Evidence of capillary hyperdiffusion in two-phase fluid flows

J.-C. Bacri, C. Leygnac, D. Salin

► To cite this version:

J.-C. Bacri, C. Leygnac, D. Salin. Evidence of capillary hyperdiffusion in two-phase fluid flows. Journal de Physique Lettres, 1985, 46 (11), pp.467-473. 10.1051/jphyslet:019850046011046700 . jpa-00232546

HAL Id: jpa-00232546

<https://hal.science/jpa-00232546>

Submitted on 1 Jan 1985

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Classification

Physics Abstracts

47.55K — 47.55M — 51.20

Evidence of capillary hyperdiffusion in two-phase fluid flows (*)

J.-C. Bacri, C. Leygnac and D. Salin

Laboratoire d'Ultrasons (**), Université Pierre et Marie Curie,
Tour 13, 4 place Jussieu, 75230 Paris Cedex 05, France

(Reçu le 1^{er} mars 1985, accepté le 15 avril 1985)

Résumé. — Nous suivons l'évolution dans le temps et l'espace des profils de saturation lors d'écoulements non miscibles dans un grès poreux au moyen d'une technique acoustique. Au cours de l'injection d'un fluide totalement mouillant (eau-imbibition) dans un échantillon prémouillé (à saturation irréductible en eau), nous avons observé que la diffusion capillaire était hyperdiffusive, c'est-à-dire que le profil de concentration s'étend plus loin que dans le cas de la diffusion classique.

Abstract. — By means of an acoustic technique we record the saturation time and space dependence in a porous medium consisting of sandstone into which immiscible flows are driven. When a totally wetting fluid (here water) injection is performed (imbibition), in a prewet sample (i.e. a medium at irreducible water saturation), the capillary diffusion presents symptoms of hyperdiffusion, which means that the saturation profile extends more widely than in classical diffusion.

1. Introduction.

The economic importance of oil recovery has been stimulating for a long time both theoretical and experimental research on two-phase fluid flows in porous media [1-3]. Recent theoretical studies [4] have clarified the physical phenomena involved in a frontal advance process : convection coupled with diffusion. Convection, which is dominant at large velocities, gives out straight displacement profiles. Interfacial tensions between the two non miscible fluids are responsible for their mutual diffusion, which leads to a broadening of the saturation profile. This broadening is typical of the physical conditions involved when a displacement is being performed :

— if the two fluids partially wet the porous medium, the shape of the edges of the saturation profile can be accounted for in a percolation-type description [4-6] ; the profile thus obtained corresponds to a process called « hypodiffusive » [4, 7] (in comparison with the more diffusive classical diffusion) ;

— on the other hand, if a film or (and) interconnected pockets of a totally wetting fluid (here water) are present in the medium (« prewet » the medium) before the imbibition is started, no proper front will be observed, but a profile extending over the whole sample, and decreasing less rapidly than the classical diffusion profile : the process is then called « hyperdiffusion » [7].

(*) Partially supported by the Institut Français du Pétrole.

(**) Unité Associée au Centre National de la Recherche Scientifique, N° 040789.

Of course, a suitable analysis of these phenomena requires a precise knowledge of how saturation varies with time and space, which discards an effluent analysis technique. An acoustic technique [8] enables us to record the saturation time-dependence in different cross-sections of a rectangular block of sandstone. A precise description of the calibration of the tool and of the first imbibition and drainage processes will be given in a forthcoming paper. Here we concentrate on the second imbibition process (performed on a porous medium at irreducible water saturation, with a constant water injection rate), where, in our opinion, we have observed the evidence of a case of hyperdiffusion.

