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THEORIES OF « SUBHARMONIC » GAP STRUCTURES IN SUPERCONDUCTING JUNCTIONS

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Résumé. — Nous présentons deux interprétations possibles de la structure due aux fréquences subharmoniques de la fréquence du « gap ».

1) Un effet tunnel de multiparticules conduit à une structure due aux sous-harmoniques et explique la présence de satellites induits par le rayonnement microondes. Il permet de déterminer l'influence de la puissance microonde sur ces satellites.

2) Le courant Josephson alternatif provoque la naissance d'une tension alternative à la fréquence Josephson. L'effet tunnel assisté par cette tension peut permettre d'expliquer la structure due aux sous-harmoniques et l'influence de la puissance d'excitation mais pour obtenir des résultats corrects il est nécessaire de tenir compte de la destruction de paires provoquée par la tension alternative.

Abstract. — Two possible explanations of the subharmonic gap structures are presented : multiparticle tunneling and self-coupling.

1) Multiparticle tunneling yields the subharmonic gap structure and the microwave induced satellites. It also yields the observed power dependence for the satellites.

2) The ac Josephson current excites an rf voltage across the junction with the Josephson frequency. Microwave assisted tunneling due to this rf voltage could account for some of the subharmonic gap structure with the observed power dependence. In order to get all the structure one has to include pair-breaking due to the ac voltage.

1. Introduction. — For subharmonic gap structures observed in superconducting junctions at voltages given by $eV_{dc} = 2\Delta/m$, where $-e$ is the electronic charge, Δ the gap parameter, and m a positive integer, two mechanisms have been considered as the cause : multiparticle tunneling and/or self-coupling. The self-coupling is due to an rf voltage across the junction induced by the ac Josephson current.

We are going to investigate these two theories of subharmonic gap structures when externally microwaves are applied to the junction.

Experimentally it is found [1] that microwaves induce satellites around the subharmonic gap structures appearing at voltages given by

$$eV_{dc} = \frac{2\Delta - n\hbar\omega}{m} \quad (1)$$

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where $\omega/2\pi$ is the microwave frequency and n an integer, and that the magnitude of the n 'th satellite around the m 'th subharmonic is proportional to

$$J_n^2(m\alpha) \quad (2)$$

where J_n is the ordinary Bessel function of order n and $\alpha = eV_{rf}/\hbar\omega$ is a measure of the microwave power level, V_{rf} being the rf voltage across the junction due to the applied microwaves.

2. Multiparticle tunneling. — The Tien-Gordon expression [2] for the single particle tunneling current through a junction with applied microwaves is

$$I(V_{dc}) = \sum_n J_n^2(\alpha) I_0 \left(V_{dc} + n \frac{\hbar\omega}{e} \right) \quad (3)$$

where $I_0(V_{dc})$ is the current without applied microwaves. $I_0(V_{dc})$ has structure at the voltage given by $eV_{dc} = 2\Delta$ since here the energy gain when one electron passes the barrier eV_{dc} is equal to the excitation energy of the superconducting ground state 2Δ .

Eq. (3) suggests that $J_n(\alpha)$ is the probability amplitude for an electron to absorb microwave energy of $n\hbar\omega$ in passing the insulating barrier.

When m electrons pass the barrier one would expect one such probability amplitude factor for each electron so that the total probability amplitude for the first electron to absorb $n_1 \hbar\omega$, the second — $n_2 \hbar\omega$, etc., becomes

$$J_{n_1}(\alpha) J_{n_2}(\alpha) \dots J_{n_m}(\alpha). \quad (4)$$

If the net absorption of microwave energy is $n\hbar\omega$ we have

$$n_1 + n_2 + \dots + n_m = n. \quad (5)$$

Before squaring eq. (4) in order to get the probability for the m electrons to absorb the microwave energy $n\hbar\omega$, we must sum over all intermediate states i. e. sum eq. (4) with the constraint of eq. (5). The result of this summation is

$$J_n(m\alpha). \quad (6)$$

Squaring eq. (6) we immediately get the experimental result of eq. (2).

The location of the satellite structure follows from the fact that the m electrons gain an energy $meV_{dc} + n\hbar\omega$ in passing the barrier, and that the excitation energy is 2Δ . This yields the experimental result of eq. (1).

The procedure outlined here is almost correct. It turns out that some of the matrix elements entering the detailed derivation [3] of eq. (6) become dependent on $n_1, n_2, \dots, n_m$. But if $\hbar\omega \ll 2\Delta/m$ this dependence can be neglected.

