PHYSICAL INTERPRETATION OF THE cos φ TERM AND IMPLICATIONS FOR DETECTORS (*)

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Résumé. — Des calculs théoriques du courant total traversant une jonction entre deux supraconducteurs, fondés sur l'Hamiltonien conventionnel pour l'effet tunnel, font ressortir 3 termes : le supracourant Josephson ou courant de paires, le courant de quasiparticules et le courant d'interférence paires-quasiparticules, ou courant en « cos φ ». Les théories existantes, quand elles se recoupent, prédisent des signes pour les courants de quasiparticules et d'interférence qui sont en contradiction avec les résultats d'expériences récentes mises en œuvre pour démontrer l'existence et les propriétés de ces derniers. La contradiction n'est donc pas levée. On ne doit pas s'attendre à ce que le courant en « cos φ » (quel que soit son signe) engendre des phénomènes nouveaux et remarquables dans les dispositifs Josephson, il peut toutefois affecter quantitativement le comportement dynamique et le bruit de ces dispositifs de diverses façons.

Abstract. — Theoretical calculations of the total current in a superconductor-barrier-superconductor device based on the conventional tunneling Hamiltonian yield three terms, conventionally termed the Josephson supercurrent or pair current, the quasiparticle current, and the quasiparticle-pair-interference or « cos φ » current. When reconciled, existing theories predict relative signs of the quasiparticle and quasiparticle-pair currents which are opposite to the results of recent experiments which purport to demonstrate the existence and properties of the latter current. This discrepancy remains unresolved. The cos φ current (whatever its sign) is not expected to give rise to qualitatively new and striking phenomena in Josephson devices, but may quantitatively affect the dynamical and fluctuation behavior of these devices in a variety of ways.

Josephson's original calculation [1] of the current between two superconductors coupled by the tunneling Hamiltonian yielded three terms or components, which he later expressed in the form [2]

$$J = J_1(V) \sin \varphi + \left[\sigma_0(V) + \sigma_1(V) \cos \varphi\right] V . \quad (1)$$

The first term is the famous phase-dependent Josephson supercurrent. The second is the « normal » or quasiparticle current first studied by Giaever. The third term is variously referred to as the « cos φ » term or the quasiparticle-pair (interference) term. This term has until recently received little attention, either theoretically or experimentally. Its known experimental manifestations are rather subtle, and it has been easy to overlook in the shadows thrown by the brilliant glow of the Josephson supercurrent. However, it is now clear that it must be taken into account in any complete picture of the behavior of weakly-coupled superconductors. It is not so clear whether it plays an *important* role in the behavior of any of the potentially useful Josephson devices, e. g., the detectors on which this conference is focused. I shall attempt to describe here the current state of our theoretical and experimental understanding of the cos φ term and then indicate some of the questions which remain to be answered, particularly with respect to its role in the operation of high frequency detectors.

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The theory of the tunnel current between two superconductors separated by an insulating barrier has been presented by many authors, beginning with Josephson. I would like to outline this theory in order to set the stage for some subsequent comments.

The starting point is the situation pictured in figure 1, two superconductors separated by a barrier. States in the L(eft) superconductor are labeled with wave number k and spin index s, states in the R(ight) superconductor are correspondingly labeled with q and s. A potential difference V is assumed to exist between the two superconductors. The Hamiltonian of the system is taken to be

$$\mathcal{H} = \mathcal{H}_{\mathrm{L}} + \mathcal{H}_{\mathrm{R}} + \mathcal{H}_{\mathrm{T}} , \quad (2)$$

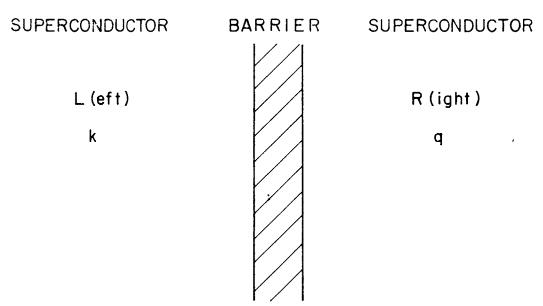

FIG. 1. — Model used in calculating the tunnel current.

