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The entry flow of a dilute suspension through a tube : Concentration redistribution effects

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Résumé. — Le profil de concentration d'une suspension diluée de sphères dures ayant la même densité que le fluide suspendant, est étudié théoriquement dans l'écoulement d'entrée d'un tube cylindrique. Les effets d'exclusion géométrique (interaction entre l'embouchure du tube et les particules) sont discutés. Le mouvement convectif des particules dans un écoulement de couche limite à travers le tube ($Re \gg 1$) est analysé. L'effet supplémentaire d'une vitesse de migration radiale induite par des effets d'inertie dans l'écoulement autour d'une particule est ensuite calculé. On montre que les effets d'exclusion géométrique, couplés à la convection des particules le long de la zone de développement de la couche limite, expliquent de façon satisfaisante la formation d'une importante couche marginale pauvre en particules. La migration par effets d'inertie amplifie ce phénomène et engendre des zones à haute concentration.

Abstract. — The concentration distribution of a dilute suspension of neutrally buoyant rigid spheres is investigated theoretically, in the entry flow of a circular tube. The geometrical effects (interaction between the tube mouth and the particles) are discussed. The convective motion of the particles in a boundary layer flow through the tube ($Re \gg 1$) is analysed. The effect of an additional inertia-induced migration velocity is then computed, in two different cases. It is found that the geometrical effects together with the particle convection along the tube inlet length can account satisfactorily for the onset of a large peripheral particle-free layer. The inertia-induced migration accentuates this phenomenon, and creates regions of high concentration.

1. Introduction. — The discovery of the special behaviour of the concentration distribution in the flow of a suspension through a tube goes back to Poiseuille in 1835 [1] who noticed, in blood flow, a particle-free layer near the walls of small vessels. Otherwise, it has been noticed by many investigators that the bulk rheological properties of a suspension flowing into a tube depends on the diameter of this tube, the apparent viscosity increasing when this diameter increases [2]. A comprehensive review about the so-called Fahraeus-Lindqvist effect can be found in [3]. This dependence of the rheological properties on the tube diameter, has been soon related with the presence near the wall of a layer which has a lower viscosity than the viscosity of the suspension [4]. The thickness of this layer may be determined indirectly by using a two-phase model (dividing the flow into a suspension core and a particle-free peripheral layer).

This has been carried out for rigid spheres [4] and more recently for a Non-Newtonian Suspension such as normal blood [5]. The thickness of the peripheral layer has also been measured directly [6] by observation of blood flow in glass capillaries under a microscope.

Many theoretical and experimental investigations have attempted to give a good explanation and description of such a phenomenon of migration of suspended particles towards the core of the flowing suspension and of the subsequent formation of a particle-free peripheral layer. This phenomenon has been shown to depend strongly on the shape of the particles, on their deformability, and on whether or not they are neutrally buoyant. We shall limit ourselves in the present paper, to the case of neutrally buoyant rigid spheres with negligible Brownian motion. From the experimental works [7, 8, 9] which dealt with dilute or semi-dilute suspensions of such spheres, it was concluded that the particles migrate radially and that they attain an equilibrium position which is

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approximately 60 % of the way from the axis to the wall of the tube (Segre-Silberberg's Tubular pinch effect) [7, 9] or near the axis [8]. Those results were obtained either by studying the migration of a single particle in a Poiseuille velocity field [9], or by measuring the concentration of a dilute suspension across tubes of different cross-sectional shapes [7, 8], at various distances from the entry. On the other hand, several theoretical attempts to understand the radial migration of the particles have been made. It has first been shown that the creeping flow approximation for low Reynolds Number applied to the particle motion cannot account for any radial migration (see [10, 11]), because of the linearity of the Stokes equations. Most investigators thus studied the first-order inertial non-linearities in the low Reynolds number flow around a particle.

