Magnons and their interactions with phonons and photons

C. Kittel

MAGNONS AND THEIR INTERACTIONS WITH PHONONS AND PHOTONS

By C. KITTEL,
Department of Physics, University of California, Berkeley 4, California (U. S. A.).

Résumé. — Cette étude est une mise au point de la théorie des ondes de spin et des effets physiques liés à ces ondes. On y montre que la condition aux limites à la surface $m_x = m_y = 0$ permet l'excitation des ondes de spin par un champ uniforme de fréquence radio. Waring et Jarrett, ainsi que Seavey et Tannenwald, ont observé ces excitations.

Abstract. — A review is given of the theory of spin waves and of the principal physical effects associated with spin waves. It is shown that the boundary condition $m_x = m_y = 0$ at the surface permits the excitation of spin waves by a uniform rf field. These excitations have been observed by Waring and Jarrett and by Seavey and Tannenwald.

A magnon is a quantized spin wave, just as a photon is a quantized electromagnetic wave and a phonon is a quantized elastic wave. Magnons exist only in an idealization, just as photons and phonons exist only in the idealization that terms in the complete hamiltonian representing interactions or collisions among the particles are neglected. Two phonons do not exist independently in the actual anharmonic potentials in real crystals, nor do two magnons exist entirely independently, but for both particles it can be shown that in the limit of small excitation amplitudes the interaction corrections are small. For magnons this has been shown [1] by Bethe, Hulthén, and Dyson; Bethe calculated explicitly the exact energy eigenvalues for two magnons on a line of atoms, and the interaction terms are found to be very small. The bearing of the electron gas in metals on the existence of phonons and magnons [2] probably does not upset the wave aspect of the phonon and magnon fields. This is explained by the fact that the plasma frequency is very much greater than the Debye frequency and the exchange frequency ; experimentally, it is known [3] from neutron and x-ray work that elastic waves have a reality in metals up to the top of the frequency spectrum, approximately 10^{13} cps. Neutron diffraction measurements [4] at Harwell give evidence of the existence of spin waves in iron.

It is perhaps more convincing to begin the treatment of magnons from a macroscopic viewpoint, just as the idea of a phonon originated with Debye in the quantization of a macroscopic elastic wave in an essentially homogeneous medium. We assert first that work must be done to distort or twist the local direction of magnetization from one point to another in a ferromagnetic specimen, just as work must be done to set up a mechanical deformation or strain in an elastic medium. The most convincing macroscopic evidence for our assertion comes from the observation of Bloch walls of finite thickness in domain patterns and, further from the observations of Rado [5] and others on exchange effects in ferromagnetic resonance in metals. Granted that the energy of a non-uniform distribution of magnetization directions is higher than for a uniform distribution, we are led directly by a well-known argument to the Landau and Lifshitz [6] expression

$$f_{ex} = A\left\{ (\nabla\alpha_x)^2 + (\nabla\alpha_y)^2 + (\nabla\alpha_z)^2 \right\} \tag{1}$$

for the exchange energy density in a cubic crystal ; here α is the unit vector in the direction of the local magnetization. Granted Eq. (1), we are then led by purely macroscopic reasoning to a theory of magnon fields.

The classical exchange hamiltonian density equivalent to Eq. (1) in the limit of small amplitudes (α_x, $\alpha_y \rightarrow 0$) is

$$\mathcal{H} = (A/M_s^2)\,[\omega_s^2(\nabla\pi)^2 + (\nabla\psi)^2], \tag{2}$$

where $\psi \equiv M_x$, and $\pi = M_y/\gamma M_s$ is the conjugate momentum density [7] ; here $\gamma = ge/2mc$. The Zeeman lagrangian leading to this result for π gives the correct spin resonance frequency for a uniformly magnetized ferromagnet in a magnetic field. The equations of motion in hamiltonian form are

$$\frac{\mathrm{d}\pi}{\mathrm{d}t} = -\frac{\partial\mathcal{H}}{\partial\psi} + \sum_\alpha \frac{\partial}{\partial x_\alpha}\frac{\partial\mathcal{H}}{\partial(\partial\psi/\partial x_\alpha)} ; \tag{3}$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = \frac{\partial\mathcal{H}}{\partial\pi} - \sum_\alpha \frac{\partial}{\partial x_\alpha}\frac{\partial\mathcal{H}}{\partial(\partial\pi/\partial x_\alpha)}. \tag{4}$$

Thus, using Eq. (2),

$$\mathrm{d}\psi/\mathrm{d}t = -2\gamma^2 A\nabla^2\pi ; \tag{5}$$

$$\mathrm{d}\pi/\mathrm{d}t = 2(A/M_s^2)\,\nabla^2\psi. \tag{6}$$

If we look for solutions of the form $e^{i(\omega t - k.r)}$, we find from Eqs. (5) and (6) that

$$\omega = (2\gamma A/M_s)\,k^2, \tag{7}$$

which is the dispersion relation for spin waves. Our discussion has been entirely parallel to the theory of phonon fields in a homogeneous medium. We have not assumed any particular atomic model.

