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► To cite this version:

O.G. Martynenko, R.I. Soloukhin. Thermo-gasdynamics control in laser beam guidance. Revue de Physique Appliquée, 1978, 13 (7), pp.329-341. 10.1051/rphysap:01978001307032900 . jpa-00244458

HAL Id: jpa-00244458

<https://hal.science/jpa-00244458>

Submitted on 1 Jan 1978

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REVUE DE PHYSIQUE APPLIQUÉE

THERMO-GASDYNAMIC CONTROL IN LASER BEAM GUIDANCE

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(Reçu le 15 février 1978, accepté le 4 avril 1978)

Résumé. — On étudie les résultats des études des lentilles thermiques à gaz, le principe de leur fonctionnement et de leur utilisation pour la focalisation et le guidage des faisceaux lasers à hautes puissances. On développe une méthode d'analyse généralisée des systèmes thermo-optiques typiques, permettant de lier la distribution de densité de gaz avec les caractéristiques principales des lentilles thermiques à gaz, ceci dans différentes conditions thermo-gazodynamiques. En particulier, les caractéristiques optiques efficaces des lentilles à gaz sont proposées et analysées tandis que les résultats typiques de leur calcul numérique sont discutés.

Abstract. — A survey is given on thermal gas lens developments and of their principles and applications for focusing and guidance of intense laser beam. The generalized analysis of typical thermo-optical systems is used to relate the gas density distribution to characteristic features of thermal gas lens designs in accordance with various thermo-gasdynamic conditions. In particular, effective optical gas lens characteristics are proposed and emphasized, and typical results based on solutions of mathematical models are discussed.

1. Introduction. — Recent progress in high-power laser development has stimulated much interest in research for efficient methods of controlling optical parameters of powerful laser beams. The traditional solid-state optical elements fail to be practicable owing to the high thermal stresses induced by the propagating beams of high-intensity. The use of gas optical elements by monitoring optical characteristics of a medium along the beam path appears to be a promising alternative in such situation. To date, there is a rather wide choice of gas analogous of solid optical elements, namely, gas lenses of different types [1] or focusing elements; deflecting devices or prisms (wedges) [2], aerodynamic windows or plane plates [3]. The gas analogous of the solid-state optical elements are better candidates at least for two reasons. First of all, with gases the threshold breakdown intensity under standard conditions is of the order of 10^9 - 10^{10} W/cm² and the effects of non-linearity associated with the nonlinear susceptibility of the medium [4], giving rise to self-focusing [5, 6], occur at powers exceeding by several orders those in condensed media (i.e. in crystals and liquids). Under subthreshold conditions, a gaseous medium can be easily renewed and is practically permanent, while a copper mirror subjected to such intense irradiation is damaged in 10^{-9} s. In the case of gas optics, only temperature field redistributions and a

subsequent change in optical parameters of the gas optical device are generated due to nonlinear effects involving energy absorption of the beam at power levels that would cause damage in solid optical elements.

On the other hand, energy losses due to reflection on a solid mirror surface constitute not less than 0.01 per cent of the incident light energy, so that multiple light reflection causes a pronounced decrease in the total light flux intensity. The reflection losses at an interface are vanishingly small in optically non-uniform gaseous media, with a refractive index not much differing from that of a surrounding gas.

Even when a medium has been chosen, there are a great many ways of devising optical systems. Of particular importance is the development of wave guidance systems for transmission of high-power laser energy over long distances with negligible losses and distortions.

In what follows, consideration is given to some common properties of inhomogeneous gaseous media and, in more detail, to thermal gas lenses (TGL), their operation principles and optical characteristics.

2. General laws. — The possibility of monitoring the optical parameters of a propagating laser beam relies on the dependence of the refractive index of a gaseous medium on its thermodynamic parameters.

The dielectric constant, ε , of gases with non-polar atoms and molecules obeys the well-known relation

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{1}{3} N\alpha \quad (1)$$

or, with reference to the equation of an ideal gas state, $p = NkT$, the refractive index, $n = \sqrt{\varepsilon}$, is

$$n = 1 + \frac{1}{2} \frac{p\alpha}{kT}, \quad (2)$$

At constant pressure, the refractive index relates to temperature as

$$n - 1 = \frac{n_0 - 1}{T} T_0 \quad (3)$$

where n_0 is the refractive index of the gas at T_0 .

A medium with a definite inhomogeneous refractive index profile is referred to as a lenslike medium. Consider some optical properties of thermo-gasdynamic lenslike media. Generally, temperature and/or pressure profiles and, consequently, the refractive index distribution are a function of the cylindrical coordinates r , φ and z . Within the approximations of geometrical optics, the trajectory in a medium with the refractive index, $n(r, \varphi, z)$, is determined by the ray equation :

$$\frac{d}{ds} \left(n \frac{d\mathbf{r}}{ds} \right) = \nabla n \quad (4)$$

where the unit vector, $\frac{d\mathbf{r}}{ds} = \mathbf{s}$, is tangent to the ray and a change in the ray vector, $\mathbf{q} = n\mathbf{s}$, follows, according to equation (4), the ∇n direction. Thus, one may guide a light beam by suitable design of the spatial distribution of the refractive index, $n(r, \varphi, z)$.

