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HYSTERESIS LOSSES ALONG OPEN TRANSFORMATIONS

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Résumé. — Si la courbe d’aimantation et le cycle à saturation sont connus, on démontre que la théorie analytique de l’hystérésis, qui détermine la valeur de J à la fin d’un parcours arbitraire de H, peut être étendue au calcul des pertes causées par l’hystérésis au cours d’une transformation arbitraire, ouverte ou fermée, dans le plan (J, H). Les données théoriques ont été calculées pour un échantillon de fer doux et pour un matériau du type Alnico. Ces dernières ont été comparées avec les données expérimentales de Bates et Simpson obtenues pour un matériau analogue. L’accord quantitatif entre les courbes expérimentale et théorique est satisfaisant.

Abstract. — It is shown that the analytical theory of hysteresis, which determines the value of J at the end of an arbitrary path of H if the magnetization curve and the saturation loop are known, can be extended to compute the energy losses due to hysteresis along an arbitrary transformation, open or closed, in the (J, H) plane.

The theoretical data have been computed for a soft iron specimen and for an Alnico type material. The latter have been compared with experimental data taken by Bates and Simpson on a similar material. The quantitative agreement between theoretical and experimental curves is quite satisfactory.

1. Introduction. — In a recent paper [1] the formal theory of hysteresis given by Preisach [2] has been considered in a more general way. It is assumed that a ferromagnetic specimen can be represented as composed of infinitesimal volume elements, each of them being characterized by a rectangular loop of sides a, b, J_s and — J_s. Hence a function (a,b) is defined which is essentially the probability density of finding an elemental volume having a loop with given a and b, that is (a,b) da db is the probability of finding a loop having one side falling between a and (a + da) and the other between b and (b + db). (a,b) has therefore the dimensions of H^-2. This function (a,b) should not depend on the macroscopic magnetic state of the specimen, but should be an intrinsic property of the material. All the hysteresis properties of the specimen should be deducible from this function. In fact, given an arbitrary path of H it is possible to find which elemental volumes have undergone a magnetization inversion (or several inversions). It is easy to prove that the volume elements whose magnetization has been reversed an odd number of times fill a surface S in the (a,b) plane. The integral of over S gives the intensity of magnetization at the end of the path of H, apart from a factor J_s. In the paper quoted [1] it has been shown that the function (a,b) of a specimen can be determined from the knowledge of the magnetization curve and saturation loop, including the irreversible heat exchanges, are contained in the magnetization curve and saturation loop.

At this point it is useful to recall that along an open transformation in the (J, H) plane reversible and irreversible heat exchanges are observed. The reversible heat transfer is due to the variation with temperature of the intrinsic magnetisation and anisotropy constant. The irreversible processes have been widely studied by Stoner [3], Bates [4] et al. The irreversible heat exchanges are due to irreversible displacements of the Bloch walls.

The model of elemental rectangular loops gives information only on the irreversible heat exchanges.

2. Computation of the hysteresis losses. — Let us consider a volume element dv, and the corresponding loop (fig. 1). If we start from F (H = 0) and apply increasing field strengths up to a value H > a, and we then go back to H = 0, the hysteresis energy loss is obviously given by
2J_s (a - b) dv. On the other hand there are no energy exchanges along the transformations FB, CG, GD, AF (fig. 1). The energy changes only along the transformations BC and DA. As far as the energy losses are concerned, it does not seem plausible that transformation BC and DA are different. It appears reasonable to believe that half of the total energy 2J_s (a - b) dv is lost along BC, and the other half along DA. This means that in the volume dv the inversion of magnetization from — J_s to J_s is associated with the same energy loss as the inversion from J_s to — J_s.

Now let us consider an arbitrary path of H. As stated above, it is possible to find a surface S in the (a,b) plane containing all the elemental loops which have undergone an odd number of inversions. Those loops suffice to determine J at the end of that path of H, but the losses along that path also depend on the loops which have undergone an even number of inversions. Hence one has to consider a surface S_1 which contains all the loops which have undergone inversions during the process, and has to keep in mind that each part of S_1 has to be taken in account many times as the number of inversions of the corresponding loops. Finally, the expression of the energy losses along that path of H is

$$ W = J_s \int_{S_1} \varphi(a, b) (a - b) \, da \, db. \tag{1} $$

In particular, if one describes the saturation loop, the surface S_1 is the whole (a,b) plane, counted twice. In fact, when the material goes from + J_s to — J_s the whole plane is covered by straight lines parallel to the a-axis (as MN in Fig. 2a), and going back from — J_s to + J_s the whole plane is covered again by lines parallel to the b-axis (as PQ in fig. 2b). Hence for the saturation loop eq. (1) gives

$$ W_L = 2 \cdot J_s \int_A \varphi(a,b) (a - b) \, da \, db \tag{2} $$

A being the whole (a,b) plane.