Rubinow and Keller [12] considered the case of a spinning sphere moving through an unbounded viscous fluid at the velocity V . Its angular velocity being Ω , they found that the particle is submitted to a lift force : $F_L = \pi \rho a^3 \Omega \times V [1 + O(\text{Rep})]$ a being the radius of the sphere, ρ the fluid density, and Rep the particle Reynolds number. This was called the *Slip-Spin* migration mechanism.

Then, Saffman [13] studied a sphere translating and spinning through an unbounded simple shear flow, the magnitude of its velocity gradient being $\dot{\gamma}$. He gave a result showing that, in this case, the lift force is independent of the rate of rotation of the particle. Its magnitude is :

$$F_L = 81.2 V a^2 (\dot{\gamma}/\nu)^{1/2},$$

ν being the fluid kinematic viscosity. The direction of this force (called *Slip-Shear*) is such that a sphere lagging behind the undisturbed flow migrates towards the larger velocity, while it migrates in the opposite direction if the sphere leads the flow.

The complete problem of a sphere in a Couette flow or a Poiseuille flow, including the effect of the presence of walls and of the non-uniformity of the shear was solved quite recently [14, 15, 16]. The comparison between the theoretical results and experimental particle trajectories, when a single particle is placed either in a Couette flow or in a two-dimensional Poiseuille flow, shows a very good agreement [15]. In particular, the *tubular pinch effect* is obtained as a result of the computations.

In those theoretical works, and in some experimental investigations, the entrance effects which take place close to the mouth of the tube are completely neglected. Such an assumption is generally justified by an evaluation of the inlet length of the flow, which can be expressed approximately as $L = 0.1 \text{ Re } R$ (R being the radius of the tube), when $\text{Re} \gg 1$ [17], and tends to R when $\text{Re} \leq 10$ [18]. But the fact that the inlet length is generally short when compared to the tube length does not mean necessarily that the initial state imposed to the suspension near the entrance of the

tube is not important. Maude and Whitmore [19] emphasized this idea, and then [20], Maude and Yearn found experimentally some effects (such as the *wall peak* of concentration) which can only be explained by an interaction between the tube entrance and the particles.

The present work is thus concerned by a theoretical analysis of the concentration redistribution of a dilute suspension, mainly in the region of the tube (near its entrance) where a boundary layer develops at high Reynolds number. In the next part, we discuss some geometrical features of the mouth of the tube, and their effect on the concentration distribution. In the third section of the paper we study the pure convective motion of the suspended particles in a boundary layer velocity field along the inlet length of the tube.

The fourth section considers the problem of an additional motion of the particles across the streamlines, due to an inertia-induced radial lift force. Finally the results are discussed and some ideas towards an extension of this work to high concentrations and low values of the Reynolds number are proposed.

2. Geometrical effects. — Some geometrical interactions between the mouth of the tube and the spherical particles may arise from the fact that they have a finite radius a .

Indeed, their centre cannot lie at a distance less than a from any part of the bounding walls.

Let us now consider that the tube entrance has merely the shape of a corner (Fig. 1). The initial concentration before the entrance being considered as uniform, and the particle centres being assumed to have the same velocity as the undisturbed flow, two cases may be distinguished. In the first case (1), the particle centre lies initially at a distance to the wall which is higher than a , but following a streamline the particle reaches a point where this distance becomes equal to a . From this point the sphere, having a

Fig. 1. — Schematic diagram of the paths of two particles following streamlines of the suspending fluid round a corner. The case (1) corresponds to a streamline whose minimal distance to the wall is smaller than the particle radius, while in the case (2) this minimal distance is higher than the particle radius.

velocity component parallel to the wall, crosses some streamlines and attains the streamline whose minimal distance to the wall is equal to a . Then, the spherical particle can get over the corner. In the case (2), the particle never contacts the wall : it merely follows a streamline. These phenomena implicate a very typical concentration curve just after the corner. Following the previous assumptions, the streamlines whose minimal distance d to the wall is smaller than a are particle-free, while the streamline whose distance d is equal to a , carries more particles than the others. The figure 2 shows such a concentration curve. The