To specify the basic properties of lenslike media, we restrict ourselves to the case of axially symmetric profiles of the dielectric constant $\varepsilon = \varepsilon(r, z)$, so that the ray equation (4) acquires the form [7] :

$$\frac{d^2 r}{dz^2} = \frac{1}{2\varepsilon} \left[1 + \left(\frac{dr}{dz} \right)^2 \right] \left[\frac{\partial \varepsilon}{\partial r} - \frac{\partial r}{\partial z} \frac{\partial \varepsilon}{\partial z} \right]. \quad (5)$$

If we assume the dielectric constant to be an arbitrary function of either the radius $\varepsilon = \varepsilon_1(r)$ or the longitudinal coordinate $\varepsilon = \varepsilon_2(z)$, the contributions of radial and longitudinal temperature gradients can be examined separately. For media of the former type equation (5) becomes

$$\frac{d^2 r}{dz^2} = \frac{1}{2\varepsilon_1} \left[1 + \left(\frac{dr}{dz} \right)^2 \right] \frac{d\varepsilon_1}{dr}. \quad (6)$$

Integrating equation (6) yields the expressions for a ray trajectory [8] :

$$z = C_2 \pm \int \frac{dr}{\sqrt{C_1 \varepsilon_1 - 1}}. \quad (7)$$

The arbitrary constants C_1 and C_2 are found from the initial conditions

$$r|_{z=0} = r(0), \quad \left. \frac{dr}{dz} \right|_{z=0} = \dot{r}(0). \quad (8)$$

Given the distribution $\varepsilon = \varepsilon_1(r)$, the determination of the ray trajectories is reduced to the evaluation of an integral.

Let a beam fall onto a lenslike medium parallel to its axis, $\dot{r}(0) = 0$ [8]. The index distribution is considered ideal if there exists a point at which the rays intersect the z -axis independent of the input coordinate, $r(0)$. Otherwise the medium suffers aberration.

For some space distribution of the dielectric constant, for instance,

$$\varepsilon = 1 - \alpha r^2 \quad (9)$$

integrating equation (7) gives

$$r = \sqrt{\frac{C_1 - 1}{\alpha C_1}} \cdot \sin \sqrt{\alpha C_1} (z - C_2) \quad (10)$$

where

$$C_1 = \frac{\dot{r}^2(0) + 1}{1 - \alpha r^2(0)};$$

$$C_2 = - \frac{1}{\sqrt{\alpha C_1}} \text{Arc sin } r(0) \sqrt{\frac{\alpha C_1}{C_1 - 1}}.$$

Here, the trajectory is recognized as a sine function with the amplitude

$$A = \sqrt{\frac{1}{\alpha} - \frac{1 - \alpha r^2(0)}{\alpha [\dot{r}^2(0) + 1]}} \quad (11)$$

and the period

$$\tau = \frac{2\pi}{\sqrt{\alpha}} \sqrt{\frac{1 - \alpha r^2(0)}{1 + \dot{r}^2(0)}}. \quad (12)$$

Spherical aberrations arise as a direct consequence of the dependence of the period on $r(0)$.

Among the peculiar properties of most thermo-gasdynamic lenslike media is the presence of higher powers of r in $\varepsilon_1(r)$ as compared to equation (9). As one more typical case, consider a medium having the following $\varepsilon_1(r)$ distribution

$$\varepsilon_1(r) = 1 - \alpha r^2 + \beta r^4, \quad (13)$$

where α and β are constants which determine the specific convergence and the spherical aberrations, respectively. Substituting equation (13) into equation (7) results in

$$z = C_2 \pm \int \frac{dr}{\sqrt{C_1(1 - \alpha r^2 + \beta r^4) - 1}}. \quad (14)$$

The integration constant is

$$C_1 = \frac{\dot{r}^2(0) + 1}{1 - \alpha r^2(0) + \beta r^4(0)}. \quad (15)$$

If $C_1 < 1$, the integral (14) may be reduced to the form

$$z = C_2 + \frac{2}{\sqrt{\alpha C_1}} \int \frac{dr}{\sqrt{(r^2 + a^2)(r^2 - b^2)}} \quad (16)$$

while for $C_1 > 1$

$$z = C_2 + \frac{2}{\sqrt{\alpha C_1}} \int \frac{dr}{\sqrt{(a_1^2 - r^2)(b_1^2 - r^2)}}. \quad (17)$$

Let us analyse, for example, solution (16). The expressions for a^2 and b^2 are found on comparing (16) and (14). As shown in [9], transformations converting the elliptical integral (16) to a normal Legendrian form are different for different intervals of integration. Provided $\eta > b > 0$ and on account of the functional relationships

$$F(-\varphi, k) = -F(\varphi, k) \quad (18)$$

$$F(m\pi \pm \varphi, k) = 2mK(k) \pm F(\varphi, k)$$

where $K(k) = F\left(\frac{\pi}{2}, k\right)$, we find the maximal deviation of a ray from the z -axis to occur at the points

$$z_m = \frac{(2m \pm 1) \cdot K(k) \cdot 2}{\sqrt{\alpha C_1} \sqrt{a^2 + b^2}} + C_2. \quad (19)$$

Note that the ray traverses the z -axis at

$$z_n = \frac{2nK(k) \cdot 2}{\sqrt{\alpha C_1} \sqrt{a^2 + b^2}} + C_2 \quad (20)$$

and the oscillation period is

$$\tau_1 = z_m - z_{m-2} = \frac{4K(k) \cdot 2}{\sqrt{\alpha C_1} (a^2 + b^2)}. \quad (21)$$

For rays entering the lenslike medium parallel to its axis, the period is

$$T = r(0) \cdot z = 4K, \quad (22)$$

where $K = F\left(\frac{\pi}{2}, k\right)$ is the full elliptical integral of a first kind. For $\varepsilon_2 = \varepsilon_2(z)$, equation (5) becomes

$$\frac{d^2 r}{dz^2} + \frac{1}{2\varepsilon_2} \left[1 + \left(\frac{dr}{dz} \right)^2 \right] \frac{dr}{dz} \frac{d\varepsilon_2}{dz} = 0 \quad (23)$$

with the solution :

$$r = \int_0^z \frac{dz}{\sqrt{C_3 \cdot \varepsilon_2 - 1}} + C_4. \quad (24)$$

Here, C_3 and C_4 are integration constants specified by the initial conditions (8). If we assume, for instance, that

$$\varepsilon_2(z) = 1 + \gamma z \quad (25)$$

then

$$C_3 = \frac{1 + \dot{r}^2(0)}{\dot{r}^2(0)}.$$

Integrating (24) with regard to equation (25) gives the ray trajectory as

$$r = \frac{\dot{r}(0) [1 + \gamma(1 + \dot{r}^2(0)z)]^{1/2}}{\gamma(1 + \dot{r}^2(0))} + C_4. \quad (26)$$

