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Mechanical percolation : a small beam lattice study

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Résumé. — Nous étudions numériquement la dégradation des propriétés mécaniques d'un réseau carré de barres (obtenu par perforations périodiques d'une plaque) avec des angles de liaison constants entre barres en fonction du nombre d'éléments supprimés au hasard (problème de percolation de liens). En vue de limiter les fluctuations dues à la petite taille des réseaux, on a évalué simultanément le module élastique et la conductance électrique en fonction du taux de dégradation. Les résultats pour des faibles et pour des forts taux sont consistants respectivement avec les modèles de champ moyen et de percolation mécanique. En variant l'allongement des barres et, par suite, le rapport entre les effets de compression et de flexion, on modifie le rapport des énergies associées à ces deux mécanismes, rapport qui devient critique au seuil de percolation.

Abstract. — We numerically study the degradation of the mechanical properties of a two dimensional square lattice of beams with rigid bond angles as a function of the elements removed at random (bond percolation problem). In order to limit the fluctuations due to the small sample sizes, the elastic modulus is plotted *versus* the electrical conductance on the same lattices. The results for weakly and strongly degraded lattices are consistent with EMA and mechanical percolation results. By varying the aspect ratio of the beam and, thus, the ratio of compressive to bending effects, we modify the ratio of overall bending to compressive energies which becomes critical near threshold.

The mechanical behaviour of tenuous structures that can be studied using percolation concepts has received much attention recently.

In this Letter we present a simple simulation using a lattice of beams which reduces to a mechanical bond percolation problem.

The original approach [1] used a direct correspondance between the scalar electrical problem and the mechanical one; it was found inexact in various numerical simulation [2-4] and experiments [5, 6]. However, considering the interplay of force and momentum effects, the models that have been studied are rather poor. Two kinds of Hamiltonians were used :

- a Born Hamiltonian adding the effect of a scalar energy term plus a spring-like elasticity [2, 4]. However this Hamiltonian is not rotationally invariant, which is unphysical;

- a second Hamiltonian form containing a spring-like elastic energy and an angular force term [3]. This model has a minor drawback despite its physical relevance to describe the elasticity of partially connected grid structures : it mixes sites and bonds.

In two dimensional triangular lattices, it was shown [2] that, if there were no angular elasticity, the mechanical percolation threshold was larger than the conductive (or connexity) one as the presence of fully triangulated elements was necessary to insure mechanical rigidity. Similarly

in a space dimension equal to 3, simple connexity does not imply any elastic stiffness because tension does not cost energy.

We consider here an incomplete network of beams as a model for mechanical percolation which does not suffer from the above drawbacks [12]. The behaviour of an individual beam is dictated by the classical theory of elasticity of one dimensional media [7]. In its generality, the model includes 4 contributions in the elastic energy :

$$2 H = \int \frac{N^2}{ES} ds + \int \frac{M_F^2}{EI} ds + \int \frac{T^2}{GJ} ds + \int \frac{M_T^2}{GJ} ds. \quad (1)$$

The integrals are taken along the beams ; E and G are the Young's and shear moduli, N is a *normal force* (extension-compression), M_F a *flexion* (bending) torque ; T a *shear force* ; M_T a *torsion torque*.

S is the area of a beam section, I and J are two moments of inertia for flexion and torsion. The Hamiltonian is rotationally invariant and takes into account the displacement and rotation degrees of freedom at each node. The torsion term disappears in 2 dimensions [12].

We have studied a bond percolation problem on a 2D square lattice of beams, both experimentally and numerically. Only the numerical results are presented here. The angles between beams at the nodes are assumed to be rigidly fixed to 90° . A computer simulation has been made on rather small lattices (up to 20×21) ($n \times (n + 1)$ in order to preserve self-duality of the lattice). We used a relaxation algorithm. At the same time we calculate the electrical conductance and elastic modulus of the whole network under extension treating separately the contributions of normal force free energy (extension-compression) and moment energy (flexion). We neglect the energetic contribution of shear forces. These terms are usually much smaller than flexion ones in beam elasticity. This is even more so in the present case near percolation threshold because the lever effect of long arms (measured by the critical correlation length) reinforces the rôle of flexion terms with respect to normal and shear forces which are of the same order of magnitude near threshold.

The model has one free parameter : the ratio I/S which gives the relative importance of bending *versus* extension effects. We call α the dimensionless ratio $I/(Sa^2)$ where a is the length of one beam. If b is the beam width, then [7] :

$$\alpha = (1/12) (b/a)^2. \quad (2)$$

The larger α , the harder it will be to bend a beam for the same normal elasticity E .