Fig. 2. — Concentration curve just past a corner. In this case, the ratio of the particle diameter and the tube radius R is supposed to be approximately 0.1. Φ is the particle centre concentration, Φ_0 is the initial particle centre concentration (before the tube entrance), and r is the dimensionless radial coordinate within the tube :

previous geometrical arguments lead to think that the particle centre-free layer, just past the tube mouth, has a thickness which is of the same order of magnitude as the particle radius. Beside the peripheral layer, the wall peak of concentration has also a width which is of the same order of magnitude as the radius (or the diameter) of the particle. Those features have been observed experimentally by Maude and Yearn [20], who have paid a special attention to the conditions which are necessary to the formation or the destruction of the wall peak. For instance, these authors have shown that a turbulent flow prevents the onset of any wall peak. Moreover, at relatively high concentration this wall peak disappears at a given distance from the tube entrance, while it increases when the concentration is low (a critical volume concentration for these effects is approximately 0.2 %).

Though our reasoning about the geometrical entrance effects has been made for a corner-shaped entrance, the same consequences must be expected (at least qualitatively) in other cases. For instance, the results of Maude and Yearn were obtained past

an entrance mouthpiece to the tube which is a cone of angle 45° . A small wall-peak was also observed [8] past a parabolic shaped inlet. Anyway, it would be very interesting to perform a quantitative theoretical study of this effect to know exactly the influence of the entrance shape on the intensity and width of the wall peak. The initial concentration curve of figure 2 being assumed just past the tube entrance, we may compute its evolution along the inlet length of a steady flow through a circular tube.

3. Convective effects. — In order to compute the concentration redistribution along the tube inlet length, we must make an assumption about the particle motion relative to the suspending fluid. In the present section, we assume that the particles are purely convected by the fluid, i.e. that they follow streamlines, and have the fluid velocity. Moreover, we choose an initial concentration and a particle to tube diameter ratio which are low enough to assume that the flow field is only slightly disturbed by the presence of the particles. We can thus distinguish two parts in the problem : first to compute the velocity field of the suspending fluid into the entrance region of the tube and then to find the resulting concentration profiles.

3.1 FLOW FIELD ALONG THE TUBE INLET LENGTH. — For the sake of simplicity, we use the Schiller's model quoted by Goldstein [21], in which a parabolic velocity profile is assumed within the boundary layer. It is of course necessary in this case to suppose that the tube Reynolds number is large enough to make the fundamental boundary layer approximation (i.e. to neglect $\partial^2 \hat{u}/\partial \hat{x}^2$ when compared to $\partial^2 \hat{u}/\partial \hat{r}^2$ and to $(1/\hat{r}) \partial \hat{u}/\partial \hat{r}$, and to assume that $\partial \hat{p}/\partial \hat{r} = 0$, $\hat{u}$ being the longitudinal component of the velocity, $\hat{x}$ the longitudinal coordinate, $\hat{r}$ the radial coordinate, and $\hat{p}$ the pressure).

Let us define now some dimensionless variables :

$$x = \hat{x}/R, \quad r = \hat{r}/R, \quad y = 1 - r, \quad \delta = \hat{\delta}/R$$

$$u = \hat{u}/u_m, \quad u_1 = \hat{u}_1/u_m, \quad k = u_1 - 1, \quad \text{Re} = \frac{2 R u_m}{\nu}$$

R is the tube radius, $\hat{\delta}$ is the boundary layer thickness, u_m is the fluid velocity at the entrance of the tube (a blunt profile being assumed), and $\hat{u}_1$ is the velocity in the core of the flow ($\hat{u}_1$ does not depend on $\hat{r}$, but only on $\hat{x}$).

ν is chosen (as an order of magnitude) to be merely the suspending fluid kinematic viscosity. Indeed, an effective medium viscosity would be very close to it, according to the low concentration assumption.

