node at time Δt , we can determine it once we know the temperatures at time 0, for each node.

Fig. 2. — The electrical equivalent circuit for an external wall.

From a mathematical point of view, the problem to be solved is the Fourier equation, i.e. a differential equation of II order, parabolic type. It can be solved numerically by many methods. The simplest one is the Milne method while a more sophisticated one is the Crank-Nicolson method.

Both methods can be solved by N equations

$$T(j, \Delta t) = \alpha_j T(j-1, \Delta t/2) + \beta_j T(j, \Delta t/2) + \gamma_j T(j+1, \Delta t/2)$$

but in the Milne method it is

$$T(j, \Delta t/2) = T(j, 0) \quad j = 1, \dots, N$$

while in the Crank-Nicolson method we put

$$T(j, \Delta t/2) = [T(j, 0) + T(j, \Delta t)]/2 \quad j = 1, \dots, N.$$

As it is well known, the Milne method is very easy to solve but gives high errors; on the contrary, the errors we get using the Crank-Nicolson method are smaller but the solution of the system of N equations requires a longer calculation time.

In our case it is impossible to use the Milne method. In fact, as is well known, in the Milne method α_j , β_j , and γ_j must satisfy the following conditions

$$0 \leq \alpha_j \leq 1; \quad 0 \leq \beta_j \leq 1; \quad 0 \leq \gamma_j \leq 1;$$

$$\alpha_j + \beta_j + \gamma_j = 1.$$

These conditions imply, for typical masonry walls,

$$\Delta t < 60 \times 10^4 \Delta x^2.$$

This is a value of $\Delta t \ll \Delta T$ (see section 1).

We can use a linear hypothesis on the external temperatures to simplify the Milne method. To show such a procedure we present the method in a vectorial formalism.

If $(T(j, t))$ is the temperature of the j -th node at time t , we can define the vectors

$$X(t) = \begin{pmatrix} T(1, t) \\ T(2, t) \\ \vdots \\ T(N, t) \end{pmatrix} \quad (N \text{ elements})$$

$$Y(t) = \begin{pmatrix} T_{\text{ext}}(t) \\ 0 \\ \dots \\ 0 \\ T_{\text{int}}(t) \end{pmatrix} \quad (N + 2 \text{ elements}).$$

The set of N equations, representing the solution of the Milne method, can be written as

$$X(\tau) = AX(0) + BY(0)$$

where τ is the chosen time interval and A and B are the transfer matrices, depending only on the physical-geometrical properties of the wall.

At time $t = i$, we have

$$X(i\tau) = AX[(i-1)\tau] + BY[(i-1)\tau]. \quad (1)$$

If the climatic data are known at $t = 0$ and $t = \Delta T = M\tau$ we can write the linear hypothesis as

$$Y(i\tau) = Y(0)(1 - i/M) + Y(\Delta T)i/M$$

and, therefore, by iterating equation 1 M times, we obtain

$$X(M\tau) = X(0)A^M + Y(0) \sum_{i=1}^M A^{M-i} \left(1 - \frac{i}{M}\right) + Y(\Delta T) \sum_{i=1}^M A^{M-i} \frac{i}{M}.$$

If we use the vector

$$Z(t) = \begin{pmatrix} T_{\text{ext}}(t) \\ T(1, t) \\ \vdots \\ T(N, t) \\ T_{\text{int}}(t) \\ T_{\text{ext}}(t + \Delta T) \\ T_{\text{int}}(t + \Delta T) \end{pmatrix} \quad (N + 4 \text{ elements})$$

we, finally, obtain

$$X(M\tau) = SZ(M\tau)$$

where S is an appropriate matrix $(N + 4)(N + 2)$ elements depending only on the physical-geometrical properties of the wall and on the integer M .

This equation can be easily solved by a computer.

To prove the accuracy of the Milne modified method we have used the last equation to solve the Fourier equation for a standard wall, with a sinusoidal external temperature.

In figure 3, the maximum percentage of error on the temperature provided by this method is shown as a function of the number of nodes N . When N increases the error rapidly decreases.

For comparison we have analysed the same wall by using the Crank-Nicolson method. In table II, the maximum values of the percentage of errors on the temperatures and on the thermal fluxes, and the computer times need to solve 1 year (17520 temporal steps) are shown for the Milne modified method (columns 1, 2 and 3 respectively) and for the Crank-Nicolson one (columns 4, 5 and 6 respectively).

Table II. — *The maximum percentage of errors on the temperatures, on the thermal fluxes and the computer times need to solve 1 year for the Milne modified method (columns 1, 2 and 3) and for the Crank-Nicolson method (columns 4, 5 and 6).*

Nodes number	ε_1 (%)	ε_2 (%)	Time (ms)	ε_3 (%)	ε_4 (%)	Time (ms)
2	35.38	313.83	25.3	29.48	311.90	34.0
3	34.71	207.21	26.7	29.08	302.21	47.5
4	15.21	150.43	28.3	9.89	112.49	74.8
6	5.64	53.02	32.5	4.42	42.74	207.2
11	1.41	13.15	49.5	1.42	11.71	1 884.3
16	0.62	5.74	74.8	0.86	6.37	9 455.2
21	0.37	3.44	109.8	0.71	4.98	32 576.0

Fig. 3. — *The maximum percentage of error on the temperature provided by the Milne modified method as a function of the node number N .*

It must be remarked that the precisions are comparable while the computer times are much higher for the Crank-Nicolson method than for the Milne modified one [2].