On account of the initial conditions, it follows from equation (26) that

$$C_4 = r(0) - \frac{2\dot{r}(0)}{\gamma[1 + \dot{r}^2(0)]}.$$

The distance, z_F , at which the ray intersects the z -axis, $r = 0$, is defined by

$$z_F = \frac{\left[\frac{\gamma r(0) [1 + \dot{r}^2(0)]}{2\dot{r}(0)} - 1 \right]^2 - 1}{\gamma[1 + \dot{r}^2(0)]^2} \quad (27)$$

provided

$$\frac{\gamma r(0) [1 + \dot{r}^2(0)]}{2\dot{r}(0)} > 1.$$

Available thermal gas lenses have both longitudinal and transverse temperature gradients. The combined effect of the ensuing longitudinal and transverse refractive indices

$$n = 1 - bz - \alpha r^2 + \beta r^4 \quad (28)$$

has been investigated through numerical solution of equation (5). Figure 1 shows the trajectories and figure 2 the amplitudes of the ray oscillation for different $r(0)$ and β at fixed α and at $\dot{r}(0) = 0$. Inspection of these graphs reveals that negative spherical aberrations ($\beta < 0$) result in a decrease while positive values of β result in an increase of the oscillation amplitude compared to the aberration-free case ($\beta = 0$). The amplitude growth at $\beta = 0$ is attributed to a presence of a negative longitudinal gradient.

FIG. 1. — The trajectories of rays entering a lenslike medium pattern at different input ordinates (Eq. (28)) : (a) $r(0) = 0.3$; (b) $r(0) = 0.5$; (c) $r(0) = 0.7$. The curves 1, 2, 3 are appropriate for $\beta = 0.5$; -0.5 ; -1.0 .

FIG. 2. — The oscillation amplitude as a function of the input ordinate of a ray entering a lenslike medium pattern (Eq. (28)). (a) $r(0) = 0.3$; (b) $r(0) = 0.5$; (c) $r(0) = 0.7$. The curves 1, 2, 3 correspond to $\beta = 0.5$; 0 ; -1.0 .

The rate of change of the amplitude increases with the ordinate of a ray entering the lenslike medium. The examples above clearly demonstrate that the simplified gas lens theory can provide various approaches in predicting basic features of the light beam propagation in a non-uniform gaseous medium created by thermal or gasdynamic means, evaluations of principal gas lens characteristics being certainly feasible.

3. Typical optical gas lens designs. — We describe now briefly the main types of gas lenses. Isothermal concentration-type gas lenses [10] are sketched in figure 3. Two gases of different refractive indices, n_1 and n_2 , flow continuously from opposing tubes, 2 and 3, into a mixing chamber, 5, to be exhausted via a tube 4 (Fig. 3a). The chamber is designed so that an interface surface, where the gases meet, acts as a lenslike medium and focuses or defocuses a

FIG. 3. — Gas concentration-regulated gas lenses.

light beam travelling through it, depending on the flow rates and the ratio of gas concentrations and densities as well as the flow mixing conditions. The simplest version of the concentration-type gas lens is depicted in figure 3b, here flows of two different gases are mixed, i.e. a gas with a greater index of refraction (tube 2) and a gas of a smaller refractive index injected through a porous insert, 4, positioned in chamber 3. As a result, an inhomogeneity in the distribution develops, and a lenslike medium is created in the tube 2 which focuses a light beam propagating along the tube axis. When the tube 2 is filled with a gas of a smaller refractive index, we have a diverging lens. Experiments have demonstrated that such a lens allows a wide variation of the optical parameters by a proper choice of gases and rates of their injection [11].

Following these is a helical thermal gas lens [12] without injection (Fig. 4a). It consists of a heater, helix 2, held by thermally insulating supports 3 in pipe 4. The helix temperature is a few degrees higher than that of a medium that induces convective streams oriented in a gravitational field and non-uniformly distributed along the helix. The focusing action of the lens is based on the property of the on-axis refractive index, averaged over one cycle of the system, to exceed the averaged off-axis refractive

FIG. 4. — Thermal gas lenses without injection : (a) the helical gas lens ; (b) the modified *quadrupole* helical gas lens ; (c) the hyperbolic-type gas lens.

index. Steady-state lens operation is achieved by the thermal balance in the system. The temperature field inside such a system is periodic along the pipe axis, whereas the refractive index variation over the lens radius is approximately described by the expression (9). Experimentally tested were two helical lenses with 4 mm and 10 mm aperture sizes. The gases used were CO_2 , air and C_4F_{10} at different pressures and varied temperatures of the helix and an outer pipe. With CO_2 at 1 atm, the focal length for rays near the axis was found to be approximately twice that for marginal rays. Such aberrations were almost eliminated by mixing (1 : 1) helium and CO_2 . Similarly, much less aberration was observed in air compared to C_4F_{10} .

A modified helical gas lens, the so-called *quadrupole* lens (Fig. 4b), is made of four pipes with two of them having a temperature lower and the other two higher than the ambient temperature. The helical pipes constitute a quadrupole system of heat sources in any plane normal to the z -axis of the system. The refractive index of this lens is approximately described by

$$n(r, \varphi, z) = 1 - (n_0 - 1) \cdot \frac{\Delta T}{T_0} \times \frac{J_2\left(\frac{2\pi r}{\psi}\right)}{J_2\left(\frac{2\pi r_0}{\psi}\right)} \cos\left(\frac{2\pi z}{\psi} - 2\varphi\right) \quad (29)$$

where r_0 is the pipe radius ; 2ψ is the helix period ;

J_2 is the modified Bessel function of the 2nd order.