When beams are cut at random, the elastic modulus variation appears to be highly correlated with that of the conductance of the same lattice. So we drastically reduce the fluctuations due to the finite size of the lattice in both problems by analysing the mechanical results as a function of the conductance ones rather than plotting them separately as a function of the fraction $q (= 1 - p)$ of missing bonds. In particular, at the threshold value of a particular lattice, which can be different from the large sample threshold ($p_c = 1/2$), both elastic modulus K and conductance G go to zero. Figure 1 gives the result for this variation for one value of aspect ratio normalized to the values G_0 and K_0 , for an intact network (curve A). Near $p = 1$, we can confirm the calculation by comparing with the behaviour dictated by the self consistent approach. For the conductance we have the classical result [8]

$$G = G_0(2p - 1). \quad (3)$$

A similar calculation (see e.g. [14]), whose principle is sketched in figure 2, gives :

$$K = K_0[(1 + \lambda)p - \lambda] \quad (4)$$

Fig. 1. — (A) Evolution of the elastic modulus (K) versus the conductance (G) of a 20×21 grid during its degradation; (B) part of K due to tensile elasticity; (C) part of K due to flexion elasticity; (D) prediction of E.M.A.

Fig. 2. — In an homogeneous lattice (strength of a bond : k_H) if we remove the bond A-B, set instead a bond of strength k and then apply at the end of the grid a force F , we observe a displacement $u = F/k_H$ $\phi(k/k_H)$. The E.M.A. states that the equivalent lattice of the structure under random degradation with a fraction p of intact bonds (k_0), is obtained if the average displacement of A-B is the one of the complete homogeneous lattice. So $(1 - p) \phi(0) + p \phi(k_0/k_H) = \phi(1)$ determines $k_H(k_0, p)$.

λ is a function of α and is defined in the following way. Let us consider an infinite homogeneous network out of which one single bond (A-B) has been cut.

This system behaves like a spring with regard to the relative displacement of A and B. Its elastic constant is written k/λ ; k is the strength of an elementary bond.

Equations (3) and (4) lead to :

$$K/K_0 = [(1 + \lambda)/2] G/G_0 + (1 - \lambda)/2. \quad (5)$$

The linear variation (D) plotted on figure 1 is coherent with our results.

An inspection of our calculations carried over several values of α shows that the larger the value of α (or the shorter the beam), the larger the validity of the linear law. This can be understood as follows : for large α the flexion energy remains small in comparison with the total energy and most of the bonds are in a traction (or compression) state. This corresponds to the linear approximation above, which deals with a redistribution of tensile stresses. Conversely, for low α , the removing of a bond creates a larger amount of bending energy in the structure. The compensation should take into account the mean strain on the beam.

In figure 3, we have plotted the ratio of energy stored as a flexion term over the total elastic energy as a function of the conductance (which is an indirect evaluation of the degree of degradation of the lattice) we see that, for a given aspect ratio, the bending energy increases as one approaches percolation threshold. As initially discussed by Kantor and Webman [8], the existence of large arms in the percolating structure near p_c gives a stronger weight to bending moments with respect to normal forces as one approaches p_c . This effect takes place farther from p_c for smaller α (larger aspect ratio). Thus, close enough to p_c , the critical elasticity is governed by flexion. Such deformations take place on singly connected bonds and one can assume that the clusters (or blobs) between such bonds rotate rigidly at no energetic cost. The effect of a traction applied at the two edges of the network is to bend the singly connected bonds and to bring them in a straighter conformation. This is the essence of the argument in Kantor and Webman work which can be translated directly in the language of beam elasticity.

Fig. 3. — Ratio of the fraction of the elastic modulus due to flexion energy over the total modulus *versus* the conductance of the structure.

The resulting effect is an increase of the elasticity critical exponent τ ($K \propto (p - p_c)^\tau$) with respect to the conductance one t ($G \propto (p - p_c)^t$) by a value 2ν , $\tau = t + 2\nu$ [15] which is the larger scale over which the linkblob description applies (the macro length of lever arms). We do not expect to get an accurate determination of τ from our small lattices. However if we plot K versus G near p_c in log-log units, we can get an estimate of the ratio τ/t .

The results of figure 4 are for individual realizations with aspect ratio 1 and 10. From a least square fit on the 10 last points on the upper right (smaller K and G), we get a slope between 2.2 and 2.4. Similar values were obtained by considering larger data averages and were also found on smaller 10×10 lattices. Using the known values $t = 1.29$ we get a value of $\tau \simeq 3$ which is clearly much larger than t but still smaller than values obtained on larger lattices $\tau \simeq 3.5$ [10]; 3.96 ± 0.04 [11]; 3.2 ± 0.4 [12].

Fig. 4. — Evolution of the logarithm of the elastic modulus versus the logarithm of the total conductance. The upper set of points refers to $\alpha = 10$ whereas the lower one is obtained for $\alpha = 1$.

Experiments are in progress on small (20×21) metallic square grids which support qualitatively the above description. They also indicate that non-linear mechanical effects take place for decreasing applied forces as the percolation threshold is approached. Extensive work is also carried in the laboratory using degraded honeycomb lattices [15].

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