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where $\mathcal{H}_L$ and $\mathcal{H}_R$ are the full many-body Hamiltonians for the left and right superconductors. (In practice, these are usually replaced by the appropriate BCS reduced Hamiltonians.) $\mathcal{H}_T$ is the tunneling Hamiltonian of Bardeen and Cohen, Falicov, and Phillips,

$$\mathcal{H}_T = \sum_{kqs} (T_{kq}\, c_{ks}^+\, c_{qs} + T_{kq}^*\, c_{qs}^+\, c_{ks}) . \tag{3}$$

The objective is then to calculate to leading (second) order in the tunneling matrix element T_{kq} the total tunneling current. This can be done by a variety of techniques. However it is done, it is important to take into account properly the phase coherence of the superconducting pair state and the associated time dependence. It is this essential feature which Josephson was the first to recognize, and which yields the phase-dependent terms in the tunnel current, most notably the Josephson supercurrent.

The result of such a calculation is

$$I = I_p + I_{qp} + I_{qpp} ,$$

where

$$I_p = -\frac{4\,\mathrm{e}}{\hbar} \sin\varphi \sum_{kq} |\,T_{kq}\,|^2\, u_k\, v_k\, u_q\, v_q \left\{ \begin{aligned} &(f_q - f_k)\left[\frac{\mathbf{P}}{E_q - E_k + \mathrm{e}V} + \frac{\mathbf{P}}{E_q - E_k - \mathrm{e}V}\right] \\ &+ (1 - f_q - f_k)\left[\frac{\mathbf{P}}{E_k + E_q + \mathrm{e}V} + \frac{\mathbf{P}}{E_k + E_q - \mathrm{e}V}\right] \end{aligned} \right\} \tag{4}$$

$$I_{qp} = \frac{4\,\pi\mathrm{e}}{\hbar} \sum_{kq} |\,T_{kq}\,|^2 \left\{ \begin{aligned} &(f_q - f_k)\begin{bmatrix} u_k^2\, u_q^2\, \delta(E_q - E_k + \mathrm{e}V) \\ -\, v_k^2\, v_q^2\, \delta(E_q - E_k - \mathrm{e}V) \end{bmatrix} \\ &+ (1 - f_q - f_k)\begin{bmatrix} -\, u_q^2\, v_k^2\, \delta(E_k + E_q + \mathrm{e}V) \\ +\, u_k^2\, v_q^2\, \delta(E_k + E_q - \mathrm{e}V) \end{bmatrix} \end{aligned} \right\} \tag{5}$$

$$I_{qpp} = \frac{4\,\pi\mathrm{e}}{\hbar} \cos\varphi \sum_{kq} |\,T_{kq}\,|^2\, u_k\, v_k\, u_q\, v_q \left\{ \begin{aligned} &(f_q - f_k)\begin{bmatrix} \delta(E_q - E_k + \mathrm{e}V) \\ -\, \delta(E_q - E_k - \mathrm{e}V) \end{bmatrix} \\ &+ (1 - f_q - f_k)\begin{bmatrix} \delta(E_q + E_k + \mathrm{e}V) \\ -\, \delta(E_q + E_k - \mathrm{e}V) \end{bmatrix} \end{aligned} \right\} . \tag{6}$$

Here, u_k and v_k are the BCS coherence parameters, $E_k = (\varepsilon_k^2 + \Delta_k^2)^{1/2}$ is the quasiparticle excitation energy, $f_k = (\mathrm{e}^{\beta E_k} + 1)^{-1}$ and $\mathbf{P} \equiv$ « principal part ». Since, for reasons which will shortly become apparent, I am particularly concerned about the signs in these expressions for the current components, I would like to be explicit about the following definitions : (1) The potential difference or voltage $V \equiv V_L - V_R$; (2) The electron charge has been taken as $-$ e, so $\mathrm{e} = |\,\mathrm{e}\,|$ in the above equations ; (3) The current has been calculated from $I \equiv -\,\mathrm{e}(\mathrm{d}/\mathrm{d}t) < N_R >$, where N_R is the total particle number on the right. The positive direction of current flow is therefore from left to right ; (4) The pair phase and electrochemical potential differences have been defined by $\varphi \equiv \varphi_L - \varphi_R$ and $\mu \equiv \mu_L - \mu_R$. It follows that the time dependence of the pair phase difference is given by