The velocity profile in the boundary layer can be written :

$$\frac{u}{u_1} = \frac{2y}{\delta} - \frac{y^2}{\delta^2}. \quad (1)$$

The equation of continuity and the momentum equation give a relation between x and k :

$$x/\text{Re} = f(k)$$

$$f(k) = \frac{1}{8} \left\{ \frac{58}{15} k - \frac{22}{5} \text{Log}(1+k) - \frac{17}{15} \sqrt{4+2k-2k^2} - \frac{16}{5} \left(\frac{4-2k}{1+k} \right)^{1/2} - \frac{37\sqrt{2}}{10} \sin^{-1} \left(\frac{2k-1}{3} \right) + \frac{26}{3} - \frac{37\sqrt{2}}{10} \sin^{-1} \left(\frac{1}{3} \right) \right\}. \quad (2)$$

The function $f(k)$ has approximately a parabolic shape, tends to 0 when $k = 0$, and is such that $x = 0.0575 \text{ Re}$ when $k = 1$ (i.e. when the flow reaches its fully developed state). This value of the inlet length ($L = 0.0575 \text{ Re } R$) is slightly underestimated when compared with more recent data [17]. But these data also indicate that, at a distance $0.0575 \text{ Re } R$ from the tube entrance, the centre line velocity is 90 % of fully developed flow : the Schiller's model can thus be considered as satisfactory enough for our purpose. We must now calculate the radial component of the velocity. From the continuity equation, we obtain :

$$v = -\frac{1}{r} \int_0^r r \frac{\partial u}{\partial x} dr \quad (3)$$

$u(r, x)$ is given by (1) in the boundary layer and is equal to u_1 in the core. Thus we can write :

$$v = -\frac{1}{2r} \left\{ \frac{\partial}{\partial x} [(1-\delta)^2 u_1 + 2u_1 I] \right\} \quad (4)$$

with

$$I = \int_{1-\delta}^r \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2} \right) r dr = \frac{y^3}{3\delta^2} + \frac{2y^3}{3\delta} - \frac{y^2}{\delta} - \frac{y^4}{4\delta^2} + \frac{2}{3}\delta - \frac{5}{12}\delta^2.$$

The derivative $\partial/\partial x$ may be computed numerically if we know $\delta(x)$. This function can be obtained from an integral form of the mass conservation equation :

$$\int_{1-\delta}^1 u 2\pi r dr + \pi(1-\delta)^2 u_1 = \pi \quad (5)$$

the integration of (5) gives a relation between δ and k :

$$\frac{1}{6}\delta^2 - \frac{2}{3}\delta + \frac{k}{k+1} = 0. \quad (6)$$

Having the functions $u(r, x)$ and $v(r, x)$, we are able

to calculate the streamline paths from the following differential equation :

$$\frac{dr}{dx} = \frac{v(r, x)}{u(r, x)}. \quad (7)$$

This implicit differential equation was solved numerically by a mere first-order finite difference scheme. The streamlines, and thus the particles paths being now known, we come to the determination of the resulting concentration curves.

3.2 PARTICLES DISTRIBUTION. — The continuity equation for the suspended phase is :

$$\nabla \cdot (\Phi \mathbf{U}_p) = 0 \quad (8)$$

where Φ is the particle centre concentration and $\mathbf{U}_p$ is the velocity of a particle. This equation may be developed, and it yields :

$$\mathbf{U}_p \cdot \nabla \Phi + \Phi \nabla \cdot \mathbf{U}_p = 0. \quad (9)$$

$\mathbf{U}_p$ being in this case equal to the fluid velocity, $\nabla \cdot \mathbf{U}_p = 0$, and $\mathbf{U}_p \cdot \nabla \Phi = 0$.

This last equation simply means that there are no changes in concentration along a streamline.

