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► To cite this version:

G. Deutscher. Critical current of granular superconductors. *Revue de Physique Appliquée*, 1973, 8 (2), pp.127-129. 10.1051/rphysap:0197300802012700 . jpa-00243667

HAL Id: jpa-00243667

<https://hal.science/jpa-00243667>

Submitted on 1 Jan 1973

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Classification
 Physics Abstracts
 14.24

CRITICAL CURRENT OF GRANULAR SUPERCONDUCTORS

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(Reçu le 29 novembre 1972)

Résumé. — Nous présentons le calcul du courant critique d'un système multicouches composé de films minces séparés par des barrières Josephson. On montre que pour des barrières très minces le courant critique est inversement proportionnel à la racine carrée de la résistivité effective, tandis que pour des barrières plus épaisses il est inversement proportionnel à la résistivité.

Abstract. — The transverse critical current of a multilayer system made of thin films separated by Josephson barriers is calculated. It is shown that for very thin barriers, the critical current is inversely proportional to the square root of the effective resistivity, whereas, for thicker barriers it is inversely proportional to the resistivity.

1. Introduction. — The structure of granular superconductors has been recently studied in some detail [1]. They consist of small cristallites, whose dimension (typically of the order of 50 Å) can be fairly uniform, and of a usually amorphous dielectric. Even for a rather low dielectric concentration, the normal state resistivity of granular materials ($> 100 \mu\Omega\cdot\text{cm}$) is much higher than that of a typical alloy ($\sim 10 \mu\Omega\cdot\text{cm}$). It is therefore assumed that the small cristallites are surrounded by a thin dielectric coating which acts as a tunneling barrier between neighbouring cristallites.

This representation of a granular superconductor as a collection of weakly coupled small cristallites has already proved useful for the study of fluctuation effects. We shall use it here for the calculation of the critical current density, which is a matter of some practical interest since high normal state resistivity superconductors are required to build satisfactory power switches.

For this purpose, we can assume that the superconductor is made of metallic layers of thickness L and surface S separated by dielectric barriers whose resistance is called R_n ; the current is supposed to flow along the direction perpendicular to the layers. We shall neglect all fluctuation effects. i. e. we assume that the pair potential in each layer has a well defined amplitude Δ and phase, and therefore the dielectric

layers parallel to the current flow don't play any role in this problem.

In short, we assume that the granular material can be represented by a series of Josephson junctions. It is well known that the critical current of one individual junction is inversely proportional to R_n , and it would seem natural to assume that the same result applied to the granular material. However, we shall show here that, when L is smaller than the coherence length, this result is in fact only obtained in the limit of large R_n . In the limit of small R_n [2], [3], the critical current density is inversely proportional to $(R_n)^{1/2}$. This calculation is presented in part 2, whereas its practical implications are discussed in part 3.

2. Critical current of a series of Josephson junctions. — We assume that $L < \xi(T)$, where $\xi(T)$ is the temperature dependent coherence length in each individual metallic layer. The pair potential has therefore a constant amplitude Δ_i and phase φ_i within each layer. If we further neglect all thermodynamic fluctuation of the Δ_i 's and φ_i 's, all the Δ_i 's are equal and the phase difference φ between adjacent layers is given by

$$J = \frac{\pi}{4 e R_n} \frac{\Delta^2}{k_B T_c} \sin \varphi \quad (1)$$

where J is the current, flowing perpendicular to the functions.

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The free energy density is given by

$$F_s = F_n + A\Delta^2 + \frac{B}{2}\Delta^4 + \frac{1}{LS} \frac{1}{R_n} \frac{\pi\hbar}{8e^2} \times \frac{\Delta^2}{k_B T_c} (1 - \cos \varphi). \quad (2)$$

For a given value of φ , we now write that $\partial F_s / \partial \Delta = 0$ to obtain $\Delta(\varphi)$:

$$A + B\Delta^2 + \frac{1}{LS} \frac{1}{R_n} \frac{\pi\hbar}{8e^2} \frac{1}{k_B T_c} (1 - \cos \varphi) = 0. \quad (3)$$

Using now (1) to express φ as a function of J and Δ , we have

$$J^2 = -J_j \frac{4e}{\hbar} LS A \Delta_0^2 f^4 (1 - f^2) \left(\frac{2e}{\hbar} \right)^2 \times (LS)^2 \Delta_0^4 A^2 f^4 (1 - f^2)^2 \quad (4)$$

where J_j is the maximum Josephson current

$$J_j = \frac{\pi}{4eR_n} \frac{\Delta_0^2}{k_B T_c} \quad (5)$$

Δ_0 being the equilibrium value of Δ in zero current, and

$$f = \frac{\Delta}{\Delta_0}. \quad (6)$$

Remembering that

$$A = -\frac{1}{2} N(0) \left(\frac{\xi(0)}{\xi(T)} \right)^2$$

where $\xi(T)$ is the temperature dependent coherence length [5], it is useful to introduce the quantity

$$J(T) = \frac{2e}{\hbar} LS \frac{1}{2} N(0) \Delta_0^2 \left(\frac{\xi_0}{\xi(T)} \right)^2 \quad (7)$$

which is closely connected to the condensation energy $LS \frac{1}{2} N(0) \Delta_0^2$ per layer. Then (4) takes the simpler form

$$J^2 = J_j J(T) f^4 (1 - f^2) \left[2 - \frac{J(T)}{J_j} (1 - f^2) \right]. \quad (8)$$

The critical current J_M is obtained by writing that $dJ^2/df^2 = 0$. Defining $c = J(T)/J_j$ this condition reads:

$$f_M^2 = \frac{3(c^2 - c) + c\sqrt{c^2 - 2c + 9}}{4c^2}. \quad (9)$$

Since f^2 has to be positive, only the + sign applies in (9).