$$\dot{\varphi} = 2\,\mu/\hbar = -\,2\,\mathrm{e}V/\hbar .$$

The sums in eq. (4)-(6) can be converted to integrals and evaluated in the usual fashion. The (at least partly) familiar results are sketched in figure 2 for the case where the two superconductors are identical. (It is interesting to note that, although more than a decade has elapsed since Josephson's original paper, the complete dependences of the tunnel current amplitudes on voltage and temperature for both identical and nonidentical superconductors have been evaluated only recently [3], [4], [5]). It will be noted that the supercurrent amplitude (eq. (4) with the $\sin\varphi$ factor deleted) is the negative of the amplitude usually presented. A reversal of the sign of φ would restore the conventional sign here and also convert the equation for the time dependence of the pair phase to the conventional $\dot{\varphi} = +\,2\,\mathrm{e}V/\hbar$. However, such a sign reversal would leave unchanged the form of the quasiparticle-pair amplitude (eq. (6) with the $\cos\varphi$ factor deleted) : This is positive below the gap voltage and negative above.

What can be said about the physical origin of the $\cos\varphi$ term on the basis of such a calculation ? Suppose we were to set out to calculate the quasiparticle tunnel current, and suppose we began by asking the question, « What are the processes by which we can add a quasiparticle to the left superconductor in state k ? » There are four such possible processes ; they are illustrated in figure 3 (V assumed positive). The simplest is the direct transfer of a quasiparticle from state q on the right to state k on the left (A). The requirement of energy conservation associates the delta function $\delta(E_q - E_k + \mathrm{e}V)$ with this process. This process and its reverse combine to yield the contribution to the current given by the first term

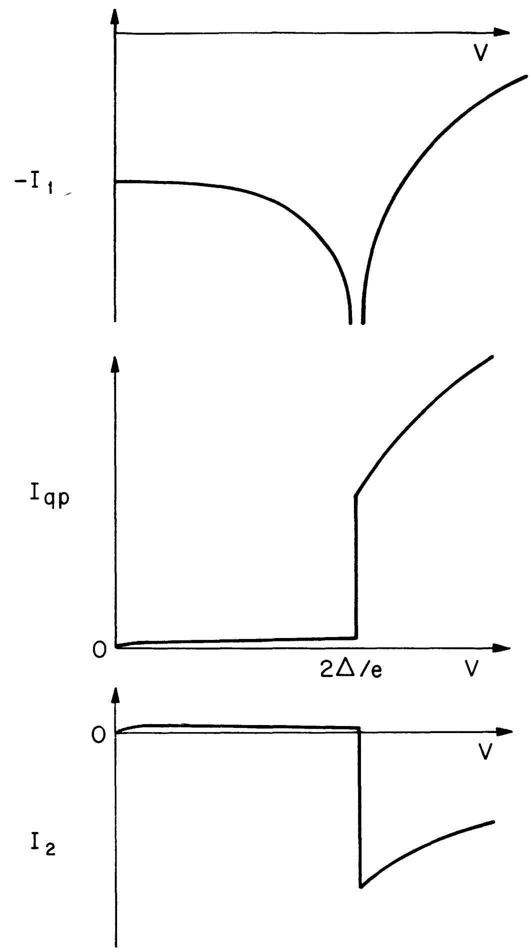

FIG. 2. — Amplitudes of the tunnel current components. The current is written in the from $I = -I_1 \sin \varphi + I_{qp} + I_2 \cos \varphi$. Theory gives a negative amplitude for the sin φ term; this has been written as $-I_1$ to conform with common usage of the symbol I_1.

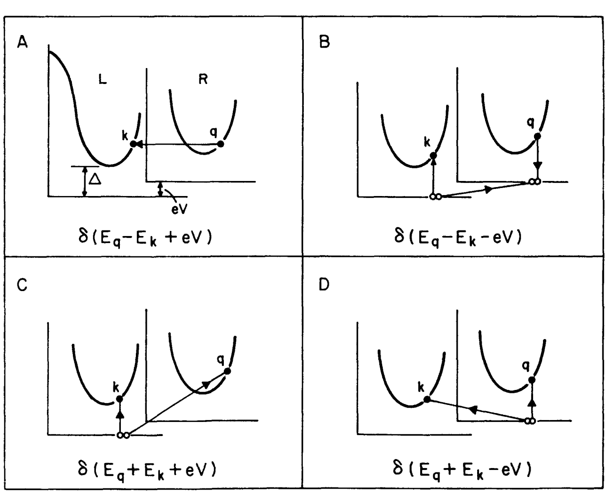

FIG. 3. — Processes contributing to the quasiparticle current.