The resulting concentration profiles are plotted in figure 3. We can observe there some important features. First, the particle-free layer which appeared in the close vicinity of the tube entrance because of geometrical interactions, becomes thicker and thicker along the inlet length of the tube. Its final thickness, when the flow is fully developed ($k = 1$) is approximately 3.4 times larger than the initial one in the case (a). Moreover, most of the convection process which induces and inward drift of the concentration curve is achieved in the first stages of the boundary layer development ($k = 0.25$, which corresponds to $\hat{x}/L$ less than 10 %). These phenomena do not depend on the flow Reynolds number, except for the length over which they take place (this conclusion is of course only available in the range $\text{Re} \gg 1$, in which the boundary layer approximation can be used, but it would be interesting to investigate whether or not it can be extended to an arbitrary Reynolds number).

Finally the effect of the particle to tube diameter ratio can be understood by comparing figures 3a

Fig. 3. — Concentration profiles along the inlet length of the tube, when the particles follow streamlines. The parameter is $k = u_1 - 1$, which is related to x by the formula (2). In (a), $2a/R \cong 0.1$, and in (b) $2a/R \cong 0.02$.

and b . In a , $2a/R \cong 0.1$, and in b , $2a/R \cong 0.02$. In the case b , the initial wall peak and particle free layer are narrower than in the case a . Moreover, the radial inward migration of the concentration profile is less in b than in a . This is due to the fact that the radial component of the fluid velocity vanishes near the wall.

It is now necessary to study the effect of an inertia-induced drift force, when it adds to the convective processes.

4. Inertial migration effects. — 4.1 GENERAL EQUATIONS. — It has been assumed in the previous section that the particles are purely convected by the suspending fluid, i.e. that $\mathbf{U}_p = (u, v)$. In this section, we assume that $\mathbf{U}_p = (u, v + v^*)$, v^* being a particle radial velocity relative to the surrounding fluid, induced by inertial lift forces. In this case, we must first analyse the consequences of the additional velocity v^*

on the continuity equation (8) and then estimate at least an approximate magnitude of v^* in some typical experimental situations, in order to obtain the resulting concentration curves.

The equation (9) may be written :

$$\mathbf{U}_p \cdot \nabla \Phi = - \Phi \nabla \cdot \mathbf{U}_p .$$

where :

$$r \nabla \cdot \mathbf{U}_p = \frac{\partial(ru)}{\partial x} + \frac{\partial r(v + v^*)}{\partial r} = \frac{\partial(rv^*)}{\partial r} . \quad (10)$$

To study an instance, we assume that v^* is proportional to the local shear rate. This is obviously true in Rubinow's and Keller's case since their lift force formula [12] is :

$$\mathbf{F}_L = \pi \rho a^3 \boldsymbol{\Omega} \times \mathbf{V} [1 + O(\text{Rep})] ,$$

while the radial drag force is (approximately)

$$|\mathbf{F}_d| = 6 \pi \nu \rho a |v^*| .$$

Indeed, we can write $\boldsymbol{\Omega} = \frac{1}{2} \nabla \times \hat{\mathbf{U}}$ where $\hat{\mathbf{U}}$ is the local velocity of the suspending fluid, and the only non-zero component of $\boldsymbol{\Omega}$ in an axisymmetric flow is the azimuthal one :

$$\left(\frac{\partial \hat{v}}{\partial \hat{x}} - \frac{\partial \hat{u}}{\partial \hat{r}} \right) .$$

In the boundary layer approximation, $\partial \hat{v} / \partial \hat{x} \ll \partial \hat{u} / \partial \hat{r}$ and we can thus write : $\Omega \cong -\frac{1}{2} \frac{\partial \hat{u}}{\partial \hat{r}}$. But in Saffman's analysis [13], v^* is proportional to the square root of the shear rate, while in more complete computations [14, 15, 16] the dependence of v^* on the shear rate is not a simple one (v^* depends in these cases on the squared shear rate, but also on the spatial derivative of the shear rate [15]). In any case, a formula such as $v^* = G(r) \frac{\partial u}{\partial r}$ can be assumed, the expression for $G(r)$ depending on the inertia induced migration effect which is considered. We can now calculate (10) using (1) :

$$\frac{\partial(rv^*)}{\partial r} = u_1 \frac{\partial}{\partial r} \left\{ G(r) \left[-\frac{2r}{\delta} + \frac{2(1-r)r}{\delta^2} \right] \right\}$$

and thus, putting $E(x, r) = \nabla \cdot \mathbf{U}_p$

$$E(x, r) = \frac{u_1}{r} \left[G(r) \left(\frac{2}{\delta^2} - \frac{4r}{\delta^2} - \frac{2}{\delta} \right) + \left(\frac{2r - r^2}{\delta^2} - \frac{2r}{\delta} \right) \frac{\partial G(r)}{\partial r} \right] . \quad (11)$$