2. Basic equations.

Let us first recall some aspects of the theory described by P. G. de Gennes [4]. In the quasi-static limit, the flow equations are derived from water J_w and oil J_o currents (dimensions of velocity) through the incompressible fluid conservation equations :

$$\nabla \cdot \mathbf{J}_w + \frac{\partial \Phi}{\partial t} = 0 \quad (1)$$

$$\nabla \cdot \mathbf{J} = 0 \quad (2)$$

where $\mathbf{J} = \mathbf{J}_w + \mathbf{J}_o$ is the total fluid current and Φ is the wetting fluid (water) volume fraction of the pores. The most general linear relations between the currents and local pressures (p_w and p_o) of the two fluids are :

$$\mathbf{J}_w = \Sigma_w(\Phi) (-\nabla p_w) - \Lambda(\Phi) (\nabla p_o) \quad (3)$$

$$\mathbf{J}_o = -\Lambda(\Phi) (\nabla p_w) + \Sigma_o(\Phi) (-\nabla p_o) \quad (4)$$

where Σ_w and Σ_o are the permeability to viscosity ratios for each fluid. The cross term coefficient is expected to be zero if each pore is occupied by a single fluid, i.e. in the case of partial wetting ; if one of the fluids completely wets the solid, Λ can be of importance. The last ingredient in the theory is the capillary pressure $\Pi(\Phi) = p_w - p_o$ which can be determined experimentally [1-3]. When neglecting gravity but including the cross term (Λ) for a one-dimensional flow and with an imposed injection rate, the following differential equation in $\Phi(x, t)$ can be drawn from the set of equations (1) to (4) :

$$\frac{\partial \Phi}{\partial t} = -J \frac{\partial}{\partial x} \rho(\Phi) + \frac{\partial}{\partial x} \left(\Sigma(\Phi) \frac{\partial \Pi(\Phi)}{\partial x} \right) \quad (5)$$

with

$$\rho(\Phi) = \frac{\Sigma_w(\Phi) + \Lambda(\Phi)}{\Sigma_w(\Phi) + \Sigma_o(\Phi) + 2\Lambda(\Phi)} \quad (6)$$

and

$$\Sigma(\Phi) = \frac{\Sigma_o(\Phi) \Sigma_w(\Phi) - \Lambda^2(\Phi)}{\Sigma_w(\Phi) + \Sigma_o(\Phi) + 2\Lambda(\Phi)} \quad (7)$$

$\rho(\Phi)$ is the fractional permeability of the wetting phase and $\Sigma(\Phi)$ the mutual permeability of the two fluids.

To bring out the main features of (5), let us rewrite it as :

$$\frac{\partial}{\partial t} \Phi(x, t) = -J \frac{d\rho(\Phi)}{d\Phi} \frac{\partial \Phi}{\partial x} + \frac{\partial}{\partial x} \left(\Sigma(\Phi) \frac{d\Pi(\Phi)}{d\Phi} \frac{\partial \Phi}{\partial x} \right). \quad (8)$$

Ignoring the second term of the right-hand side in (8) we have a simple convection equation : the profile $\Phi(x, t)$ is then controlled by a local concentration-dependent drift velocity $c(\Phi) = J \frac{d\rho}{d\Phi}$. Generally speaking ($\Lambda = 0$), $c(\Phi)$ vanishes for the two extremal values of Φ which are $\Phi = \Phi_{\text{IR}}$ (irreducible water-saturation) and $\Phi = \Phi^*$ (residual oil saturation); $c(\Phi)$ reaches a maximum in between. As $\Phi(x, t)$ cannot be double-valued for any (x, t) Buckley and Leverett [1-3] introduce a discontinuity (« shock wave ») corresponding to a vertical profile drifting at a velocity $c(\Phi_s)$ which can be defined here through the Weldge construction [1-3]. Let us notice that this velocity is directly proportional to the injection flow rate.

The second term of the RHS of (8) corresponds to a non-linear diffusion [10], with a concentration-dependent diffusion coefficient $D(\Phi) = \Sigma(\Phi) \frac{d\Pi(\Phi)}{d\Phi}$; this term is responsible for the broadening of the shock front, but in very different ways depending on how $D(\Phi)$ varies with Φ . Let us emphasize that the water current J_w is strictly zero for $\Phi \leq \Phi_{\text{IR}}$ as well as the oil current J_o for $\Phi \geq \Phi^*$, whereas $\frac{d\Pi}{d\Phi}$ (derivative of capillary pressure) diverges at Φ_{IR} . Two cases are to be distinguished :

— if $\Lambda = 0$, $\Sigma(\Phi)$ vanishes at both ends of the saturation range, generally more drastically than $\frac{d\Pi}{d\Phi}$ diverges. Thus $D(\Phi)$ falls down to zero for both Φ_{IR} and Φ^* [1] saturation values : this is the hypodiffusive case (h) for which the profile is sharper than in the classical case.