in eq. (5). It vanishes at $T = 0$, but for $T > 0$ contributes for all (positive) voltages. We can also get a quasiparticle in k by breaking a pair on the left, exciting one of its members to k, and tunneling the other to the right, where it may either recombine with a thermally excited quasiparticle to form a pair (B), or it may go into a quasiparticle state q (C). Energy conservation gives us the indicated delta functions, and these processes lead to the second and third terms in eq. (5). Like process A, process B contributes for all positive V provided $T > 0$. Process C contributes for no positive V. Finally, we can get a quasiparticle in k by breaking a pair on the right, tunneling one member to k and promoting the other to q(D). This process contributes even at $T = 0$, provided $eV > 2\Delta$, and is responsible for the familiar sharp jump in the quasiparticle current at the « gap voltage ».

Now note that three of these processes involve pairs and hence the superconducting condensed state. This suggests that the phase coherence of the condensed state might play some role in a complete theory of the quasiparticle current, and so it does. Terms appear in the calculation we have outlined which correspond to each of the four processes with a *pair* transfer superimposed. Consider for example process A: If we add a pair transfer from left to right, the combined process has the same initial and final configurations as process B, and energy conservation gives the same delta function as in process B. Similarly, superimposing a pair transfer makes B look like A, C like D, and D like C. We thus see that there is another way of viewing these quasiparticle current processes which involves a pair transfer in each one. Since pair transfer must depend on the pair-phase difference across the barrier, we should expect a phase-dependent contribution to the quasiparticle current. This is the cos φ term. We also see that the cos φ term should not be regarded as a separate (and perhaps mysterious) part of the tunnel current, but, in Josephson's own words [1], « ... can be regarded as fluctuations in the normal current due to coherence effects ». I would prefer « modulation » to « fluctuation », but Josephson's characterization (as usual) hits the right nail on the head. I would also like to underscore the aptness of Libchaber's view [6] that the phase-dependence of the quasiparticle current implies a breakdown of the « two-fluid » model we have all been using in our descriptions of Josephson phenomena.

What is the experimental situation *vis-à-vis* the cos φ term ? Experimental confirmation of its existence was a long time coming. Dahm *et al.* [7] first suggested that it might be observed by studying the damping of the plasma resonance in Josephson tunnel junctions. The experiment was subsequently carried out by Pedersen, Finnegan, and Langenberg [8], [9]. In essence, it involves driving a tunnel junction biased in the zero-dc-voltage mode with a small microwave signal at frequency ω. The phase φ can execute small amplitude ($\varphi \ll 1$) oscillations about an equilibrium value φ_0 which correspond to plasma-like longitudinal pair charge oscillations across the junction or, in the conventional pendulum model, simple harmonic oscillations of the pendulum. The characteristic frequency of this mode is $\omega_p = \omega_J(\cos \varphi_0)^{1/2}$, where

$$\omega_J^2 = 2\,eI_1/\hbar C\,,$$

I_1 is the junction critical current, and C is the junction capacitance. When $\omega_p = \omega$, the junction responds resonantly. The width or Q of this resonance is dominated by damping due to the quasiparticle current. If this current is approximated for small voltages by $(V/R)\,(1 + \zeta \cos \varphi)$, then in the small amplitude limit, the Q is given by