Otherwise the equation (9) can be solved easily by the method of characteristics, if one notices that :

$$\mathbf{U}_p \cdot \nabla \Phi = \frac{d\Phi}{dt} \quad (12)$$

where t is a parameter defined by :

$$\frac{dx}{dt} = u \quad \text{and} \quad \frac{dr}{dt} = v + v^* . \quad (13)$$

We finally obtain from equations (9), (10), and (12) :

$$\frac{d\Phi}{dt} = -\Phi E(x, r) \quad (14)$$

where $E(x, r)$ is defined by (11). Equations (13) and (14) were solved numerically, by a first-order finite difference scheme.

4.2 APPLICATION TO SOME TYPICAL EXPERIMENTAL SITUATIONS. — The experimental situation which was examined by Brandt and Bugliarello [8] is geometrically rather different from our theoretical model : they have analysed a rectangular channel of aspect ratio 1/16, while we study a circular tube. It is anyway interesting to estimate the order of magnitude of the lift forces in their device and to examine the consequences of such a force when experienced by particles in a circular tube.

One important feature of the experiment we are considering is the high aspect ratio of the rectangular channel, coupled together with the fact that its minor dimension, D , is only 1.6 times larger than the particle diameter. Therefore, the particle migration will take place along the main cross-sectional dimension D' of the channel, while the drag effects will be exerted on the particles along the minor dimension D . As a first effect, a particle moving radially (along the main dimension) experiences a drag force that can be obtained from a formula given by Oseen (see [10]) :

$$F_d = \frac{6 \pi \nu \rho a V_d}{1 - \frac{9a}{16} \left(\frac{1}{l_1} + \frac{1}{l_2} \right)} \quad (15)$$

where V_d is the *radial* component (along the main cross-sectional dimension) of the particle velocity, l_1 is the distance from the particle centre to a wall, and l_2 is the distance to the other wall, along the minor dimension.

Moreover, concerning the longitudinal component of the velocity, a particle lags behind the undisturbed fluid velocity. An expression has been found for this effect [15] for a particle moving between two plane walls, when a Poiseuille flow takes place there. If the particle centre lies at an equal distance from the two walls, the relative velocity between the undisturbed fluid and the sphere is :

$$V = \frac{4}{3} u_2 \left(\frac{a}{D} \right)^2$$

where u_2 is the undisturbed centre-line velocity and D is the distance between the two plane walls. In the case we are considering, we have $D/2a = 1.6$,

and thus $V = 0.13 u_2$. Such a considerable slip velocity implies that the *slip-spin* force (or Magnus effect) is a dominant effect to explain the *radial* migration of a sphere across the streamlines. Indeed, let us equate Rubinow's and Keller's formula to the drag force (15). We obtain $V_d = 0(\text{Rep}) \frac{a}{12} \dot{\gamma} (1 + 0(\text{Rep}))$.

In this case, $\text{Rep} = \frac{aV}{\nu}$ and γ is of the same order of magnitude as u_2/D' , D' being the main cross-sectional dimension of the rectangular channel. Moreover,

$$\text{Rep} \approx \frac{D' u_2}{\nu} \cdot \frac{a}{D'} \left(\frac{a}{D} \right)^2$$

and thus,

$$\frac{V_d}{u_2} \cong \frac{D' u_2}{\nu} \left(\frac{a}{D'} \right)^2 \left(\frac{a}{D} \right)^2 .$$

