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COUPLED JUNCTIONS AND PARAMETRIC AMPLIFICATION

INTERACTIONS IN SMALL SYSTEMS
OF COUPLED JOSEPHSON JUNCTIONS
AT MICROWAVE FREQUENCIES (*)

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Abstract. — A major factor in the practical utilization of any Josephson device for microwave and higher frequency applications (such as the generation and detection of electromagnetic radiation) is the success with which the junction device can be coupled to an external transmission line. As part of an experimental investigation of the electrodynamics of small arrays of Josephson tunnel junctions, the properties of the coherent radiation emitted both by individual junctions and by coupled junction systems have been studied at frequencies between 2 and 12 GHz. Near 9 GHz, more than $10^{-9}$ W of coherent radiation has been detected from a single Pb-Pb oxide-Pb junction coupled via waveguide. The development of thin film devices in which the tunnel junctions form part of a microstrip line have made it feasible to observe directly (in a single device) the radiation emitted both at the fundamental geometrical frequency and at the low order harmonics. The observed temperature dependence of the junction cavity $Q$ indicate the low temperature losses are primarily due to cavity loading of the external transmission line.

1. Introduction. — The possibility of using thin-film Josephson tunnel junctions as generators and detectors of electromagnetic radiation at microwave and submillimetre frequencies has been recognized for some time [1]. However, the poor impedance matching generally attained between the junction and the external transmission line to which it is coupled (usually a waveguide structure) has limited the practicality of these applications. Recently, motivated in part by the work of Clark [2] and Tilley [3] on the coherent radiative properties of large arrays of coupled Josephson junctions, Finnegan and Wahlsten [4] have directly observed a superradiant state in pairs of coupled resonant tunnel junctions in which the emitted radiation from the two junctions added coherently.

In this paper we present the results of additional experiments on coupled junction systems and of some new experiments on Josephson devices in which the junctions are directly coupled to a microstrip line circuit. The latter experiments have helped to clarify the relevant electromagnetic properties of a Josephson tunnel junction when used as a cavity or stripline element and as an oscillator.

2. Theory. — A long tunnel junction may be regarded as an open-ended section of transmission line having electromagnetic resonances at frequencies

$$v_n = \frac{n\pi v}{L}$$

(1)

where $v$ is the phase velocity in the dielectric barrier, $L$ is the junction length, and $n$ is an integer. In general $v$ is both temperature and frequency dependent. The

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capacitance $c$, inductance $l$, and resistance $r$ per unit length of the junction stripline can be readily derived via the Maxwell-London equations and are, respectively,

$$c = \frac{\varepsilon_0 \varepsilon_s \ell}{\ell}$$  \hspace{1cm} (2a)$$

$$l = \frac{\mu_0(2 \lambda + \ell)}{\ell}$$  \hspace{1cm} (2b)$$

$$r = \frac{2 R_s}{\ell}.$$  \hspace{1cm} (2c)$$

Here $\varepsilon_s$ is the relative dielectric constant of the barrier layer; $\ell$ is the junction width; $\lambda$ is the London penetration depth; and $R_s$ is the surface impedance of the superconducting films assumed to be identical. If the junction cavity losses are small, the characteristic impedance $Z_J$ of the junction transmission line operating in the transverse electromagnetic (TEM) mode is

$$Z_J = \sqrt{\frac{l}{c}} = \frac{1}{\nu c}.$$  \hspace{1cm} (3)$$

and is independent of frequency. For typical junction parameters ($\nu = 1.5 \times 10^7$ m/s, $L_w = 0.10 \times 0.03$ cm and $C = 10$ nF), $Z_J \approx 0.01$ $\Omega$. If the junction stripline is directly coupled to an external transmission line with characteristic impedance $Z_e$, the transmission coefficient $T$ between the two lines is [5]

$$T = \frac{4 Z_J Z_e}{(Z_J + Z_e)^2}.$$  \hspace{1cm} (4)$$

For $Z_J \ll Z_e$, $T \approx 4 Z_J/Z_e$. When $Z_e$ is the impedance of free space ($\sim 377$ $\Omega$), $T$ is about $10^{-4}$.