With $\frac{\pi r}{\psi} \ll 1$, we have for the refractive index

$$n - 1 = (n_0 - 1) \frac{\Delta T}{T} \frac{r^2}{r_0^2} \cos\left(\frac{2\pi r}{\psi} - 2\varphi\right). \quad (30)$$

The dependence of the refractive index on the angle, φ , makes the ray spiral about the z -axis. This type of gas lens is distinguished by small focusing power per unit length. A possible enhancement of the optical power by increasing the temperature difference of the pipes is hampered because of the buoyancy that distorts the focusing configuration of a steady temperature field.

Figure 4b shows schematically a hyperbolic-type gas lens [14]. The lens is assembled of a heated and a cooled pair of pipes. In this case the refractive index profile is given by

$$n = n_0 \sqrt{[1 - (qx)^2 + (qy)^2]} \quad (31)$$

where

$$q = \frac{1}{r_0} \sqrt{(n_0^2 - 1) \frac{\Delta T}{n_0^2 T}}. \quad (32)$$

T_0 is the gas temperature on the axis ; ΔT is the temperature difference between the wall and the lens axis. With such a design, the focusing properties of the medium are convergent in the x -direction and divergent in the y -direction (the picture is reversed when the heaters and coolers exchange places). Thus, when developing lenslike media, these systems should be set in pairs with an angle of $\frac{\pi}{2}$ between them and spacing $l_0 \ll l$.

A *chimney*-type gas lens [12, 15] is schematically shown in figure 5a. The focusing action of the lens proceeds in a heated horizontal pipe. Here the gas flow is sustained due to a gas density difference in

FIG. 5. — The gas circulation-type lens (a) and the laminar flow-type thermal gas lens (b).

vertical pipes. In a central pipe, 4, the gas flows upward after being heated electrically by helices, 3, whereas in lateral channels, 2, it is cooled and flows downward. Berreman [12] has proposed the following expression for the focal length in such a lens

$$\frac{1}{f} = \frac{hr_0^2}{4(l+l')} \frac{(n-1)}{x\mu} \rho \left(\frac{\Delta T}{T} \right)^2 \quad (33)$$

here ρ , μ are the gas density and dynamic viscosity based on the temperature, T , respectively; x is the thermal diffusivity; h is the central pipe height; l the length of each diverging pipe section; l' the additional length specified by the condition that a straight pipe with the radius r and length $l + l'$ has the same hydraulic resistance as the system shown in figure 5a.

A laminar flow-type thermal gas lens (Fig. 5b) is simply a circular tube that is kept at a temperature higher than that of an incoming laminar air flow. The following approximate expression is suggested in [21] for a focal length of the thermal lens with a gas blown through it at constant temperature

$$f = 0.596 \frac{r_0^2 T_0}{l(T_w - T_0)(n_0 - 1)} \quad (34)$$

where l is the lens' length. The expression is derived by neglecting the temperature profile asymmetry induced by the free convection effect. Such lenses are advantageous because of their simple design and the possibility of control lines separately the gas injection rate and temperature. Many theoretical and experimental studies are devoted to such kind of gas lenses [16-21].

The operational principle of a gas diverging lens relies on the mechanism that heat is generated by viscous dissipation in a high-speed gas or fluid flow in a pipe [22]. The refractive index distribution in an adiabatic compressible gas flow in this case is derived as

$$n(r, z) = 1 - \frac{r^2}{r_0^2} (n_0 - 1) \frac{C_p - C_v}{2 C_p} R \frac{p_1}{p_0} M_1 \cdot M(z) \quad (35)$$

where r_0 is the pipe radius; n_0 the refractive index at atmospheric pressure p_0 ; p_1 the inlet pressure; M_1 and $M(z)$ the Mach numbers at the beginning and along the pipe, respectively; $R \approx 0.9$, the so-called recovery factor.

In acoustic gas lenses [23], the gas density changes in generated acoustic waves cause variations in the index of refraction. Let an acoustic wave of the pressure amplitude, p_m , frequency, ω , and speed of propagation, v , be excited inside a pipe of a radius r_0 , filled with a gas that has a refractive index n_0 . The refractive index profile in such a medium is described by :

$$n(r, z) = n_0 \alpha J_0(kr) \cos \beta z, \quad (36)$$

where

$$\alpha = (n_0 - 1) \frac{p_m}{\gamma p_0}; \quad \beta = \sqrt{\left(\frac{\omega}{v}\right)^2 - \left(\frac{3.83}{r_0}\right)^2}$$

$\gamma = \frac{C_p}{C_v}$ is the heat capacity ratio and $J_0(k, r)$ is the Bessel function of zero order. Near the optical axis, $kr \ll 1$, equation (36) is simplified to the form :

$$n(r, z) = n_0 + (n_0 - 1) \frac{p_m}{8 p_0} \left(1 - \frac{k^2 r^2}{4}\right) \cos \beta z. \quad (37)$$

The focal distance of a lens with length l is

$$f = \frac{v}{2\pi r_0} \left[\left(\frac{3.83}{r_0}\right)^2 - \left(\frac{2\pi}{l}\right)^2 \right]^{1/2}. \quad (38)$$

There is experimental evidence of the focusing effect of acoustic gas lenses with a wave amplitude of 280-650 N/m². To compensate the optical (diffractive) divergence of a beam of radius

$$a_0 = 3.5 \times 10^{-3} \text{ m}$$

an acoustic wave with a pressure of $\Delta p = 4.2 \times 10^4 \text{ N/m}^2$ is required.

In electromagnetic gas lenses, a light beam may be guided due to the dependence of the dielectric constant upon electromagnetic field intensity [24-26]. Many dielectrics, ferroelectrics, plasmas and other materials possess nonlinear properties at sufficiently high electromagnetic field intensities. The following polynomial expression can be taken for the dielectric constant as a function of the electric field intensity in nonlinear dielectric media

$$\varepsilon = \varepsilon_0 + \varepsilon_1 E + \varepsilon_2 E^2 + \dots + \varepsilon_n E^n + \dots \quad (39)$$

In the general case, the ε_n are tensors and represent contributions of various physical effects. The electric field dependence of the dielectric constant makes possible to affect it either by an external electric field or, at sufficiently high intensities, by the propagating wave itself. By imposing an inhomogeneous distribution in an external electric field, for instance via an appropriate geometry of electrodes, one may attain the required inhomogeneity of the refractive index distribution.