We can compare this result to the terms considered by Ho and Leal [15] in a pure two dimensional case. With the same notations, we would have :

$$\frac{V_d}{u_2} \cong \frac{D' u_2}{\nu} \left(\frac{a}{D'} \right)^3 .$$

We can therefore conclude that, a/D' being approximately equal to 0.02 in [8], while $a/D \cong 0.3$, the dominant effect is the one with squared a/D , while the effect with cubed a/D' is of the same order of magnitude or even negligible. The very special geometrical cross-section we are considering, thus lead us to use a Magnus-type lift force. The expression we use for v^* (which is the dimensionless equivalent to V_d in axisymmetrical geometry) is :

$$v^* \cong 0(\text{Rep}) \frac{a}{12 R} \frac{\partial u}{\partial r} (1 + 0(\text{Rep})) .$$

When $\text{Rep} \cong 0.5$ (we must limit ourselves as for the value of Rep , to be able to use the Rubinow's and Keller's formula, though Rep is higher than 0.5 in [8]), and for $a/R = 0.05$, we obtain $v^* \cong 10^{-3}$ to $10^{-2} \frac{\partial u}{\partial r}$.

This velocity v^* being used, the figure 4 shows a comparison between the particles paths and the streamlines along the inlet length, when the tube Reynolds number is 40.

One can observe that the particle paths move away from the streamlines, to the inward direction. The migration across the streamlines is especially considerable near the walls. This is due :

- i) to the high shear rate near the wall
- ii) to the fact that the boundary layer shear rate reaches the peripheral particles sooner than the particles which lie near the centre line.

On the other hand, the equation (14) indicates that the concentration is not constant along a particle

Fig. 4. — Comparison between streamlines along the inlet length of a tube and the paths followed by particles when they experience a Magnus-type lift force. The migration velocity is $v^* = 10^{-2} \partial u / \partial r$. Here, the tube Reynolds number is 40. The continuous lines are streamlines, while the dotted lines are particle paths.

path. This effect is shown in figure 5. One can notice there that from $k = 0$ to $k = 0.25$, the phenomena are dominated by the convective processes : these two first curves are very similar to the corresponding ones in figure 3.

For $k > 0.25$, the processes are more and more dominated by the inertia-induced migration : the radial inward drift of the concentration curves is approximately constant and the concentration peak increases when k increases. Moreover, a comparison between figures 5a and 5b show that this effect is increased when the tube Reynolds number increases, though we have kept $v^* = 10^{-2} \frac{\partial u}{\partial r}$.

For a given velocity, this dependence on the tube Reynolds number is merely due to the dependence of the inlet length on Re : the longer the inlet length is, the more the particles have time to migrate. Finally, one can notice the qualitative agreement of these theoretical results with the first stages of Brandt and Bugliarello's experimental ones [8], which are plotted in figure 6. The late stages of the migration process are very very influenced, in the experimental situation,

Fig. 5. — Concentration profiles along the inlet length of the tube when the particles experience a Magnus-type lift force. In (a), the tube Reynolds number is 80, while in (b) it is 120.

by high concentration effects : the volume concentration reaches 10 % near the centre line of the main cross-sectional dimension of the rectangular channel.

The most common experimental situation (and also the present theoretical model) deals with the flow of a suspension through a circular tube. In this case, an analysis about the order of magnitude of the inertia-induced migration effects must take into account the fact that the cross-sectional shape of the channel is no more rectangular, but rather has an aspect ratio of 1. We can thus take the same formulas as previously for the drift velocity, but with $D = D'$ and we obtain : $\frac{V_d}{u_2} \cong \frac{D' u_2}{v} \left(\frac{a}{D'} \right)^4$ for the *slip-spin* mechanism, and $\frac{V_d}{u_2} \cong \frac{D' u_2}{v} \left(\frac{a}{D'} \right)^3$ for the mechanisms retained by the recent works [15, 16]. The dominant effect is now represented by the second

