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recherche français ou étrangers, des laboratoires publics ou privés.
ACOUSTIC WAVE PROPAGATION IN MATERIALS WITH STRAIN DEPENDENT DIELECTRIC CONSTANT

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Abstract. — It is shown that the application of an external temperature gradient across the materials with strain dependent dielectric constant can lead to amplify an acoustic wave even when the drift velocity of the electrons is less than the velocity of sound.

1. Introduction. — A decade ago Gulyaev and Épshtein [1] proposed in their investigation that the propagation of an ultrahypersonic wave in semiconductors leads to the generation of temperature gradient. Recently Sharma and Singh [2] investigated the inverse of the phenomenon proposed by Gulyaev and Épshtein. In their work Sharma and Singh revealed that at large temperature gradient the acoustic wave may be amplified. However, their analysis is applicable only to piezoelectric semiconductors and in the range \( ql \ll 1 \) (\( q \) is the wave number of sound wave and \( l \) is the mean free path of electrons). In the support of thermoelectric amplification of sound wave Épshtein [3] has also developed a theory which is applicable in the range \( ql \gg 1 \).

Pekar [4] and Ogg [5] have pointed out that in the presence of a dc electric field, ferroelectric semiconductors with high and strain dependent dielectric constant (SDDC) can have a much larger effective electron-acoustic wave coupling than piezoelectric semiconductors. It is, therefore, of much interest to explore the effect of temperature gradient on the propagation of acoustic waves in SDDC materials. Following hydrodynamic approach which is valid at low frequencies (\( ql \ll 1 \)) we calculate the loss in the energy of the wave and hence the attenuation coefficient. A distinguished conclusion drawn from the present theory is that the acoustic wave propagating in SDDC materials can be amplified even when the drift velocity (due to dc electric field) of electrons is less than the velocity of sound.

2. Theory. — In a material in which the only coupling between the acoustic wave and electrons is due to the dependence of its dielectric constant on strain, the constitutive relations governing the interaction of the acoustic wave with charge carriers in the presence of the temperature gradient are [2, 5]

\[
T' = cS + \frac{1}{2} \varepsilon g E^2, \quad (1)
\]

\[
D' = \varepsilon E - \varepsilon g SE, \quad (2)
\]

\[
J = neV, \quad (3)
\]
Where $T$ is the stress, $S = \partial u/\partial x$ is the strain, $c$ is the appropriate elastic constant, $\varepsilon$ is the dielectric constant in the absence of any strain, $g = \varepsilon/3$ is the coupling constant [4, 5], $E$ is the electric field, $D'$ is the electric displacement, $J$ is the electronic current density, $e$ is the electronic charge, $V$ is the electronic velocity, $m$ is the effective mass of the electron, $n_0$ is the time-independent part of electron concentration, $k_B$ is the Boltzmann constant, $T$ is temperature, $\nu(T) = n_0 f_1$ is the collision frequency of electrons and $u$ is the elastic displacement. The total electric field can be expressed as

$$E = E_0 + E_1 \exp \{ i (qx - \omega t) \},$$

where $E_0$ is the dc electric field, $E_1$ is the field accompanying the acoustic wave, $q$ is the wave number and $\omega$ is the frequency of the acoustic wave. The electron velocity and concentration can also be expressed in time-independent and time-dependent parts as follows:

$$V = V_0 + V_1 \exp \{ i (qx - \omega t) \},$$
$$n = n_0 + n_1 \exp \{ i (qx - \omega t) \}.$$

