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SOME CONSEQUENCES OF THE ANALYTICAL THEORY OF THE FERROMAGNETIC HYSTERESIS

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Abstract. — Some properties of the transformations in the \((J, H)\) plane of a ferromagnetic material are described. They are consequences of the Preisach model which considers the material as composed of independent elements of volume whose magnetic behaviour is wholly described by a rectangular loop, and assumes that the distribution of elemental loops, statistically independent of \(J\) and \(H\), is a property of the material.

The most interesting property concerns the determination of the region of the plane \((J, H)\) where the magnetization curve can be found, once the loop is known, and the region where the loop can be found, once the magnetization curve is given. The coercive force of a symmetrical loop cannot be larger than the field strength corresponding to \(J_\nu/2\) on the magnetization curve, \(J_\nu\) being the intensity of magnetization at the vertex of the loop.

1. — A formal theory of ferromagnetic hysteresis, based on a generalization of Preisach’s theory [1], has been proposed recently [2]. It is assumed that the magnetic behaviour of an infinitesimal volume of a ferromagnetic material is wholly described by a rectangular loop of sides \(H = a\), \(H = b\), \(J = \pm J_\nu\). Then it is possible to deduce, from the knowledge of the magnetization curve and the saturation loop, a function \(\varphi(a, b)\), which is assumed to be unique. This function is essentially the probability density of finding an elemental loop of sides \(a\) and \(b\). If an arbitrary path of \(H\) is given, it is possible to determine a surface \(S\) in the plane \((a, b)\) which corresponds to that path. In fact, when the material is demagnetized, the loops contained in the triangle OBC, Fig. 1a, are positively magnetized, the others negatively. When the field \(H_1 > 0\) is applied, the magnetization of those volume elements which have \(a \leq H_1\) becomes positive (Fig. 1b). If the field strength is now reduced to \(H_2 < H_1\), the magnetization of those volume elements which have \(b \geq H_2\) again becomes negative (Fig. 1c). The net magnetization at the end of the process is due to the loops contained in the surface ONMQ.
procedure is similar when more complicated processes are considered. The integral of \( \varphi(a,b) \) over the surface \( S \) gives the intensity of magnetization at the end of that path of \( H \). Several experimental checks of the validity of the theory have been reported [2].

Here we shall present a few general properties of the ferromagnetic hysteresis which follow from the existence and uniqueness of \( \varphi(a,b) \). We shall refer only to the basic property of \( \varphi(a,b) \) of being positive or zero in the whole \((a,b)\) plane, owing to its nature of probability density. Moreover, we recall that in general the intensity of magnetization at the end of a given path of \( H \) is equal in magnitude and opposite in sign to the intensity of magnetization at the end of a path of \( H \) obtained from the preceding by changing \( H \) into \(-H\) everywhere. This means that

\[
\varphi(a,b) = \varphi(-b,-a).
\]

2. — The main properties concern the mutual relationships between magnetization curve and symmetrical loops. Let us consider a symmetrical loop with maximum field strength \( H_r \) (Fig. 2a) ; and examine the intensity of magnetization corresponding to the points 1, 2, 3, by means of the \((a,b)\) plane, Fig. 2b. The surface \( S_1 \) corresponding to point 1 is ADEBA, that \( (S_2) \) corresponding to point 2 is ADFBA, that \( (S_3) \) corresponding to point 3 is the triangle ADGA. It turns out easily that :

\[
S_1 - S_2 = DEBFD = DELFD + FLBF
\]
\[
S_2 - S_3 = FBGF.
\]

Because of the mentioned symmetry of \( \varphi(a,b) \) with respect to the line \( OB \), it is easily proved that \( (J_1 - J_2) > (J_2 - J_3) \). In fact \( J_1 - J_2 \) includes the integral of \( \varphi(a,b) \) over the rectangle DELFD which does not appear in \( J_2 - J_3 \), whereas the other surfaces are in common. This integral is positive, or at least zero. Furthermore \( J_2 > J_3 \), because \( J_2 \) includes the integral of \( \varphi(a,b) \)
over the triangle FBGF. Therefore we can conclude that: The magnetization curve does not cross any symmetrical loop (except at their vertexes). Furthermore the magnetization curve lies below (or at least coincides with) the mean curve of the loop. By mean curve of the loop we mean the locus of the middle points of the chords of the loop parallel to the \( J \)-axis. Hence, given the loop, the region where the magnetization curve can be found is known (Fig. 3a). We can also note that all points of the loop between the vertex and the remanence cannot be higher than the vertex itself. In fact, \( \varphi(a, b) \) is positive, or at least zero, in the triangle \( \text{OH}_C \text{CO} \) (Fig. 2b). This consideration, together with the preceding ones, enables us to determine the region of the \((J, H)\) plane where a symmetrical loop can be found, once the magnetization curve and \( H_a \) are known (Fig. 3b). In fact the intensity of magnetization \( J_v \) at the vertex corresponds to \( H_a \) on the magnetization curve. Then let us draw a straight line parallel to the \( H \)-axis from \( J_v \) to \( J_r \). From each point \( A \) of \( J_vJ_r \) let us draw the parallel to the \( J \)-axis. It crosses the magnetization curve in \( B \). The other point of the loop cannot be lower than \( C \), such that \( AB = BC \). The locus of the points \( C \) determines a curve which is certainly external to the real loop. The point \( H_{CM} \) where this curve crosses the \( H \)-axis is therefore at the right of the real coercive force of the loop having the vertex at \((H_a, J_v)\). The point \( H_{CM} \) can be easily identified. In fact for it, the following relationship holds: \( H_{CM}P = PQ \). Therefore we conclude that: The coercive force of a symmetrical loop having the vertex at \((H_a, J_v)\) is lower than the field strength corresponding, on the magnetization curve, to the intensity of magnetization equal to one half of the intensity of magnetization \( J_v \) of the vertex.