The quality factor $Q$ of the junction cavity will include contributions from the following sources: (1) the surface impedance of the superconducting films, (2) dielectric losses in the barrier region, and (3) radiation losses so that the resultant junction $Q$ will be

$$Q^{-1} = Q_s^{-1} + Q_d^{-1} + Q_r^{-1}.$$  \hspace{1cm} (5)$$

These loss terms are respectively; $Q_s = \omega l/\nu$, $Q_d = \omega c/g$ where $g$ is the shunt conductance of the junction dielectric barrier, and $Q_r \approx 1/T$. $Q_s$ can be calculated using the results of Mattis and Bardeen [6] for the surface impedance $R_s$. The dielectric loss factors in thin oxides (e. g. those of lead, tin, and niobium) at low temperatures are not known, however, $Q_d$ is expected to be independent of temperature in the region of interest. The radiation losses will significantly contribute to the total $Q$ at low temperatures where the temperature-dependent surface impedance losses are negligible.

Experiments involving photon-assisted tunneling [7], [8], microwave-induced Josephson steps [9], and various self-induced step phenomena [10], [11] associated with radiation emission can be used to probe the electrodynamics of externally-coupled resonant tunnel junctions. For example, the frequency response of the microwave induced steps can be used to determine the temperature-dependent center frequency and $Q$ of the junction cavity. A detailed understanding of the self-induced step phenomena in coherently-radiating tunnel junctions has yet to be achieved. The current-voltage ($I$-$V$) characteristic of such a junction exhibits current singularities or steps at voltages $V_n = n h v_n / 2 e$ where $v_n$ is the fundamental geometrical resonant frequency, and it is only when deep biased on such steps that detectable amounts of coherent radiation are observed. The intrinsic linewidth of this radiation is usually quite narrow, i. e. $\ll 1$ part in $10^7$ at 10 GHz. Theoretically, expressions for the linewidth have been solved for the case in which only the fundamental cavity mode is excited [12], [13]. However, it has never been established experimentally that this assumption is valid for physically realizable systems.

The Josephson junction oscillator is treated here phenomenologically. The step structure and associated radiation emission result from the non-linear interaction between the Josephson supercurrent and the electromagnetic fields excited in the junction cavity. Inasmuch as the junctions are non-linear oscillators, the occurrence of frequency-pulling and locking phenomena when two or more such oscillators are coupled to one another is not surprising. The most dramatic characteristic of the locking-phenomena in pairs of coupled junctions is the existence of two stable locked-configuration [4]. In the first, the radiation from the two junctions is observed to add coherently in-phase while in the second, the radiation is observed to add exactly out-of-phase (i. e. with a phase shift of $\pi$). The aim of the present work (which is still in progress) has been directed toward better characterizing the coupled junction interactions by investigating various device geometries, the junction cavity parameters, and the harmonic content of the emitted radiation.

3. Experiment. — 3.1 Josephson Devices. — A variety of Josephson device geometries were fabricated and studied and several are shown in figure 1. All devices considered here were made by sequentially depositing $\sim 2,000$ Å thick lead films on $2.54 \times 2.54 \times 0.12$ cm glass substrates and thermally oxidizing the desired films. Figures 1a and 1b show the two possible orientations of the classic rectangular in-line geometry. The longest in-line junction we studied was $3.84 \times 0.08$ mm and had a fundamental resonance frequency of about 2.4 GHz. The in-line geometry is particularly simple and is ideally suited for use in microstrip line circuits when mounted above a ground plane.

The two junction device shown in figure 1c was designed specifically for coupled junction experiments. For these experiments, it was necessary that the fundamental resonant frequencies of the two junctions...
be equal. In addition, with this geometry it was possible to break the superconducting series connection between the two junctions and completely isolate the two dc bias circuits. Typical junction dimensions in this case were 0.9 x 0.1 mm. The distance between junctions was about 1 mm.

The device shown in figure 1d was designed as a prototype junction-microstrip line element with (1) relatively good coupling of the emitted radiation to an external 50 Ω cable and (2) a geometry which could be readily modified to include several identical junctions. The wide strip at the bottom of figure 1d is not part of the junction structure itself, but is designed to form a 50 Ω microstrip transmission line when the substrate is mounted directly on a ground plane. A few double-junction devices of the type shown in figure 2 have also been successfully fabricated. The microstrip line is capacitively-coupled to the junctions by means of fine scratches (1 to 10 μm wide) which are made after all the thin-film depositions have been completed. A typical pattern of scratches is indicated in the insert of figure 2.