In the limit where $c \gg 1$ (very weakly coupled grains), J_M may be written as

$$J_M = J_j \left(1 - \frac{1}{c} + \frac{1}{2} \frac{1}{c^2} + 0 \left(\frac{1}{c^3} \right) \right) \quad (10)$$

which shows how J_M goes to the Josephson value in the weak coupling limit, where the product $J_M \cdot R_n$ is a constant.

However, in the limit where $c \ll 1$ one obtains

$$J_M^2 = \frac{8}{27} J_j J(T) \left(1 - \frac{2}{9} c + 0(c^2) \right). \quad (11)$$

Here, it is the product $J_M^2 \cdot R_n$ which is constant. This result is similar to that obtained for a homogeneous superconductor containing non magnetic impurities. Figure 1 gives the shape of the general solution of (8), (9).

FIG. 1. — Critical current J_M of a granular superconductor as a function of the intergrain coupling parameter c . In the weak coupling limit ($c \gg 1$), J_M tends to the Josephson value J_j , whereas in the good coupling limit ($c \ll 1$) J_M is equal to the critical current of an homogeneous alloy with the same resistivity.

3. Application to the case of granular superconductors. — Let us first evaluate the effective resistivity, ρ_c , of a granular film where $c = 1$:

$$\frac{\pi}{4eR_n} \frac{\Delta_0^2}{k_B T_c} = \frac{2e}{\hbar} LS \frac{1}{2} N(0) \Delta_0^2 \frac{\xi_0^2}{\xi(T)} \quad (12)$$

where L is now the grain size. Assuming that the grains are essentially pure, the effective resistivity of the granular material is $\rho_n = R_n (S/L)$ and we obtain at low temperatures

$$\rho_c = \frac{\pi\hbar}{4e^2} \frac{L}{n} \frac{E_F}{k_B T_c} \quad (13)$$

where n is the number of electrons per grain. For a grain size $L \sim 50 \text{ \AA}$, $T_c \sim 2 \text{ K}$ and $E_F \sim 3 \text{ eV}$, one obtains for ρ_c the typical value $10^{-2} \Omega \cdot \text{cm}$.

Hence, if $\rho_n \ll 10^{-2} \Omega \cdot \text{cm}$ the granular film falls into the good coupling limit and eq. (11) applies. The critical current density J_c can then be written as

$$J_c^2 = \frac{H_c^2}{17.64 \rho_n} \cdot \frac{k_B T_c}{\hbar} \cdot \frac{T_c - T}{T_c} \quad (14)$$

where H_c is the bulk thermo-dynamical critical field at the temperature under consideration. Eq. (14) predicts that with high T_c superconductors, values of $\rho_n J_c^2$ up to $10^{11} \text{ W} \cdot \text{cm}^3$ could in principle be obtained, with $\rho_n \leq 10^{-2} \Omega \cdot \text{cm}$. This result might

be quite interesting in practice [6]. The product $\rho_n J_M^2$ should be constant for a given temperature, and J_M should vary as $(T_c - T)^{3/2}$ near T_c .

For $\rho_n > 10^{-2} \Omega \cdot \text{cm}$, the situation is more complicated, since fluctuation effects cannot be neglected, as shown previously. Generally speaking, these fluctuation effects will reduce the critical current density and eq. (10) should be considered only as an upper bound for T_c . More specifically, two cases have to be considered

1) If

$$L^3 > \frac{k_B T_c}{2 N(0) |A(0)|^2},$$

only the fluctuations in the phase difference between adjacent grains are important, but the order parameter has a well defined amplitude in each grain. The main square value of this phase difference is given by

$$\langle \Phi^2 \rangle = \frac{k_B T_c}{|A_0|^2} \frac{8 e^2 \rho_n}{\pi \hbar L}. \quad (15)$$

At low enough temperatures, $\langle \Phi^2 \rangle$ becomes small and the result (10) should be recovered.

2) If

$$L^3 < \frac{k_B T_c}{N(0) |A(0)|^2},$$

there are very large fluctuations in the amplitude of the order parameter and superconductivity essentially vanishes.

4. Conclusion. — In the good coupling limit, the critical current density of a granular superconductor is identical to that of a homogeneous alloy with the same resistivity, as already found for other properties by Parmenter [2] and Abeles *et al.* [3]. In the weak coupling limit, the Josephson result is recovered, provided that the grain size is large enough to prevent the destruction of the superconducting state by fluctuation effects. The crossover region occurs for normal state resistivities of the order of $10^{-2} \Omega \cdot \text{cm}$.

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