The dispersion relation $\omega = (2\gamma A/M_s)\,k^2$ is identical with that of de Broglie waves for a non-relativistic particle of mass

$$m = \hbar M_s/4\gamma A, \tag{8}$$

and we may take over to magnons many of the results we know concerning the motion of electron wave packets. The group velocity of a spin packet is

$$v_g = \frac{\partial\omega}{\partial k} = \frac{4\gamma A}{M_s}\,k. \tag{9}$$

If $A \sim 2.10^{-6}$ erg/cm, $M_s \sim 10^3$, $\gamma \sim 2 \times 10^7$ $\sec^{-1}$ œrsted^{-1}, we have

$$\omega \sim 10^{-1}\,k^2\,\sec^{-1}, \tag{7}$$

$$m \sim 10^{-25}\ \mathrm{gm}, \tag{8a}$$

$$v_g \sim 10^{-1}\,k\ \mathrm{cm/sec}. \tag{9a}$$

For $\omega \sim 10^{11}\ \sec^{-1}$, we have $k \sim 10^6\ \mathrm{cm}^{-1}$ and $v/g \sim 10^5$ cm/sec. Their dispersive properties make it difficult to observe directly the motion of a packet of pure spin waves. The Sixtus-Tonks and related experiments deal with the propagation of Bloch walls.

Interaction of magnons and long wavelength phonons. — It is interesting to consider the interaction of spin waves and ultrasonic waves in ferromagnetic crystals [7]. This problem is treated most simply by setting up in hamiltonian form the equations of motion of the two types of fields, including the first-order magnetoelastic interaction as deduced from magnetostrictive measurements. The hamiltonian density $\mathcal{H}$ may be written as the sum of Zeeman, exchange, magnetoelastic interaction and phonon terms :

$$\mathcal{H} = \mathcal{H}_z + \mathcal{H}_e + \mathcal{H}_1 + \mathcal{H}_p. \tag{10}$$

Assuming (a) small deflection amplitudes of the magnetization from the z-axis and (b) elastic isotropy, we have

$$\mathcal{H}_z = (\omega_0/2\omega_2)\,(\omega_s^2\,\pi^2 + \psi^2), \tag{11a}$$

$$\mathcal{H}_e = (A/M_s^2)\,[\omega_s^2(\nabla\pi)^2 + (\nabla\psi)^2], \tag{11b}$$

$$\mathcal{H}_1 = (2b_2/M_s)\,(\omega_s\,\pi\,S_{yz} + \psi\,S_{zx})\ ; \tag{11c}$$

$$\mathcal{H}_p = (1/2\rho)\,(\pi_x^2 + \pi_y^2 + \pi_z^2) + \alpha\sum_{i,j} S_{ij}^2 + \beta(\Sigma\,S_{ii})^2 \tag{11d}$$

Here $\omega_0 = \gamma H_0$; $\omega_s = \gamma M_s$; b_2 is a magnetoelastic constant ; ρ is the density ; α and β are elastic constants ; π_i is a conjugate phonon momentum density ; and

$$S_{ij} = \frac{1}{2}\left(\frac{\partial R_i}{\partial x_j} + \frac{\partial R_j}{\partial x_i}\right). \tag{12}$$

is a strain component.

The strongest effects of the magnon-phonon coupling occur when both the wavelengths and frequencies of the magnons and phonons are equal. We have found a number of interesting properties of the motion :

(*a*) It appears that thin ferrite crystal sections might make effective magnetostrictive oscillators at microwave frequencies if the spin system is driven at the spin resonance frequency.

(*b*) The absorption of ultrasonic waves is very large when the resonance conditions are satisfied. This is probably the explanation of the effect discovered by Friedberg that the thermal conductivity of ferrites at helium temperatures depends on the magnetic field.

(*c*) The rotatory dispersion of ultrasonic waves is large. Even quite far from the spin resonance frequency the rotation of the plane of polarization of degenerate transverse elastic waves is large. The rotation of the plane of polarization is the basis for application as a non-reciprocal mechanical circuit element. A whole class of new devices, such as acoustic gyrators, becomes possible in principle.

Excitation of magnons by a uniform rf field [9]. — The excitation of magnetostatic modes by a non-uniform rf field has been discussed by White and Solt, Walker and others. We are not concerned here with the White-Solt effect, but with the excitation of exchange modes by a uniform rf field. The latter effect has probably been observed by Jarrett and Waring [10] in a crystal of $NiMnO_3$. We have proposed an explanation along the following lines : It is possible to excite exchange and magnetostatic magnons in a ferromagnet by a uniform rf field provided that spins on the surface of the specimen experience anisotropy interactions different from those acting on spins in the interior. The concept of surface anisotropy was first introduced by Néel.