For instance, in the simple case of a rectangular loop material, where $b = -a =$ const = $H_s$, eq. (2) gives correctly:

$$ W_L = \frac{1}{2} J_s H_s \int_A \varphi(a,b) \, da \, db = \frac{1}{2} H_s J_s $$

since the integral of $\varphi(a,b)$ over the whole plane is equal to unity. As for the way of applying eq. (1) let us consider, as an example, the following path of H, starting from demagnetized material (Fig. 2a) : from $H = 0$ to $H = H_1 > 0$ and then from $H_1$ to $H_2$ (0 < $H_2$ < $H_1$). Going from $H = 0$ to $H = H_1$ the magnetization of the loops contained in the triangle OPQ (fig. 2b) becomes positive. The losses in this process are given by eq. (1), $S_1$ being the triangle OPQ. Going from $H_1$ to $H_2$ the magnetization of the loops contained in the triangle PMN also again becomes negative. Hence the losses in this part of the process are given by eq. (1), $S_1$ being now the triangle PMN. The losses along the whole transformation are thus:

$$ W = J_s \int_{OPQ} \varphi(a,b) (a - b) \, da \, db + J_s \int_{PMN} \varphi(a,b) (a - b) \, da \, db. $$
3. Experimental results. — Making use of the \( \varphi(a, b) \) of the iron examined in the quoted paper [1], we have computed the losses along an open symmetrical path, with vertexes at \( \pm 1.5 \text{ Wb/m}^2 \). They are shown in figure 3.

We have also examined a material of the Alnico type, because for this material good experimental data on heat exchanges along open symmetrical transformations are available from the literature [5]. The function \( \varphi(a, b) \) (computed as explained in the appendix of [1]), is given in figure 4. As a check of the \( \varphi(a, b) \) we have computed and compared with experimental data a loop with the vertex at 40 000 A/m. The results, shown in Figure 5, are quite satisfactory.

Using the method explained here we have computed the hysteresis losses along three symmetrical open transformations with extreme field strengths of about \( \pm 2400 \text{ Oe} \); \( \pm 1750 \text{ Oe} \) and \( \pm 750 \text{ Oe} \). The starting point has been reached from the demagnetized state through continuously increasing field strengths.

The results are given in figure 6. They can be compared to those reported by Bates and Simpson [5]. To make this comparison it is necessary to subtract from the experimental curves of Bates and Simpson, the reversible heat exchanges, by the procedure suggested by themselves. As one can see from the figure, the reversible heat exchanges
are small compared to the irreversible ones for materials of this type.

Considering that the material on which we have worked is not exactly the same as that of Bates and Simpson, the agreement between our theoretical curves and their experimental data is very satisfactory.

![Diagram showing heat exchanges along open symmetrical transformations in Alnico type materials.](image)

**Fig. 6.** Heat exchanges along open symmetrical transformations in Alnico type materials:
- Broken line: experimental from Bates and Simpson [5]; Solid line: the same curves after subtraction of the reversible heat exchanges; Points: computed irreversible losses.

**REFERENCES**


**DISCUSSION**

*M. Bates.* - Je désire demander si vous avez fait des calculs pour les autres matériaux mentionnés dans la communication de Bates et Simpson.

*M. Ferro.* - We have computed the losses also for an iron specimen. For the other materials mentioned by Bates and Simpson we have not yet found the magnetization curve and hysteresis loops which we need for our computations.

*M. Kondorski.* - The relation between magnetization and hysteresis curves was previously considered in some papers (E. Kondorski, JETF, 1940, 10, 420; Doklady, 1940, J. Phys., U. S. S. R., 1942, 6, 93 and N. Popcov, L. A. Černikova, J. Phys., U. S. S. R., 1945). There are many interesting results in Dr. Montalfant's, Dr. Pescetti's, and Dr. Preisach's papers. However it seems to me that the Preisach's model, although it is very useful in some cases, is also very artificial.

In a real ferromagnetic body we have principal domains, closure domains, and the walls between them as predicted in Landau-Lifshitz's and Neel's theories, and discovered by Bozorth, Williams and Shockley. I think that the further development of the theory of magnetization processes will be found in more detailed study of domains patterns and in calculations related with the real domain structures.

*M. Ferro.* - The model is formal, and has no direct connection with the domain structure. The volume having a given rectangular loop can be thought as an infinitesimal volume, without any relationship with domain volume.

The model is mainly a sort of general tool to describe by differential calculus irreversible processes of this type.

*M. Rhodes.* - Might it not be useful to apply this formal method of analysis to the theoretical hysteresis curve of Stoner and Wohlfarth for an assembly of randomly oriented single-domain particles? The derived irreversible thermal changes could then be compared with those previously calculated directly (Rhodes, Proc. Leeds Phil. Soc., 1948, 5, 116).

*M. Biorci and M. Ferro.* - Yes, it might. In effect it should be interesting to compare the theoretical ϕ(a,b) obtained from the theory you have quoted with the ϕ(a,b) obtained from several experimental curves.