Solving eqs. (1) — (8) along with Maxwell's equations for $J_1$ and $E_1$, the $\omega$-dependent parts of the current density and the electric field, respectively, we obtain

$$J_1 = -\frac{\partial \mathbf{D}'}{\partial t} = \frac{\nabla \mathbf{E}_1}{m} - \nu(T) \left( V - \frac{\partial u}{\partial t} \right).$$

$$E_1 = \frac{gE_0}{\varepsilon} \left[ 1 - \frac{V_0}{V_s} + \frac{i \omega T}{\omega D T_0} \right] \frac{\omega \nu_1}{1 - \frac{V_0}{V_s} + i \left( \frac{\omega T}{\omega D T_0} + \frac{\omega}{\omega_c} \right)},$$

where

$$V_0 = \frac{1}{f_T} \left( V_d + \frac{k_B}{m n_0} \frac{\partial T}{\partial x} \right); \quad V_d = \mu E_0, \quad p = f_T \frac{q V_0}{n_0} + i q V_0,$$
$$\omega D = V_d^2 (\mu k_B T_0/e), \quad \omega_c = \frac{n_0 q^2}{\varepsilon} \frac{V_0}{\varepsilon}.$$
Our analysis is applicable in the region of lower frequencies \((q_l \ll 1)\) and, therefore, the approximation \(\nu (T) \gg \omega\) is justifiable. In this condition \(R_1, R_2, R_3\) and \(R_4\) reduce to

\[
R_1 = \frac{1}{f_T} \left( 1 - \frac{V_s}{V_o} \right),
\]

\[
R_2 = \frac{(\omega - V_s \partial_t) \omega T}{\nu_0 f_T^2 \omega_0 T_0} \ll R_1,
\]

\[
R_3 = \left( 1 - \frac{V_o}{V_s} \right)^2,
\]

and

\[
R_4 = \frac{1}{f_T^2} \left( \frac{\omega T}{\omega_0 T_0} + \frac{\omega_0}{\omega} \right)^2,
\]

respectively. Thus, the simplified expression for the attenuation coefficient \(\alpha\) can be written as

\[
\alpha = \frac{\varepsilon_0 \omega_c f_T}{2 f_s V_s^2} \left[ f_T^2 \left( 1 - \frac{V_o}{V_s} \right)^2 + \left( \frac{\omega T}{\omega_0 T_0} + \frac{\omega_0}{\omega} \right)^2 \right].
\]

This is the general expression which is applicable to any material with strain dependent dielectric constant. It is obvious that when there is no temperature gradient (i.e., \(T = T_0\) and \(f_i = 1\)), our expression for attenuation coefficient reduces to that of Ogg [5].

3. Discussion. — We notice from eq. (16) that the presence of a dc electric field is necessary for the attenuation or amplification of acoustic waves in SDDC materials and that the condition to switch over from attenuation to amplification is \(V_o = V_s\), i.e., when

\[
\frac{V_o}{V_s} = f_T - \frac{k_0 \mu}{e \nu} \left| \frac{\partial T}{\partial x} \right|.
\]

For materials whose Curie temperature is very low, e.g., \(\text{KTaO}_3, \text{SrTiO}_3\) etc., the operating conditions will also be at low temperatures. In this region ionized impurity scattering will be the dominant scattering mechanism for which \(f_T = (T/T_o)^{-3/2}\) is less than one. Therefore, in case of such materials the threshold drift velocity \(V_d\) (due to dc field) of electrons will always be smaller than the velocity of sound \(V_s\). For the numerical appreciation of results, we consider a 0.2 mm thick specimen of \(\text{SrTiO}_3\) with \(V_s = 3.4 \times 10^5\) cm/s and \(\mu\) (at 20 K) = \(10^4\) cm²/V.s. If the two faces of the specimen are at 20 K and 77 K, temperatures, respectively, it is found from eq. (17) that the amplification of the acoustic wave will occur for all values of dc field corresponding to \(V_d \geq 0.125 V_s\).

But, for materials which have Curie temperature quite high, e.g., \(\text{BaTiO}_3, \text{KTa}_{0.65}\text{Nb}_{0.35}\text{O}_3\) etc., the value of \(f_T\) is one [7]. Again the threshold drift velocity is smaller than the velocity of sound because of the second term on the right of eq. (17) which is due to the presence of temperature gradient. Therefore, for most of the materials with strain dependent dielectric constant the attenuation of an acoustic wave crosses over to its amplification at the drift field corresponding to which the drift velocity of electrons is smaller than the velocity of sound, when a temperature gradient is applied. Furthermore, as the temperature gradient is increased, the amplification occurs at relatively low values of the dc electric field.

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References