3. Self-coupling. — An applied dc voltage V_{dc} will yield an ac Josephson current with a frequency given by the Josephson equation

$$\dot{\phi} = \frac{2eV}{\hbar} \quad (7)$$

where ϕ is the phase difference. The frequency becomes $\omega_0 = 2eV_{dc}/\hbar$.

This ac current induces an rf voltage with the same frequency, — $V_{rf}^J \cos \omega_0 t$, where V_{rf}^J is the amplitude of the self-induced rf voltage. Inserting this rf voltage in the Tien-Gordon expression eq. (3) we get

$$I(V_{dc}) = \sum_n J_n^2 \left(\frac{V_{rf}^J}{2V_{dc}} \right) I_0((2n+1)V_{dc}). \quad (8)$$

Eq. (8) yields structure in $I(V_{dc})$ at voltages given by $eV_{dc} = \frac{2\Delta}{2n+1}$ [4] i. e. only the subharmonic gap structures corresponding to odd m ($= 2n+1$). A similar contribution comes from the pair-current [4]. Also here only odd m structure occurs.

In order to get the even m structures one has to

invoke pair-breaking [4] due to the self-induced rf voltage. The energy $n\hbar\omega_0$ (n Josephson « photons » or one « photon » of the n 'th harmonic of the Josephson frequency) exciting the superconducting ground state (2Δ). This yields $eV_{dc} = 2\Delta/2n$.

The phase difference corresponding to the total voltage may be found from eq. (7)

$$\phi = \omega_0 t + \frac{V_{rf}^J}{V_{dc}} \sin \omega_0 t. \quad (9)$$

Applied microwaves — voltage $V_{rf} \cos \omega t$ — would without self-coupling yield a phase difference

$$\phi = \omega_0 t - 2\alpha \sin \omega t \quad (10)$$

where α as before is $eV_{rf}/\hbar\omega$.

Eq. (9) and (10) suggest the following phase difference with both applied microwaves and self-coupling

$$\phi = (\omega_0 t - 2\alpha \sin \omega t) + \frac{V_{rf}^J}{V_{dc}} \sin (\omega_0 t - 2\alpha \sin \omega t). \quad (11)$$

The Tien-Gordon expression eq. (3) for the tunnel current is valid only if just one frequency is involved. For the phase difference of eq. (11) one has to use the more general expression given by Werthamer [4]. Inserting eq. (11) into eq. (11) of reference [4] one obtains the observed microwave power dependence of eq. (2) and the observed location of eq. (1) for odd m .

The even m structure also follows from the phase difference of eq. (11) [3].

The phase difference of eq. (11) is essential in order to get the observed microwave power dependence of eq. (2). If for instance a phase difference

$$\phi = \omega_0 t - 2\alpha \sin \omega t + \frac{V_{rf}^J}{V_{dc}} \sin \omega_0 t$$

is used instead of eq. (11) then the power dependence becomes $J_n^2(\alpha)$ for odd m .

The phase difference of eq. (11) can be theoretically justified only for small self-coupling and $V_{rf} \ll V_{dc}$ [3].

4. Conclusion. — It is impossible to distinguish between multiparticle tunneling and self-coupling as explanations of the subharmonic gap structures if only location (eq. (1)) and microwave power dependence (eq. (2)) of the subharmonic gap structures and the microwave induced satellites are considered.

Even in the case that the junction consists of two non-identical superconductors with gap parameters Δ_1 and Δ_2 , multiparticle tunneling and self-coupling yield the same results. Eq. (1) and (2) still hold if $2\Delta \rightarrow \Delta_1 + \Delta_2$, for odd m and $2\Delta \rightarrow 2\Delta_1$ and/or $2\Delta_2$, for even m .

References

- [1] LONGACRE, A., Jr. and SHAPIRO, S., *Bull. Am. Phys. Soc.* **16** (1971) 399.
- HOFFMANN SOERENSEN, O., KOFOED, B., PEDERSEN, N. F. and SHAPIRO, S. *Revue Phys. Appl.* **9** (1974) 153.
- HOFFMANN SOERENSEN, O., KOFOED, B., PEDERSEN, N. F. and SHAPIRO, S. (to be published).
- [2] TIEN, P. K. and GORDON, J. P., *Phys. Rev.* **129** (1963) 647.
- [3] HASSELBERG, L. E., LEVINSSEN, M. T. and SAMUELSEN, M. R. (to be published).
- [4] WERTHAMER, N. R., *Phys. Rev.* **147** (1966) 255.