Fig. 6. — Experimental concentration profiles obtained in a rectangular channel (from [8]). The channel Reynolds number related to the hydraulic radius is here 1 170, and the initial volume concentration is 5 %. The particle diameter is $1/25$ of the main cross-sectional channel dimension D' (and $1/1.6$ of the minor dimension D). To allow a comparison with figure 5, the curves are located by means of the parameter $\xi = \frac{x D'}{2 L}$ (where L is $0.0575 \text{ Re } D'/2$, according to Schiller's formula), and of the parameter k which is computed from x by the expression (2), R being replaced by $D'/2$.

formula. With $\text{Re} = 40$ and $\frac{a}{R} = 0.05$, the order of magnitude of $\frac{V_d}{u_2}$ is 10^{-2} . Moreover, to take into account the complicated dependence of the drift velocity on the radial coordinate [15, 16], we assume a simple corrected formula : $v^* = 10^{-2}(r - 0.6) \frac{\partial u}{\partial r}$.

The results obtained using such assumptions are shown in figure 7. The first concentration curve is

Fig. 7. — Concentration profiles, when the particles migrate across the streamlines with a velocity $v^* = 10^{-2}(r - 0.6) \frac{\partial u}{\partial r}$. The tube Reynolds number is here 40. The curve located at $x = 2.3$ corresponds to the end of the inlet length (with Schiller's formula). After this point, a Poiseuille flow field is assumed.

the initial one (at the tube entrance). The second one is obtained at the end of the inlet length (here $x = 2.3$) and it can be seen that, at this location the essential effect is still the convective inward migration.

The last curve is the result of the computation of the inertia-induced migration far after the inlet length, a Poiseuille profile being assumed. This curve is an illustration of the *Tubular Pinch effect* first observed by Segre and Silberberg [7]. Similar results were obtained by these authors who used semi-empirical particle paths to complete their computation.

Such particle paths can now be interpreted, according to our model, as a combination of inlet convective and inertia-induced motions.

5. Conclusion. — It has been noticed previously that the present theoretical computations of the entry flow of a suspension through a tube can only be applied to a case where :

1) $\text{Re} \gg 1$.

2) The concentration and particle to tube diameter ratio are low enough to assume that the flow field is almost unaffected by the presence of the particles.

Some improvements might be added to this model in order to extend it to the physiological situation of small vessels, or to similar experimental devices.

1) The tube Reynolds number in small arteries is approximately 1 and thus, it would be interesting to study the case of relatively low values of the Reynolds number, by a numerical computation of the flow field at a tube entrance.

2) The influence of high concentrations might be taken into account by diffusive terms related to particle-particle collisions, and also by an adequate study of the dynamics of the particle clusters in a tube flow [22].

It is nevertheless possible from the present analysis to draw out some general conclusions and remarks about the flow of suspensions in situations which bear physical similarity to blood circulation within the arteriolar network. The characteristic lengths and velocities in such vessels [23] are the following : the average tube length (between two bifurcations) is approximately 0.5 cm. The tube Reynolds number is of order 1, and the averaged velocity is roughly 1 cm/s. Finally, the ratio between a red blood cell diameter and the tube diameter is generally smaller than 0.1. (It must be noticed however that the characteristic diameter of a migrating particle may be larger than the red blood cell one, because of the presence of aggregates.)

If we use, in such a situation, an order of magnitude for inertia-induced migration velocity, such

as $\frac{V_d}{u_2} = \text{Re} \left(\frac{a}{D'} \right)^3$, or if we refer to some experimental measurements [9], we are led to think that the inertia-induced migration forces are not intense enough to explain the presence of a peripheral particle free-layer (or plasma layer) along *most* of the length of a vessel. Some additional mechanisms must be supposed. Among those, the inward convective

motion combined to geometrical effects might be an important one, since it takes place along the flow inlet length which is (for $\text{Re} = 1$) of the order of magnitude of the tube diameter. The results about this effect, which were obtained in this paper for a dilute suspension of rigid spheres, and these arguments of order of magnitude, claim for a further study of the entrance phenomena in more general conditions.

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