Additional experiments have also been performed on four-junction devices of the type used in the initial coupled junction work [4]. The dimensions of these junctions were approximately 1.0 x 0.3 mm.

3.2 WAVEGUIDE COUPLING. — The usual method of coupling to tunnel junction devices at microwave frequencies has been to mount the device in a section of waveguide and use an adjustable sliding short to maximize the coupling to the external microwave system. Interactions in two-junction (Fig. 1c) and four-junction devices were investigated at X-band (8.0-12.4 GHz) frequencies by mounting the device substrates across the broad wall of an X-band waveguide so that the junctions were near the center of the waveguide. Unfortunately, the observed microwave properties of devices mounted in this way were strongly influenced by unwanted resonances associated with the dc bias lead configuration, the film strip lengths, and the waveguide holder dimensions. Although these additional resonances could be identified since they were temperature-independent, they made experimental determination of the intrinsic junction cavity response extremely difficult. Thus, in part to minimize these problems and achieve more uniform external coupling, elementary microstrip line coupling techniques were investigated.

3.3 MICROSTRIP LINE COUPLING. — A thin-film tunnel junction device can be readily integrated into a conventional microstrip transmission line by mounting the entire device on a ground plane so that the substrate itself forms the dielectric insulator. In the system we have used, each microstrip line is coupled to an external 50 Ω coaxial line (either miniature semi-rigid or flexible cable) via a 4 mm (OSM) connector to stripline launcher. The main features of the device holder are indicated in figure 3, and a Josephson device is shown mounted in place. Two devices could be simultaneously mounted on opposite sides of the copper block. When the thickness and dielectric constant of the glass substrate were taken into account, a film width of 1.8 mm was required for a 50 Ω microstrip line.

Devices with the various geometries shown in figure 1 were tested in this holder. The response of
FIG. 3. — Cryogenic holder for microstrip line coupled Josephson devices. The semi-rigid coax cable is of stainless steel with a copper plated center conductor and is 86 cm long. The dc bias wires are attached to the four contact points shown as solder blobs.

Each of the devices to externally applied radiation (coupled via the microstrip line formed by the wide film) was relatively good and essentially independent of the positioning of the external microstrip line relative to the two junction dimensions. In radiation emission experiments, on the other hand, relatively good coupling was only achieved when the external microstrip line and the junction stripline were colinear, i.e., the device geometries shown in figures 1a and 1d.

The broadband frequency response of the holder was readily demonstrated by detection of the radiation emitted by a single in-line junction near its fundamental resonance frequency at 2.4 GHz, and near its second and third harmonics.

For the Josephson device shown in figure 3, $Z_j \approx 0.025 \Omega$, $Z_e = 50 \Omega$, and the transmission coefficient $T$ is approximately $2 \times 10^{-3}$. (Since the characteristic impedance of the external microstrip line formed by the narrow film is much greater than 50 $\Omega$, the junction coupling to it is much less, and it has therefore been neglected in this analysis.) The radiated power, $P_r$, actually observed from junctions with this geometry was in surprisingly good agreement with that estimated assuming $P_r = TP_a$ where $P_a$ is the total available power, i.e., $P_a = I_{dc}V_{dc}$ for low order self-induced steps.

3.4 MICROWAVE EQUIPMENT. — The coherent radiation emitted by the Josephson device was detected with a broadband (2-12 GHz) superheterodyne receiver which has been described elsewhere [4]. When a swept backward wave oscillator (BWO) was used as the local oscillator, the video output of the intermediate frequency (IF) amplifier was displayed with an oscilloscope. For more precise measurements, the 30 MHz IF output of the receiver was directly observed with a spectrum analyzer. For linewidth measurements, a phase-locked klystron was used as the local oscillator. In all experiments, but especially for those involving the detection of radiation at frequencies $\nu$ and $2\nu$ from the same device, care was taken to avoid any spurious responses which might occur due to unwanted coupling of harmonics or subharmonics between the receiver mixer, the local oscillator, and the Josephson device. For example, when detecting at $\nu$, low pass cutoff filters were used to remove signals at $2\nu$ whereas when detecting at $2\nu$, both high pass and low pass filters were used to remove the signals at $\nu$ and $3\nu$. The resolution of the system as normally operated was better than $10^{-13}$ W and was calibrated between 3.8 and 11.0 GHz with standard microwave test sets.