3. Comparison among experimental and simulated results.

We have used the simulation program, based on the model shown in section 1, to simulate the thermal behaviour of a building which has been monitored during a heating season. In this section we present the main features of the building, the experimental

results, and the comparison among these and the calculated behaviour.

The building under study is located in Naples; it is seven stories high and consists of fourteen apartments, grouped into two parts linked by a wall; one of the two parts is supported by columns while the other one is supported by unheated basements. The building roofs are terraces.

For its characteristics this building can be considered as a typical Italian building.

The model has been run using measured physical-geometrical properties of the building and measured records of

- i) external air temperature
- ii) solar radiation on horizontal surface.

The experimental test has been made on the following quantities :

- i) internal air temperature
- ii) energy burned by the heating plant.

The efficiency of the burner was measured.

The internal air temperature was measured by using thermocouples displaced in many rooms; as the difference among the various temperatures was very small, apart from some special rooms (for the first or the last floor), we have considered their average value as the internal air temperature of the whole building.

The heating plant (a boiler heated by a gas burner) is controlled by an automatic control system which sets the outlet water temperature according to the external air temperature; the setting of the automatic control system is

$$\text{for } T_{\text{ext}} \leq 11^\circ\text{C} \quad T_{\text{mr}} = 84.5 - 3.3 T_{\text{ext}}$$

$$\text{for } T_{\text{ext}} > 11^\circ\text{C} \quad T_{\text{mr}} = 70^\circ\text{C}$$

where T_{mr} is the average value between the outlet and inlet water temperatures.

By using the model described in the previous sections, we have simulated the building behaviour.

In figure 4 the simulation results (dashed line) and the experimental measurements (full line) are shown for a period of one week. The comparison

Fig. 4. — Comparison between computed (dashed line) and measured (full line) indoor temperatures. Full solar gain and thermal capacity of the internal structures.

shows that, during the first 3 days (Feb. 12th-14th) there is no agreement. This is due to the arbitrary initial conditions imposed to the simulation. Later (Feb. 15th-17th and following days not reported) the simulated results agree with experiment within $5 \div 10 \%$.

The errors come from two main source. One should remark that on February 16th and 17th, the model overestimates the temperatures, whereas on February 15th the measured data are well simulated. Now we must emphasize that the 15th was a cloudy day whereas the 16th and 17th were clear days. It is possible, therefore, to conclude that the simulation program overestimates the amount of solar radiation entering the building. This can be explained by considering that in the simulation no scree at windows was considered, whereas it is well known that in reality curtains and blinds are partially closed by users.

In figure 5 we report the measured results (full line) and the simulated ones (dashed line) when we introduce a reduction of $1/3$ of the solar radiation entering the building. This reduction appears compatible with the user habits.

A second correction concerns the thermal capacity of the internal structures. Let us observe the temperature decrease between 10 o'clock p.m. (set off of the heating plant) and 6 o'clock a.m. (sunrise), every day; we can see that the simulated temperature decrease is always slower than the measured one. This means that the thermal capacity considered in the program is too high.

Fig. 5. — Comparison between computed (dashed line) and measured (full line) indoor temperatures. Reduced solar gain and full thermal capacity of the internal structures.

By best fitting the building capacity to the measured slope, we obtain that the optimal value of the equivalent capacity of the internal structures of the building is

$$C_{eq} = 0.80 C_{meas}.$$

By introducing such a correction, the simulated temperatures (dashed line) fit the measured ones (full line) as shown in figure 6.

It must be remarked that the correction on the thermal capacity of the internal structures, which we have obtained in this way, is numerically the same as reported in [3], on the basis of semi-theoretical considerations.

Figures 4, 5 and 6 show that our model gives results with appreciably small errors, especially if the above corrections on the input data are applied.

The great advantage of our model is the speed of calculation. For our computer, the UNIVAC 1100/80 of the Centro di Calcolo Elettronico Interfacoltà della Università di Napoli, the analysis of the building requires a CPU time of 20 seconds to describe the behaviour during 30 days.

4. Conclusions.

We have presented and discussed a model able to simulate the thermal behaviour of a building. The main features of this model can be summarized as follows :

- i) the thermodynamic system is divided in isothermal blocks;
- ii) each block is simulated by an electrical circuit chosen in accordance with the desired approximation;
- iii) the results are accurate and the computer time short.

The model has been tested by using data measured on a building located in Naples. The comparison shows that the model is able to describe the thermal behaviour of the thermal with good accuracy.

The model appears to be a powerful instrument for energy management of building. As a consequence of the short computer time, it is possible to compare easily the effects produced by different energy management operations.

Fig. 6. — Comparison between computed (dashed line) and measured (full line) indoor temperatures. Reduced solar gain and thermal capacity of the internal structures.

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