— If $\Lambda \neq 0$, Σ can be different from zero when Φ gets close to Φ_{IR} , in which case, the capillary pressure imposes its divergence on $D(\Phi)$: diffusion will then rapidly increase when getting close to Φ_{IR} . This hyperdiffusive case (H) is sketched in figure 1, as well as the hypodiffusive one (h); the dashed curve corresponds to classical Gaussian diffusion (G : constant diffusion coefficient).

3. Experiment and results.

The totally wetting fluid is a sodium chloride brine (40 g/l) which has a viscosity of $\mu_w = 1.0$ cP (at $T = 22$ °C) and a density of $\rho_w = 1.026$ g/cm³. The other fluid is an oil ($\rho_o = 0.755$ g/cm³ and

Fig. 1. — Typical space-dependent saturation profiles for different diffusion processes : G Gaussian diffusion (constant diffusion coefficient D); h (--) hypodiffusion (D vanishes at both extreme Φ_{IR} and Φ^* values); H (—) hyperdiffusion (D diverges at Φ_{IR}).

$\mu_o = 1.5$ cP). The interfacial tension between the two fluids is $\gamma = 32$ erg/cm². Our porous medium is a $4.3 \times 4.3 \times 15.5$ cm³ sandstone core, coated with epoxy. Flows are driven vertically along the larger dimension at constant flow rates varying from 0.6 cm³/h to 24 cm³/h. In this range of injection rates, no fingering instabilities will occur, owing to the fact that the viscosity contrast is weak. The porous medium has a porosity of 21.5 % and a monophasic oil permeability of 110 mD (1 mD = 10^{-11} cm²); at $\Phi = \Phi_{IR}(J_w = 0)$, the measured oil permeability is 80 mD and at $\Phi^*(J_o = 0)$, the measured water permeability is 4 mD. The ratio $\frac{\Delta\rho g \Sigma_o}{u}$ (gravity velocity *versus* injection velocity) will thus remain inferior to 3 % for the smallest injection rate. This justifies neglecting gravity in (5).

Saturation measurements are derived from the velocity variation of a sound wave in 10 cross-sections of the sample. The acoustic wave is generated and analysed thanks to 10 transmitter-receiver pairs of transducers laid every 1.5 cm along the larger side of the sample. The spatial resolution in this direction reaches 3 mm; the ten positions are scanned in less than 2 mm through automatic procedure. Our calibration curve of velocity variations with Φ and the accuracy of relative velocity measurements (better than 5×10^{-4}) provide an overall precision in Φ better than 1 %. In the experimental procedure, the dry sample is first saturated in oil with a vacuum impregnation technique ($\Phi = 0$). A first imbibition is then performed (water injection) at the end of which $\Phi = \Phi^*$. At this stage we drain (oil injection at a low flow rate which is increased step by step) in order to get the maximum of oil into the sample ($\Phi = \Phi_{IR}$). Further

Fig. 2. — Time dependence of the water saturation $\Phi(t)$ in four cross-sections of the sample (from left to right $x = 3.8, 6.8, 9.8, 12.8$ cm from the inlet onwards), recorded during three imbibition runs performed on a prewet sandstone (i.e. at irreducible water saturation $\Phi_{IR} = 46\%$) with three different constant water injection rates (a : 12 cm³/h; b : 3 cm³/h; c : 1.2 cm³/h). $\Phi^* = 64\%$ corresponds to the residual oil saturation.

imbibitions can be carried out in the same starting (Φ_{IR}) and finishing (Φ^*) conditions (we have to drain as explained before, between two successive imbibitions).

The three figures (a, b, c) in figure 2 correspond to imbibition processes at three different flow rates (12, 3, 1.2 cm³/h). The four curves in each figure correspond to the saturation time-dependence $\Phi(t)$ for four different pairs of transducers at $x = 3.8, 6.8, 9.8, 12.8$ cm from the inlet. Two main features can be deduced from figure 2 : i) when Φ tends to Φ^* , every curve presents a cusp. ii) On the opposite side ($\Phi \rightarrow \Phi_{IR}$), as the flow rate is decreased, no shock profile is observed any more, and for the smallest injection rate, the diffusion process expands all over the sample, as soon as the imbibition is started.