For junction cavity $Q$ measurements via the microwave induced steps, the BWO was used as the frequency source and operated in the continuous wave mode with the microwave output leveled externally. A precision attenuator was used to determine relative power.

4. RESULTS. — 4.1 INTERACTING JOSEPHSON OSCILLATORS. — When mounted in the waveguide holder, all coupled junction systems (in which pairs of junctions could be dc biased to radiate at and near the same frequency) exhibited frequency-pulling and locking phenomena. Most of our radiation emission experiments were carried out at temperatures near 4 K with the junctions biased on either a Fiske step or a zero-field step. The onset of frequency-pulling between the Josephson oscillators was usually observed when the two junctions were biased to radiate within about 1 MHz of one another.

In addition to reproducing the experiments described in reference [4] on four junction devices, we have also investigated the non-linear interactions in double junction devices of the type shown in figure 1e. In one such identical pair of junctions ($R_n \approx 0.4 \Omega$, $\nu_R \approx 11$ GHz) mounted with the wide center strip parallel to the waveguide axis, both in-phase and out-of-phase locking were observed at 4.3 K (via a spectrum analyzer display of the receiver IF output) as follows: If $\nu_1$ and $\nu_2$ are defined as the unperturbed oscillation frequencies of the two junctions when biased with dc currents $I_1$ and $I_2$ respectively (on the $n = 2$ steps), then as $I_1$ (and hence $\nu_1$) was monotonically increased (with $I_2$ fixed) frequency-pulling effects, out-of-phase locking of the amplitudes, in-phase locking, more frequency-pulling effects and finally no interaction were each observed in turn. Both locked states were stable over limited ranges of $I_2 - I_1$: one for $\nu_2 > \nu_1$ and the other for...
\[ v_2 < v_1. \] Near \( v_1 = v_2 \), the interaction was unstable and the linewidth very broad. Interpretation of this complex behavior was further complicated by the existence of a strong non-junction resonance in the waveguide-substrate holder near \( v \). This device exhibited essentially the same non-linear response after the superconducting series connection between the two junctions had been removed by scratching away the film. Empirically, detected powers up to \( 2 \times 10^{-10} \) W at 11 GHz were observed with this type of device coupled via waveguide.

With the four junction geometry, a maximum power of \( 1.4 \times 10^{-9} \) W at 8.94 GHz was observed from a single junction (\( R_N \sim 0.08 \Omega \)) biased on the \( n = 4 \) zero-field step associated with the narrow junction dimension \( w \). In general, however, a maximum power of about \( 2 \times 10^{-10} \) W from a single junction is the typical best we have observed for a variety of device geometries coupled via waveguide.

### 4.2 Radiation Spectrum of Junctions

In this section, we consider the results of radiation emission experiments in which Josephson devices were externally coupled via microstrip lines. The various microwave properties of two single-junction devices with the geometry shown in figure 1d were measured in detail. The junctions had the same dimensions (1.8 x 0.1 mm), normal state resistances \( R_N \) of 0.5 \( \Omega \) and 0.1 \( \Omega \), and capacitances \( C \) of 3.6 nF and 4.3 nF. The temperature dependence of the junction cavity resonances at the fundamental and second harmonic are shown in figure 4 for the 0.5 \( \Omega \) junction. The solid lines are best-fits to the data using the expression for the temperature dependence of the London penetration depth

\[ \lambda_0(T) = \lambda_0(0) \left[ 1 - \left( \frac{T}{T_c} \right)^4 \right]^{1/2}, \]

where \( T_c = 7.19 \) K for lead. The data were obtained via measurements of the microwave-induced Josephson steps.

In figure 5, the detected radiation spectrum of the same junction is shown for the \( n = 2 \) zero-field step at 4.3 K. Near \( v_R \), a maximum power of \( 1.2 \times 10^{-11} \) W was observed at 5.09 GHz. Near \( 2v_R \), the maximum detected power was about \( 0.45 \times 10^{-11} \) W at 10.17 GHz. For these measurements, the 50 \( \Omega \) coaxial line between the stripline holder and the top of the cryostat was miniature flexible cable with several adapters. At liquid nitrogen temperatures, the attenuation of this line was about 6 dB at 5 to 10 GHz.