Since ε_n is a function of temperature, there exists an additional possibility to guide light by varying this coefficient. For example, the temperature of the medium changes because of absorption of energy from the propagating light beam and this, in turn, leads to a change in the density distribution of the medium. The rays will converge or diverge if $\frac{\partial \varepsilon}{\partial T} < 0$

or $\frac{\partial \varepsilon}{\partial T} > 0$. The nonlinear properties of the medium are treated as thermal self-focusing [27-29]. It has been shown that a wave-guide channel may develop

if the beam intensity exceeds some critical quantity, P_{cr} . In such a regime, the diffraction divergence of rays is compensated by refraction if

$$P_{cr} = (1.22 \cdot \lambda)^2 \cdot \frac{C}{64 n_2} \quad (40)$$

where λ is the light wavelength; C , speed of light; $n_2 = \frac{\epsilon_2}{2\sqrt{\epsilon_0}}$ is a nonlinear term in the refractive index, n , which is associated either with the Kerr effect (i.e. field orientation of anisotropically polarized molecules) or with the electrostriction action (i.e. dielectric volume variations in an electric field). In air at 1 atm $n_2 = 0.041 \times 10^{-13}$ and critical power, $P_{cr} = 80$ W, while at 100 atm, $n_2 = 4.1 \times 10^{-13}$ and $P_{cr} = 0.8$ MW [30]. The light-guide channel in a dielectric medium is initially formed by a high-power beam, and then comparatively weak signals of a higher frequency may be transmitted over the wave guide.

A theory of light beam propagation in periodic systems of thin self-aligning elements and a discussion of the possibility of applying these elements for light beam transmission along the channels are presented in [31]. A focal length of such elements, with the nonlinearity due to the Kerr effect, is given in the ray optics approximation

$$f_k = \frac{n_0 a^4}{\left(\frac{P}{P_{cr}} - 1\right) l} \quad (41)$$

where a is the beam width; P is the beam power; l is the ray propagation length along a self-focusing element and P_{cr} is the critical self-focusing intensity. In the same approximation, the focal length of a thermal lens is

$$f_t = \frac{a^2}{\beta P l} \quad (42)$$

where $\beta = \frac{n_2 \alpha}{\pi \lambda}$; α is the small signal absorption coefficient and λ is the thermal conductivity.

Note that in addition to electrical, there exists a possibility to guide light beams by means of magnetic field variations and other effects.

The classification of different gas lenses might be extended. It seems, however, that the examples above sufficiently illustrate various gas lens principles and designs.

4. Optical characteristics. — When examining optical properties of gas lenses it has become customary to use the ray equation (5), which is the fundamental equation of geometrical optics, and justifies the concept of rays. With the given initial conditions for a ray entering a lens, this equation completely defines

an output ray coordinate. At the same time, by virtue of some reasons (for instance, utilization of gas lenses as light guide elements) it is expedient to describe gas lenses in terms of the traditional geometrical optics, i.e. focal lengths, aberration characteristics etc. This approach seems attractive since it allows a comparison with the experimental data.

Let us now introduce the definition of the focal length and the principal surfaces of a gas lens. With this end in view, consider a ray incident parallel to the lens (a plane front) and discuss only the case when the optical system has an axis of symmetry. Let a ray enter the lens parallel to its axis at a distance $r(0)$ (see Fig. 6). In the lens, the ray is deviated

FIG. 6. — An illustration to the focal distance definition of a gas lens.

by the angle $\dot{r}(l)$, l being the lens length, and behind the lens it follows the straight path. The difference between the abscissae of the point of intersection of the ray with the axis and the point of intersection of two straight lines, one of which coincides with the incident ray direction and the other with its direction when leaving the lens, is defined as the focal length. The locus of the points of intersection of the two straight lines for all rays defines the principal surface of a gas lens.

By definition, the expression for the focal length is

$$f = \left| \frac{r(0)}{\tan \varphi} \right| = \left| \frac{r(0)}{\dot{r}(l)} \right|. \quad (43)$$

If the focal length is a function of the input ray coordinate, such a lens is considered to involve aberrations, otherwise it is ideal. It is evident that this lens is ideal if $\dot{r}(l)$ is proportional to $r(0)$.

The principal surfaces are defined by formulas

$$F_+ = l + \frac{r(0) - r(l)}{\dot{r}(l)}; \quad F_- = \frac{r(l) - r(0)}{\dot{r}(0)} \quad (44)$$

where the index (+) indicates a value that belongs to rays propagating in the gas stream direction, while the index (−) refers to the backward propagating rays. If F_+ and F_- coincide, a gas lens is equivalent to an optically thin one.

The shape of the principal surface of a duct-type thermal gas lens has been investigated in [32] for different thermo-hydrodynamic regimes at constant lens walls temperature. The results of numerical calculations show that surfaces F_+ and F_- nearly coincide so that, practically, a gas lens is optically thin.

There is another approach to describe optical parameters of a gas lens that avoids solution of equation (4) which, in most practically important cases, poses mathematical difficulties. Let $\mathbf{r}_0$, $\mathbf{r}'_0$, $\mathbf{r}'_1$ and $\mathbf{r}_1$ be coordinates of the points of a ray crossing the planes of an object, an entrance and exit pupils and that of a maximum image, respectively, while $\mathbf{r}_1^*$ is the coordinates of the point of a paraxial image specified by

$$\mathbf{r}_1^* = M\mathbf{r}_0 \quad (45)$$

here $M = l_1/l_0$ is the Gaussian lateral magnification, l_0 and l_1 are the length scales in the object and image planes, respectively.