Although the proportion of surface atoms in an actual specimen may be less than $1 : 10^4$ of the total number of atoms, yet a modest surface anisotropy can have a far-reaching effect on the transition probabilities for the excitation of spin waves in the microwave region. The essential point is simple : the surface anisotropy acts to pin down the surface spins. For a line of length L with the origin on one end, the modes will tend to have the form $\sin p\pi z/L$, where p is an integer. The modes of odd p will have a non-vanishing interaction with a uniform rf field, because the instantaneous transverse magnetic moment does not sum to zero over the line.

A surface anisotropy of the order of 10^{-2} of the isotropic exchange energy suffices to pin down the surface spins, according to the detailed calculation. The separation of successive magnon excitations in a crystal of thickness 3×10^{-4} cm will be of the order of 2p oersteds, where p is the mode index. Observation of the Jarrett-Waring effect should be a very good direct method for the determination of the exchange interaction constant A in Eq. (1).

It should further be possible to observe discrete resonance lines from the microwave excitation of magnons in metal films (of thickness 10^{-5} to 10^{-4} cm). Here the exciting field may be non-uniform because of eddy currents, but the damping of the spin waves temselves must be considered. We consider the result [7] $Q \approx c^2 k^2/\sigma\gamma M_s$ for the effect of eddy current damping on the spin waves. If $k \approx 10^5$ cm^{-1} ; $M_s \approx 10^3$, and $\sigma \approx 10^{17}$ esu, as for iron at room temperature, we have $Q \approx 10^4$, which suggests that a discrete excitation spectrum might be observable in ferromagnetic metallic films under the proper conditions. Tannenwald discusses this suggestion independently at this colloque.

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[2] Herring (C.) and Kittel (C.), *Phys. Rev.*, 1951, **81**, 809 ; see also Van Kranendonk (J.) and Van Vleck (J. H.), *Rev. Mod. Phys.*, 1958, **30**, 1. One of the questions raised by the latter authors is the effect of a disordered arrangement of spins on the existence of spin waves. It may not be entirely irrelevant toquote the analogous problem of phonons in a glass, where *long* wavelength phonons propagate relatively undisturbed by the fine-scale inhomogeneity of the underlying structure.

[3] See, for example, the collection of papers in the January, 1958 issue of the *Reviews of Modern Physics*, vol. 30.

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[7] Kittel (C.), *Phys. Rev.*, scheduled to appear in the issue of May 1, 1958.

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[9] Kittel (C.), *Phys. Rev.*, in the press.

[10] Waring (R. K.) and Jarrett (H. S.), private communication.

DISCUSSION

Mr. Vonsovskij. — In connection with your interesting report, I want draw attention to the work of my collaborators Turov and Irkhin (F. M. M., 1957), who showed in a pure phenomenological way the existence of two branches in the energy spectrum of the elastic-magnetic continuum Only in the limiting case do these branches transform to pure spin-wave and phonon branches. Besides, in the new work by Akhiezer and others (JETF, May 1958) there are many interesting results, which agree with yours.

Mr. Nagamiya. — How could you pin the spins on the surface ? Are you able to control the condition of the pinning ?

Mr. Kittel. — Surface spins are naturally in a different, and less symmetric, environment than spins in the interior. Thus, as pointed out by Néel, there is an intrinsic surface anisotropy. It should, however, be possible to change and to enhance the surface anisotropy by appropriate chemical or mechanical treatment. Thus with cobalt on oxide layer on the surface has been shown by Meiklejohn and Bean to change drastically the surface anisotropy.

Mr. Wolf. — Could the mechanism of " surface pinning ", which you mentioned, provide an explanation of the variation of line width, with polishing which has recently been observed in crystals of yttrium iron garnet by Rodrigue and Jones, and others ?

Mr. Kittel. — It appears to me not unlikely that surface pinning may be involved here.

Mr. Clogston. — Would you explain why the surface effects spread the resonance over a much wider range in this case, than in the case of small spherical crystals where only a small broadening is observed.

Mr. Kittel. — The surface effects are enhanced in thin specimens ; the separation of adjacent modes goes inversely as the square of the thickness.

Mr. Schlömann. — Is the separation of the satellite lines in the Jarrett-Waring experiment determined exclusively by exchange interaction, or is dipolar interaction also important ?

Mr. Kittel. — In the flat plate geometry the magnon energy is determined by the exchange interaction. In other geometries the dipolar interaction may be important.

Mr. de Gennes. — My remark is concerned with the experiments on thermal conductivity through a coupled system of phonons and spin waves. As the collisions between both types of excitations conserve both energy and wave vector (in the temperature range where U-processes are negligible) one could set up states of finite heat transfer which would not decay (apart from impurtity effects). This implies that the conductivity is then controled either by the phonon-defect or by the magnon-defect cross sections, and not by the phonon-magnon cross sections.

Mr. Kittel. — This is a very good point. It is not unlikely that the magnon-defect cross-section will depend on the magnetic field, because at a constant energy the magnon wavelength will depend on the field.