The corresponding spectrum for the 0.1 \( \Omega \) junction is shown in figure 6. Its resonant frequency

\[ [v_R(T = 0) = 4.90 \text{ GHz}] \]

is less than that of the 0.5 \( \Omega \) junction, as expected, due to a thinner oxide layer. For this device, a 50 \( \Omega \) semi-rigid coaxial cable was used between the stripline holder and the top of the cryostat (86 cm). This cable had an attenuation of about 1 dB at room temperature. Near \( v_R \), a maximum power of \( 10^{-11} \) W was detected at 4.36 GHz, and near \( 2v_R \), \( 1.2 \times 10^{-11} \) W was observ-
FIG. 6. — Radiated power spectrum for 0.1 Ω junction
(L, w = 1.8 x 0.1 mm) biased on n = 2 step in zero applied magnetic field ; a) fundamental mode, and b) second harmonic mode.

ed at 8.25 GHz. Most of the fine structure in the spectrum was caused by standing waves in the semi-rigid cable. It is interesting to note that the powers detected in the two modes of resonance are comparable.

The impedance $Z_j$ (eq. (3)) for each of the two junctions is 0.025 Ω. With $Z_r = 50$ Ω, the transmission coefficient $T$ is $2 \times 10^{-3}$. When the miniature cable losses are taken into account, the ratio of the power radiated to the power available, $\varepsilon_p = P_r/P_a$, for the 0.5 Ω junction varied between $0.8 \times 10^{-3}$ and $4 \times 10^{-3}$ near $v_R$, and between $0.4 \times 10^{-3}$ and $1.6 \times 10^{-3}$ near $2 v_R$. The agreement between $\varepsilon_p$ and the transmission coefficient calculated with $Z_j = 1/(2 v_R C)$ is relatively good. The significantly higher harmonic content of the lower resistance junction is expected since the two superconductors forming the junction are more strongly coupled.

4.3 JUNCTION CAVITY LOSSES. — The $Q$ dependence of each junction was measured as a function of temperature using the microwave-induced step phenomena. The results for the 0.5 Ω junction are shown in figure 7. An expression of the form $Q^{-1} = Q_0^{-1} + Q_0^{-1}$ has been used to fit the data. (For the surface impedance calculations [14], the following parameters for Pb were used ; $\xi_0 = 830$ Å, $\lambda_0(0) = 370$ Å and a mean free path of 2 000 Å for each of the films.)

The temperature independent contribution to the $Q$, $Q_0$, was obtained by fitting the data at temperatures below 3.6 K. The $Q_0$ was 305 at $v_R$ and 395 at $2 v_R$. If each $Q_0$ is ascribed to cavity loading by the microstrip line, the implied transmission coefficients are $T(v_R) = 3.3 \times 10^{-3}$ and $T(2 v_R) = 2.5 \times 10^{-3}$. These results are in excellent agreement with the theoretical value of $T = 2 \times 10^{-3}$ obtained via our simple microstrip line analysis. The $Q$ results for the 0.1 Ω junction were similar to those shown in figure 7 for temperatures $> 2.5$ K. However the $Q_0$ values implied at the lowest measured temperature (2.2 K) were about 500 at $v_R$ and 700 at $2 v_R$.

5. Conclusions. — Microstrip line coupling techniques have been successfully used to study the broad-band radiation spectrum and cavity parameters of tunnel junctions. We have found that in Pb-Pb oxide-Pb junctions at low temperatures, the $Q$ is mainly determined by radiation losses into the external transmission line. Our results also have shown that the coupling between a junction stripline and an external microstrip line can be understood on the basis of elementary stripline concepts and consequently that transmission coefficients of order $10^{-1}$ into a 50 Ω line are conceivable for devices with very narrow (several micrometres wide) junctions. (Jutzi et al. have recently demonstrated that such devices can be fabricated [15]). These concepts should apply at frequencies up to 100 GHz if care is taken in designing the external microstrip line.

Our observations of the interactions in coupled systems of tunnel junctions has shown that the locking-phenomena is extremely complicated but predomi-
nantly involves the coupling of high frequency electromagnetic radiation between junctions. With classical high frequency oscillators, mutual non-linear interactions are expected to depend critically on the amplitude, harmonic content, and relative coupling of the oscillators. The results of the present experiments with Josephson oscillators have shown the same factors must be taken into account in order to achieve a detailed understanding of the coupled-junction phenomena.

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References