The ray aberration δ is measured as the difference between the coordinates of the intersection points of the image plane with the real and paraxial rays i.e.

$$\delta = \mathbf{r}_1 - \mathbf{r}_1^* = \mathbf{r}_1 - M\mathbf{r}_0. \quad (46)$$

It seems convenient to relate the ray aberration δ to the wave aberration φ identified as the optical path length between the points of intersection of the ray with the real wave front and a reference Gaussian sphere

$$\delta = -\frac{D_1}{n_1} \frac{\partial \varphi}{\partial \mathbf{r}'_1} \quad (47)$$

where D_1 is the distance between the exit pupil and image plane; n_1 is the refraction index in a space of the image. On the other hand, the optical path between the points where the ray intersects the object and image planes is defined as the integral of the refractive index along the ray path

$$V(\mathbf{r}_0, \mathbf{r}_1) = \int_{p_0}^{p_1} n(\mathbf{r}) ds. \quad (48)$$

Here $V(\mathbf{r}_0, \mathbf{r}_1)$ is the optical path recognized as a point eikonal. Thus, the function of wave aberrations may be expressed through the point eikonal as

$$\varphi = V(\mathbf{r}_0, \mathbf{r}) - V(\mathbf{r}_0, \mathbf{r}'_1). \quad (49)$$

With ordinary approximations not affecting the accuracy of the theory, $\mathbf{r}$ may be replaced by $\mathbf{r}'_1$, i.e. by the coordinate of the ray crossing the exit pupil [7]. Then

$$\varphi(\mathbf{r}_0, \mathbf{r}'_1) = V(\mathbf{r}_0, \mathbf{r}'_1) - V(\mathbf{r}_0, \mathbf{r}'_1). \quad (50)$$

Following equation (47), the ray aberration may be expressed through (50) as

$$\mathbf{r}_1 - \mathbf{r}_1^* = -\frac{D_1}{n} \frac{\partial V(\mathbf{r}_0, \mathbf{r}'_1)}{\partial \mathbf{r}'_1} \quad (51)$$

since the second term on the right-hand side is independent of $\mathbf{r}'_1$. Besides, account is taken of the fact that the refractive index of the object and image space are the same. Substituting (48) into (51) yields

$$\delta = \mathbf{r}_1 - \mathbf{r}_1^* = -\frac{D_1}{n} \frac{\partial}{\partial \mathbf{r}'_1} \int_{p_0}^{p_1} n(\mathbf{r}) ds. \quad (52)$$

Thus, the type of ray aberration in a gas lens is governed by the functional form of the index $n(\mathbf{r})$.

In order to simplify equation (52), first of all, replace the arc element ds by the element of the axis dz . On physical grounds, this substitution is justified by the great focal length of most gas lenses and by the small angles of ray incidence with respect to the lens axis. This approximation eliminates difficulties in the integration. The corresponding error usually is extremely small, $\Delta r(l) \approx \pm 10^{-7}$.

Equation (52) thus simplifies to the form

$$\delta = \mathbf{r}_1 - \mathbf{r}_1^* = \frac{D_1}{n} \frac{\partial}{\partial \mathbf{r}'_1} \int_{p_0}^{p_1} n(\mathbf{r}) dz. \quad (53)$$

Consider the important case, that the refractive index is independent of z . Taking $n(\mathbf{r})$ out of the integral sign, we arrive at

$$\delta = -\frac{D_1}{n} \frac{\partial n(\mathbf{r}'_1)}{\partial \mathbf{r}'_1} (z'_1 - z_0). \quad (54)$$

If the incident wave front is plane, then $\mathbf{r}_1^* = 0$ (its image is a point on the axis), $D_1 = f$, $z'_1 - z_0 = l$, and letting $n = 1$ and dropping the prime in $\mathbf{r}'_1$, we obtain the expression for aberration

$$\delta = \mathbf{r}_1 = -fl \frac{\partial n(\mathbf{r})}{\partial \mathbf{r}}. \quad (55)$$

Note, that in deriving the expressions (53) through (55), no particular symmetry of the optical system has been assumed. Thus, the symmetry of optical properties is fully specified by that of a refractive index.

Being equipped with the expression in a general form, that relates a refractive index field to optical parameters of thermal gas lenses, start with consideration of some properties of thermal lenses with gas injection [33-35]. The steady-state heat transfer region temperature profile in a laminar incompressible constant-properties liquid flow in a circular tube under the second-kind boundary conditions is known to be represented by the expression

$$\Theta = \frac{T - T_0}{\frac{q_c r_0}{x}} = 2x + \frac{y^2}{2} - \frac{y^4}{8} - \frac{7}{48}, \quad (56)$$

neglecting the effect of mass forces [36]. The relevant dielectric constant distribution reads

$$\varepsilon - 1 = a \left[1 - b \left(2x + \frac{y^2}{2} - \frac{y^4}{8} - \frac{7}{48} \right) \right]. \quad (57)$$

Here

$$a = \frac{2.80 \cdot p \times 10^{-6}}{T_0}; \quad b = \frac{q_c \cdot r_0}{T_0 \lambda}.$$

Taking into account that real gas lenses have large focal lengths, the term $(dr/dz)^2$ in equation (5) may be neglected, and the ray trajectory equation acquires the form

$$\frac{d^2 r}{dz^2} = \frac{1}{2\varepsilon} \left(\frac{\partial \varepsilon}{\partial r} - \frac{\partial r}{\partial z} \cdot \frac{\partial \varepsilon}{\partial z} \right). \quad (58)$$