Equation (8) (which so far has no analytical solutions) is non-linear ; as such, it cannot strictly be solved by superimposing diffusion on convection. However, we first reason in terms of a shock wave (moving at the velocity $u = c(\Phi_s)$) to which we add diffusion. A natural variable change in (8)

is then $\xi = \frac{x - ut}{\sqrt{t}}$ which leads to

$$\frac{\xi}{2} \frac{d\Phi}{d\xi} = - \frac{d}{d\xi} \left[D(\Phi) \frac{d\Phi}{d\xi} \right]. \quad (9)$$

The velocities u involved in ξ are deduced from large flow rate values for which the shock wave is dominant, and then reduced in the flow rate proportion (from $u = 50$ cm/h for 24 cc/h down to $u = 1.25$ cm/h for 0.6 cc/h).

Figure 3 displays a plot of Φ versus ξ for the two smallest flow rates. Let us notice that, at a given flow rate (u), the different plots $\Phi(\xi)$ match for every x . When x is varied, the curves do look universal (Fig. 3) for the smallest flow rates (diffusion is then dominant), but not when the flow rate is increased, the hypo- and hyperdiffusive character still remaining evident. As we are mainly interested in diffusion, ξ stands as a good variable to be chosen. Moreover it allows us to derive the saturation dependence of D from equation (9) through the relation [10] :

$$D(\Phi) = - \frac{1}{2 \frac{d\Phi}{d\xi}} \int_{\Phi}^{\Phi^*} \xi(\Phi) d\Phi \quad (10)$$

Fig. 3. — Variation of the water saturation Φ with the reduced variable $\xi = (x - ut)/\sqrt{t}$, for our two smallest flow rates (--- : $u = 1.25$ cm/h, and — : $u = 2.5$ cm/h) and respectively $x = 6.8$ cm and $x = 9.8$ cm.

Fig. 4. — Variation of the diffusion coefficient $D(\Phi)$, in a semi-logarithmic scale *versus* the water saturation Φ ; these variations are derived from figure 3 through relation (10). The continuous curve through the data is a guide for the eye.

Thus for a given Φ , $D(\Phi)$ will be computed from a slope measurement $\left(\frac{d\Phi}{d\xi}\right)$ and a surface measurement $\left(\int_0^\Phi \xi d\Phi\right)$ in figure 3. $D(\Phi)$ is plotted in figure 4.

4. Discussion.

Obviously $D(\Phi)$ diverges close to Φ_{IR} (hyperdiffusion) and it disappears close to Φ^* (hypodiffusion); this is our central experimental result. Let us compare these characteristic features with what is observed in a first imbibition process (in which Φ varies from zero to Φ^*); in this case, $D(\Phi)$ vanishes at both extremal saturation values (also hypodiffusive) and exhibits a maximum for $\Phi \sim 30\%$, $D(0.30) \sim 10^{-5} \text{ cm}^2/\text{s}$. In the second imbibition process, $D(\Phi)$ also falls off to zero when Φ is getting close to Φ^* while still displaying larger values than in the first imbibition case.

Theory assigns a critical behaviour to $\Sigma(\Phi)$ and $\Pi(\Phi)$ in Φ_{IR} and Φ^* vicinities. $D(\Phi)$ should then be described in terms of power laws in these saturation ranges :

— When Φ increases to Φ^* [11], $D(\Phi)$ behaves as $D(\Phi) \sim D^*(\Phi^* - \Phi)^{n^*}$ with $n^* = 1$ (which is typical of a hypodiffusive case) and $D^* \simeq 10^{-3} \text{ cm}^2/\text{s}$. Let us recall that a percolation type description [4] concludes in favour of a $D(\Phi) \sim (\Phi^* - \Phi)^{\tilde{m}-1}$ (with $\tilde{m} \gtrsim 4$) behaviour. Indeed, the predicted decrease is more drastic than what is observed here, but we have computed $D(\Phi)$ over a 2 to 8 % saturation range (from Φ^* downwards), and there is no way to identify a more or less drastic behaviour under 2 % [11].