$$Q^{-1} = \frac{1}{\omega_p\,RC}\,(1 + \zeta \cos \varphi_0) \qquad (7)$$

so that a plot of experimental values for $\omega_p\,Q^{-1}$ should reveal the presence or absence of the cos φ term provided ζ is not too small. Pedersen *et al.* found $\zeta = -\,0.9 \pm 0.2$ [9]. Calculations by Poulsen [3] show that this small-voltage approximation is applicable despite the rather singular behavior of the theoretical quasiparticle and quasiparticle-pair current amplitudes near $V = 0$ (they vary as $V \ln\,(kT/eV)$ for small voltage) and predict $|\,\zeta\,| = 0.93$ for the experimental conditions of Pedersen *et al.* The experimental result is thus seen to be in agreement with theory as regards magnitude and, it was initially thought [8], [9], sign. *However*, Scalapino, Schreiffer, and *I* [10] have carefully re-analyzed the theory and find that, as noted above, the theoretical quasiparticle and quasiparticle-pair amplitudes have the same sign for small voltages, and hence the theory predicts ζ to be *positive* ! One may inquire whether all of the many versions of the theory which have appeared in the literature concur in this conclusion. This question is rather difficult to answer because the phase-dependent part of the quasiparticle current has rarely been explicitly displayed in an unambiguous manner. However, « archaeological » investigations by us [10] and by Harris [5] indicate that, with several exceptions where sign errors or ambiguities appear to have occurred, there *is* a theoretical consensus that ζ is positive, in contradiction with the experimental results of Pedersen *et al.*

Our quandary is further deepened by two recent developments. Falco, Parker, and Trullinger [11] have examined the current-voltage characteristics of thin-film weak links under conditions where the zero-voltage critical current is substantially depressed or suppressed by noise. They find that quantitative interpretation of the characteristics requires a cos φ term and that the ζ parameter is *negative* and has a magnitude consistent with that found by Pedersen *et al.* One might question whether a cos φ term similar to that in tunnel junctions is to be expected at all in weak links, where the basic physical processes are presumably quite different from simple tunneling. (My personal opinion is that the tunneling Hamiltonian may very well provide a reasonable description of a weak link, with the differences in basic mechanism buried in the « tunneling » matrix element in a way which does not affect the qualitative features of the result.) In any event, this experiment appears to confirm that such a term does exist and that its sign and magnitude are like those observed in tunnel junctions.

Tinkham and Beasley [12] have developed a quite general argument based on the Kramers-Kronig relations which appears to show that the sign must be negative.

We are thus faced with an apparent disagreement between the accepted microscopic theory and experiment plus general theoretical arguments for the sign of the cos φ term. This is rather disturbing, particularly so because we are accustomed to finding that experiment confirms the theoretical predictions of Josephson. Further study is clearly called for.

What effect can the cos φ term be expected to have on the behavior of Josephson devices, particularly detectors ? I feel somewhat uneasy about discussing this question in view of the sign discrepancy problem. (A quotation from Epictetus seems apt here : « No man is able to make progress when he is wavering between opposite things. ») However, some general remarks can be made. First of all, it is almost self evident that the effects will be rather subtle : if the cos φ term gave rise to large and striking effects, it would have forced itself on the attention of experimentalists long ago. However, I believe its effects *will* appear quite generally among the details of the dynamics of Josephson devices in a variety of circumstances, including rf detection and mixing experiments. This is already apparent in existing work : Harris [5] has pointed out that Werthamer's theory [13] predicts that, for the case of an rf voltage source, the cos φ term has no effect whatever on the amplitudes of rf-induced (Shapiro) steps, because the amplitude of the cos φ term is odd under change of sign of voltage. Auracher, Richards, and Rochlin [14] have carried out numerical calculations for the standard shunted-junction model containing a Josephson element shunted by an ohmic resistor and biased from a current source, but with a cos φ term added to the shunt conductance (as in eq. (7) above). They find the current-voltage characteristic is unaffected by the cos φ term. However, when a shunt capacitance is added to the model, the cos φ term does have an effect (unfortunately, not one easily distinguishable experimentally). When an rf current bias is added, the cos φ term has small effects on the amplitude and power dependence of the rf-induced steps.

There have been a number of recent papers on the rf behavior of Josephson devices, including analytical and numerical calculations of the small and large signal impedance for the shunted-junction model [15]. None have yet included the cos φ term, an omission which probably ought to be rectified. The analysis of our own plasma resonance experiments has been based on a calculation of the small-signal impedance for the shunted-junction model in the zero-dc-voltage mode, and here the cos φ term certainly has an observable effect. Figures 4 and 5 show the real and imaginary parts of this impedance with the shunt capacitance removed from the model (thus eliminating the plasma resonance). The curves including the