Substituting (57) into (58) yields

$$\frac{d^2 y}{dx^2} - \xi \frac{dy}{dx} + q^2 \left(y - \frac{y^3}{2} \right) = 0 \quad (59)$$

where

$$\xi = a \cdot b, \quad q^2 = \frac{\xi \cdot Pe^2}{2}.$$

The contribution of a longitudinal temperature gradient may be evaluated by setting $q = 0$ in equation (59), while that of a radial gradient by setting $\xi = 0$ in (59) that results in

$$\frac{d^2 y}{dx^2} + q^2 \left(y - \frac{1}{2} y^3 \right) = 0. \quad (60)$$

The variation of q^2 in gas lenses is affected by thermodynamic parameters, flow patterns and heat transfer modes specified by the prescribed quantities of the Pe numbers and heat flux, respectively, as well as by the gas lens diameter, d . In practicable gas lenses operated in air at atmospheric pressure ($p_0 = 1013.25$ mbar) the gas temperature changes from 250 to 350 K, the temperature drop between the wall and the tube ranges between 5 and 100 °C, the heat flux is $q_c \sim 10 \div 10^3$ W/m², the Pe numbers lie within 200 to 2000 and the tube diameter varies within 0.01-0.1 m. The quantities of q^2 , therefore, may range within $0 \div 10^4$. To solve equation (60) with the chosen q^2 values, let us take the initial conditions in the form

$$x = 0, \quad y_0 = r(0), \quad (0 \leq y_0 \leq 1); \quad \dot{y}_0 = \dot{r}(0). \quad (61)$$

The values of $r(0)$ may be assumed equal to 0.2; 0.4; 0.6; 0.8 and let the values of an incidence angle vary in such a way that the ray does not touch the wall that agrees with the condition $y < 1$. Then solving equation (60) numerically at the initial conditions (61) and $q^2 = 1, 10, 50, 100, 200$ we obtain the ray trajectories which in figure 7 are presented for a light beam entering the lens parallel to its axis as a function

FIG. 7. — Ray trajectories with zero incidence angle *vs* the input ordinate, $r(0)$, (a) and the aberration curve corresponding to the predicted trajectories (b).

of the input ray ordinate with $q^2 = 100$. From figure 7 it follows that a lenslike medium pattern, with the boundary conditions $q_c = \text{const.}$ on walls, exhibits aberration properties. Spherical aberration *vs* the input ray position with $q^2 = 100$ is shown in figure 7b. Figure 8a plots the focal length as a function of effective lengths for different heat fluxes and a non-dimensional input ray ordinate, y_0 . By the effective lens length, a length without a thermal entrance length becomes clear. An observation of the graphs reveals that the focal length diminishes with an increase in l_{eff} ; here it should be noted that an essential change in the focal length is observed with lenses of small length. For lenses with $l_{\text{eff}} > 2$ m, a change in the focal length is insignificant. Figure 8b presents aberration curves, i.e. the focal length F , as a function of the input ray ordinate, y_0 , for different l_{eff} at constant Pe numbers, diameter d and heat flux q_c . From the graphs, it is clear that with input ray ordinate, the focal length increases with a ratio of the focal length for rays travelling near the wall to that of the ray near an axis being constant and approximately 2.

The magnitude of longitudinal aberration reduces when the effective length increases. For instance, from figure 8b, we find that aberration for lenses with effective lengths of 2; 1.6; 0.8; 0.4 m is 6; 8; 10; 16; 32 m, respectively. No proportional relationship between the aberration magnitude and effective length is observed here, and in order to reduce the lens aberration we should increase its effective length. However, an increase in the lens length results not only in aberration reduction but in the focal length

FIG. 8. — The focal distance *vs* the heat flux (a). The curves 1-6 correspond to $l_{\text{eff}} = 0.4; 0.8; 1.2; 1.6; 2.0; 2.8$. The aberration curves (b) : $Pe = 100$; $d = 0.04$ m; $q_w = 21.6$ W/m^2 . The curves 1 stand for $l_{\text{eff}} = 0.4; 0.8; 1.2; 1.6; 2.0$. The focal distance as a function of the lens diameter (c) : $Pe = 1414$; $q_w = 14$ W/m^2 , the non-dimensional ordinate $y = 0.4$. The curves 1-5 are appropriate for $l_{\text{eff}} = 0.5; 1.0; 1.5; 2.0; 2.5$.

decrease as well. For different l_{eff} and heat flux q_c quantities the aberrations are seen to be insignificant for rays with the input ordinates on the interval (0-0.5) and on the contrary, they increase sharply on the interval (0.5-1). Comparison of the same figures reveals that with the heat flux growth the aberrations reduce for the lenses with a fixed length. From figure 8a, it follows that the focal length is inversely proportional to the heat flux, q_c . A slight heat flux variation within 0-20 W/m^2 contributes to a change in the focal length to a greater extent than that within 20-35 W/m^2 . A further increase in the heat flux will reduce the focal length insignificantly.

Figure 8c shows the focal length as a function of the effective lens length l_{eff} for $Pe = 1414$, $q_c = 14.4$ W/m^2 , the input ray ordinate $y = 0.2$ and different lens diameters. The lenses of small diameters are seen to have small focal lengths whose change with lengths — above 1 m for the rays near the lens axis and above 1.5 m for the rays travelling close to the lens wall — is not essential. An increase in the effective lens length leads to focal length reduction

while an increase in the lens diameter results in the focal length increase.

The data reported give an insight into optical properties of a gas lens under steady-state heat transfer conditions regardless of the entrance length effect. Now consider the contribution of the thermal entrance length [37]. Figure 9a is a plot of the focal length *vs*

FIG. 9. — The focal distance as a function of b (a). The curves 1-4 correspond to $l/r_0 = 100; 150; 200; 400$. $r(0) = 0.5$; $Pe = 1000$. The focal distance as a function of the input ordinate (b) : $b = 0.03$; $Pe = 1000$. The curves 1-4 are appropriate for $l/r_0 = 100; 150; 200; 300$.

the parameter b for different lens lengths l/r_0 and $Pe = 100$, $y_0 = 0.5$ (solid curves). An insignificant variation of b within 0.01-0.03 affects the focal length to a greater extent than the b variation within 0.03-0.06. A further increase in b slightly reduces the focal length. Comparison of the curves (Fig. 9a) for a gas lens with the thermal entrance length (solid curve) and without it (dashed curve) reveals a certain quantitative but not qualitative difference in their behaviour. The difference is accounted for by a large value of the entrance length whose focusing capacity is weak. With an increase in the lens length, the entrance length effect weakens and with the lens length being $l = 1400 r_0$ it becomes insignificant (the curves almost coincide). The entrance length effect depends on heat transfer modes specified by the parameter b . With small b , the effect is greater, while with an

increase in b the effect reduces though it is still significant.