— In the neighbourhood of Φ_{IR} , the $D(\Phi)$ divergence can be directly deduced from the time and space saturation dependence : $\Phi - \Phi_{\text{IR}} \sim \frac{t}{(x - ut)^2} \sim \frac{1}{\xi^2}$ which leads to $D(\Phi - \Phi_{\text{IR}})_{\Phi \rightarrow \Phi_{\text{IR}}} \sim D_{\text{IR}}(\Phi - \Phi_{\text{IR}})^{n_{\text{IR}}}$ with $n_{\text{IR}} = -1$ (hyperdiffusive case) and $D_{\text{IR}} \simeq 10^{-4} \text{ cm}^2/\text{s}$. The only case yet known of predicted hyperdiffusion in literature [7] concerns water-film growth in a fractal material; the power law ($n_{\text{IR}} = -1$) observed close to Φ_{IR} is indeed in good accordance with this prediction, contrary to the corresponding prefactor ($10^{-10} \text{ cm}^2/\text{s}$) which is in complete disagreement with our $10^{-4} \text{ cm}^2/\text{s}$ observed value (but the predicted case concerns a single wetting film whereas we deal here with two fluids).

It is remarkable that $\left. \frac{d\Pi}{d\Phi} \right|_{\Phi \rightarrow \Phi_{\text{IR}}}$ does display a divergence similar to the diffusion coefficient one close to Φ_{IR} ($\frac{d\Pi}{d\Phi} \simeq 10$ atmospheres when $\Phi - \Phi_{\text{IR}} = 2\%$). This should prevent $\Sigma(\Phi)$ close to Φ_{IR} from falling off to zero : the permeability deduced from our diffusion coefficient measurement and the $\left. \frac{d\Pi}{d\Phi} \right|_{\Phi \rightarrow \Phi_{\text{IR}}}$ value (through $D(\Phi) = \Sigma(\Phi) \frac{d\Pi}{d\Phi}$) reaches 1 mD, which is much larger than the predicted value based on a $A = 0$ assumption (then we would have a permeability associated to Σ of less than 10^{-3} mD when $\Phi - \Phi_{\text{IR}} = 2\%$).

An evaluation of the cross-term A in the basic equations (3)-(4) might provide theoretical predictions coherent with the observed effect.

5. Conclusion.

Thanks to a local probe technique (acoustics), we follow the saturation time and space-dependence in a block of sandstone during a second imbibition process, for which a « film » and(or) interconnected pockets of a totally wetting fluid are initially present in the medium. The saturation profile thus obtained presents a hyperdiffusive behaviour for saturation values close to Φ_{IR} , whereas it remains hypodiffusive for saturation values close to Φ^* .

If the hyper- and hypodiffusion is patent in this experiment, a theoretical understanding of these phenomena would require a better knowledge of the elementary processes involved in two-phase fluid flows, in relation with their respective wettabilities.

Acknowledgments.

We are greatly indebted to Dr. Bieber and Mr. Schliter for stimulating discussions.

References

- [1] SCHEIDEGGER, A., *The physics of flow through porous media*, 3rd ed. (University of Toronto Press) 1974.
- [2] MARLE, C. M., *Multiphase flows in porous media* (Technip, Paris) 1981.
- [3] DULLIEN, F. A. L., *Porous media, fluid transport and pore structure* (Academic Press, New York) 1979.
- [4] DE GENNES, P. G., *Physicochemical Hydrodynamics* 4 (1983) 175.
- [5] GUYON, E. and DE GENNES, P. G., *J. Méc.* 17 (1978) 403.
- [6] WILKINSON, D., *Phys. Rev. A* 30 (1984) 520.
- [7] DE GENNES, P. G., to be published.
- [8] BACRI, J.-C., LEYGNAC, C. and SALIN, D., *J. Physique Lett.* 45 (1984) L-767.
- [9] LENORMAND, R., ZARCONI, C. and SARR, A., *J. Fluid Mech.* 135 (1984) 337.
- [10] CRANK, J., *The Mathematics of diffusion* (Clarendon University Press, Oxford) 1956.
- [11] In fact during imbibition processes the maximum saturation in the profile is closer to Φ_s than to Φ^* (Fig. 8 from Ref. [4]).