This property can be easily verified on many materials, from the data of the literature. In Table I we have collected the coercive forces of many materials and the corresponding theoretical limits \( H_{CM} \), obtained in the way explained above (Fig. 3b).

### Table I

<table>
<thead>
<tr>
<th>Material</th>
<th>( J_a ) or ( J_r ) kilogauss</th>
<th>( H_C ) Oe</th>
<th>( H_{CM} ) Oe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>24.2</td>
<td>1.0</td>
<td>2.5</td>
</tr>
<tr>
<td>Iron</td>
<td>9.6</td>
<td>1.0</td>
<td>1.25</td>
</tr>
<tr>
<td>Iron</td>
<td>5.2</td>
<td>0.7</td>
<td>0.87</td>
</tr>
<tr>
<td>Fe-Si 4 %</td>
<td>19.7</td>
<td>0.5</td>
<td>1.8</td>
</tr>
<tr>
<td>Mmumetal</td>
<td>6.7</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Hyperm 5 T</td>
<td>20.0</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Hyperm 50</td>
<td>15.0</td>
<td>0.1</td>
<td>0.27</td>
</tr>
<tr>
<td>Hyperm 800</td>
<td>8.0</td>
<td>0.01</td>
<td>0.09</td>
</tr>
<tr>
<td>Hyperm Co 35</td>
<td>24.0</td>
<td>2.6</td>
<td>5.1</td>
</tr>
<tr>
<td>Deltamax</td>
<td>15.1</td>
<td>0.06</td>
<td>0.09</td>
</tr>
<tr>
<td>8-79 Mo-Permalloy</td>
<td>6.2</td>
<td>0.04</td>
<td>0.43</td>
</tr>
<tr>
<td>Supermalloy</td>
<td>7.0</td>
<td>0.006</td>
<td>0.008</td>
</tr>
<tr>
<td>Alnico</td>
<td>10.3</td>
<td>4.10</td>
<td>4.40</td>
</tr>
<tr>
<td>Alnico</td>
<td>6.2</td>
<td>310</td>
<td>340</td>
</tr>
</tbody>
</table>

3. Other properties of the loops can be found by referring to the function \( \varphi(a, b) \). Let the field strength \( H \) follow an arbitrary path. Then let the field strength go from \( H_a \) to \( H_B < H_a \) and from \( H_B \) to \( H_a \) several times. If we follow these paths of \( H \) on the \((a, b)\) plane we find that the third and the following loops are identical to the second. Furthermore, when the transformation \( H_a \rightarrow H_B \), or its opposite, are made, only the elemental loops contained in the triangle \( \text{PQMP} \) (Fig. 4a) undergo inversions, while the set of the

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**Fig. 4.** a) A loop having extreme field strengths \( H_A \) and \( H_B \). The set of the loops at the right of \( MH_A \) depends on the previous path of \( H \), as shown in the following.

b) The loop is traced with the path of \( H \):

\[
O \rightarrow H_A \rightarrow H_B \rightarrow H_A \rightarrow H_B
\]

c) The loop is traced with the path of \( H \):

\[
O \rightarrow H_B \rightarrow H_B \rightarrow H_A \rightarrow H_B
\]

\( (H_B \) corresponds to saturation). d) The loop is traced with the path of \( H \):

\[
O \rightarrow H_B \rightarrow O \rightarrow H_A \rightarrow H_B \rightarrow H_B
\]

It could also be proved that, after having gone from \( H_a \) to \( H_B \) and vice versa at least twice, that is after having traced a stable loop between the limits \( H_a \) and \( H_B \), any other path of \( H \) contained
in the range \( H_A, H_B \) cannot bring the representative point in the \((J, H)\) plane outside that loop. That is: After having traced an arbitrary stable loop, any path of \( H \) in the range of the field strengths at the vertexes of the loop gives a transformation in the \((J, H)\) plane which cannot cross the loop itself, but is wholly contained inside it.

Finally, it could be easily proved that a loop between the field strengths \( H_A \) and \( H_B (H_A > H_B) \) is internal to a loop between \( H'_A \) and \( H'_B \), if \( H'_A > H \) and \( H'_B < H_B \), provided the initial path of \( H \) is the same. In particular: A symmetrical loop does not cross any other symmetrical loop.

Of the properties described in this section some have an experimental proof, as the property of not intersecting symmetrical loops. For others there are not sufficient data in the literature, and an experimental check should be useful. In particular, the property that a loop depends only on the extreme field strengths and not on the average intensity of magnetization, if experimentally confirmed, should prove that \( \varphi(a,b) \) does not depend on \( H \) and \( J \), because the shape of a loop, determined by a small region of the \((a,b)\) plane, is independent of the set of the elemental loops contained in the remaining parts of the plane (Fig. 4a, 4c, 4d). It is worth recalling that all the properties here explained proceed from the model of rectangular elemental loops, and therefore are valid only where the irreversible processes are large compared to the reversible ones.

REFERENCES