The focal length as a function of the input ray ordinate y_0 (aberration curves) for different l/r_0 , Pe numbers and $b = 0.03$ is plotted in figure 9b. The solid and dashed curves refer to lenses with the thermal entrance length and without it, respectively. The entrance length is seen to exert a particularly pronounced influence on longitudinal aberrations in lenses of small length. Because of the fact that at the entrance length the gas near the axis is heated slightly, the lenslike medium is weak and the focal lengths for rays entering near the axis are large. For lenses with a greater length, the focusing power strengthens with an increase in the input ray coordinate to terminate in a decrease and then in an increase of the focal length. Explanations about this are following. The air near a wall becomes warm thoroughly and the relationship between the focal length and the input ordinate is of the same character as in the case of the lens with no thermal entrance length, i.e. the greater the input ray distance apart the lens axis, the greater the focal length is. Moreover, the focal lengths near the very wall of the lenses with an entrance length and without it clearly coincide. This is attributed to the fact that near the wall a temperature field in a lens with the entrance length and without it does not differ. With an increase in the lens length, a shape of the curve resembles that of the gas lens when the entrance lens effect is neglected. The lower the Pe numbers, the shorter lengths, l , are required to attain it.

FIG. 10. — The focal distance as a function of the Pe number; $b = 0.03$. The curves 1-4 correspond to $l/r_0 = 100$; 150; 200; 300; (a) $r(0) = 0.2$; (b) $r(0) = 0.8$.

In figures 10a, b, focal lengths are plotted as a function of the Pe numbers for different lens lengths, l/r_0 , the input ray ordinates being $y_0 = 0.2$ and $y_0 = 0.8$. From the graphs it follows that the focal length increases with the Pe numbers. For example, if under certain conditions at $Pe = 1000$ the thermal steady-state section equals $140 r_0$ then under the same conditions for $Pe = 250$ it is smaller by a factor of 4. This indicates that for the same lens length, the temperature profile is more stretched in lenses with a smaller Pe number, i.e. this lens is stronger and its focal lengths are shorter. A comparison of figures 10a, b reveals that the focal length is most affected by the Pe numbers in the case of small

input ray ordinates. For the rays with the input ordinate being $y_0 = 0.8 - 1.0$, the Pe number effect on the focal length is insignificant. No Pe number effects are observed at all in gas lenses without the entrance length. This is explained by the fact that a temperature field in the steady-state heat transfer region is insensitive of the Pe number.

There is experimental evidence of such a combination of the operation parameters in thermal gas lenses when the focusing power of a gas lens is practically independent of the gas injection rate (i.e. of either Reynolds or Peclet numbers). This is the so-called *plateau* regime which appears to be a rather valuable operational characteristic of a thermal gas lens since in real regimes, certain gas flow rate variations thus available do not highly affect the optical parameters of a thermal gas lens (see Fig. 11).

FIG. 11. — The optical strength D_0 vs the Pe number at $l/r_0 = 100$. The curves 1-3 correspond to $\Delta t = 0.5$; 1.0; 1.5 °C/cm.

Throughout the consideration, the temperature distribution field in a gas lens has been assumed to have axial symmetry. However, it should be noted that the temperature gradient over cross-section of a gas lens causes buoyancy forces which, in a lens laid horizontal, are normal to the flow direction. As a result, the transverse gas circulation proceeds and the velocity and temperature profiles maxima leave the geometrical axis. There develops a complicated three-dimensional flow picture and, therefore, the refractive index profile in a thermal gas lens acquires an asymmetric pattern (see Fig. 12). This, in turn, causes non-symmetrical aberrations which weaken thermal gas lens characteristics to a great extent.

An effective way of suppressing the negative effect of free convection to give a temperature field symmetrization is a rotation of the gas lens about its longitudinal axis [38]. Here an additional opportunity arises which allows monitoring optical characteristics of a gas lens rotating about its axis. As the experiments have shown, a complete symmetrization of velocity, temperature and refractive index profiles initiates already at low lens rotation rates (see Fig. 13). Thus, at $Re = 10^3$, taken for the axial gas velocity, and $Gr = 10^3$ the density field symmetrization occurs at a rotational Reynolds number of about 100. It is of interest to note that with a diffuser installed the lens rotation causes axial gas flow that makes it

FIG. 12. — The stream functions at $Gr = 700$; $Re = 10^3$; $Pe = 0.73$ (a); 1) 0.000 314; 2) 0.000 58; 3) 0.000 52; 4) 0.001 45 and isotherms (b), the same conditions; 1) 0.27; 2) 0.14; 3) 0.03; 4) 0.14.

possible not to use an external compressor for adjusting constrained gas motion in a gas lens. Certainly, the dynamic character of thermal lens operation imposes additional constraints on temporal stability of thermo-hydrodynamic parameters, controlling optical characteristics of a lens, and allows variation of

FIG. 13. — The isotherms in a rotating thermal gas lens at different swirling numbers. $N_n = 0$ (a); 30 (b); 50 (c).

these characteristics if operational conditions require it.

5. Concluding remarks. — As it is evidenced by the discussion above, the available thermo-optical facilities and methods of their analysis and modeling provide a design of a wide range of optical systems that allow a thermo-gasdynamic control of light beams of different purpose and range of controllable parameters. A further development of thermo-optical methods should involve an account of non-linear optical effects, monochromatic (resonance) and spectral absorption and stimulated radiation as well as a proper choice of a gas or another media actively interacting with a laser light source at various frequencies [39]. So, we may say with a great deal of confidence that researches in the field of thermo-optics and its applications are far from being complete.

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