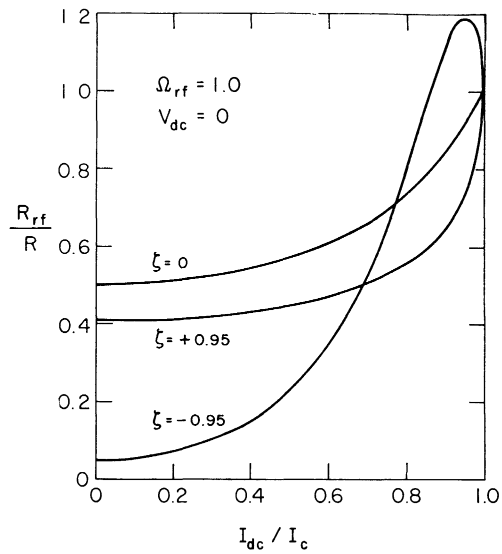

FIG. 4. — Real part of the small-signal rf impedance for zero dc voltage, calculated in the same manner as by Auracher and van Duzer (ref. [15]), but including the cos φ term (with both signs).

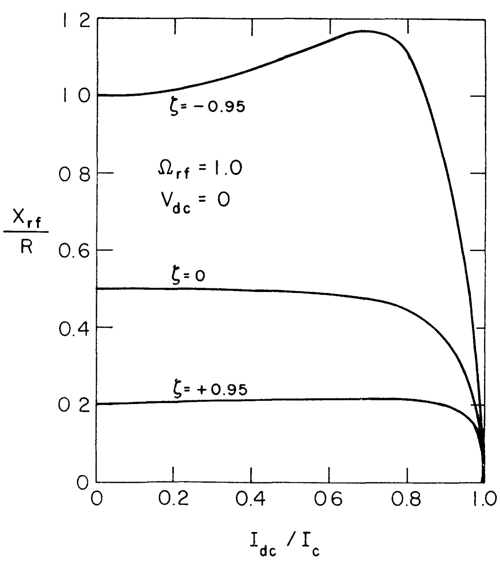

FIG. 5. — Imaginary part of the small — signal rf impedance for zero dc voltage, calculated in the same manner as by Auracher and van Duzer (ref. [15]), but including the cos φ term (with both signs).

effect of the cos φ term (for both signs of ζ) are quite different from the corresponding curves of Auracher and van Duzer [15] who omitted the cos φ term. Although this zero-voltage impedance is not particularly interesting for practical applications, the results suggest that the extension of the calculations for non-zero dc voltage to include the cos φ term might turn up substantial effects. It may also be worth noting that when our plasma resonance impedance calculations are extended away from the small signal limit, the frequency and Q of the plasma resonance appear to become different for the two directions of dc bias current, through terms arising from the cos φ term. One may speculate that similar asymmetries may also arise in other rf properties away from the small signal limit [16].

Another type of dynamical phenomenon in which the cos φ term might be expected to manifest itself is in fluctuations in Josephson devices. Indeed, it has already done so in the experiments of Falco *et al.* on current voltage characteristics of weak links in the presence of noise. Here again, the effects are likely to be more a matter of factors of two or so in quantitative details rather than qualitative differences, but these may become important when we approach the problem of wringing the last bit of performance out of Josephson detectors and other devices. An example of the « factor-of-two » effect may already have arisen in the work of Dahm *et al.* [17] on the line width of the radiation emitted by a Josephson tunnel junction. An inspection of their results shows an apparent discrepancy between theory and experiment which might possibly be due to the omission from the theory of a noise contribution from the cos φ term.

To summarize my own views then, the cos φ term is, like everything else predicted in Josephson's remarkable Letter, real. It can be viewed as a phase modulation of the quasiparticle current caused by « interference » between the quasiparticle and phase-dependent pair currents, and it probably occurs quite generally in systems of weakly coupled superconductors. At the present time there is an embarrassing discrepancy between the theoretically predicted and experimentally observed signs. This may be due to some trivial but subtle error on one side or the other, or to some fundamental conceptual difficulty ; time will tell. The role of the cos φ term in future work on Josephson phenomena and devices promises to be a little like that of a pin in one's mattress : It won't affect the basic functioning of the device much. With care and/or luck, it can probably be ignored. But we won't really feel safe until we known how to keep track of where it is. It would be particularly nice to find out whether it points up or down. Unfortunately (and here the analogy fails) we can't remove it.

Acknowledgments. — I am grateful to many of my colleagues for their efforts (both successful and unsuccessful) to lessen my confusion about the cos φ term. Particularly persistent were R. E. Harris, D. J. Scalapino, and J. R. Schreiffer.

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