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A. Gorius, Bernard Perrin, S. Fauve. Experimental study of convection between poorly conducting media. *Journal de Physique Lettres*, 1985, 46 (7), pp.295-299. 10.1051/jphyslet:01985004607029500 . jpa-00232514

HAL Id: jpa-00232514

<https://hal.science/jpa-00232514>

Submitted on 1 Jan 1985

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LE JOURNAL DE PHYSIQUE-LETTRES

J. Physique Lett. **46** (1985) L-295 - L-299

1^{er} AVRIL 1985, PAGE L-295

Classification

Physics Abstracts

47.25Q

Experimental study of convection between poorly conducting media

A. Gorius, B. Perrin and S. Fauve

Groupe de Physique des Solides de l'Ecole Normale Supérieure,
24, rue Lhomond, 75231 Paris Cedex 05, France

(Reçu le 12 décembre 1984, accepté le 8 février 1985)

Résumé. — Nous présentons une étude expérimentale de convection thermique dans une couche de mercure limitée par des plaques faiblement conductrices de la chaleur. Une comparaison avec le cas de parois parfaitement conductrices montre une nette décroissance du nombre de Rayleigh critique. Une étude du champ de températures permet l'investigation de la structure convective.

Abstract. — An experimental study of thermal convection in a mercury layer bounded by poorly heat conducting plates is reported. A comparison with the case of perfect heat conducting boundaries shows a striking decrease in the critical Rayleigh number. A study of the convective temperature field provides useful information about the convective structure.

The study of convection in a horizontal layer of fluid heated from below has been performed intensively since the first analyses of Lord Rayleigh [1].

Quite complete theories have been elaborated, first in the case where the upper and lower bounds of the layer are supposed to be perfect heat conductors, and during the last few decades, in the more realistic situation of horizontal boundaries of arbitrary conductivities [2] or almost perfectly insulating ones [3]. This so-called « fixed heat flux » convection occurs, for example, in several geophysical or astrophysical models.

From the experimental point of view, a great variety of measurements have been successfully performed in the case of almost vanishing temperature perturbations at the conducting boundaries.

We present here a first experimental investigation in a situation of nearly insulating upper and lower bounds and show that both the convection threshold and the convective heat flux are much lower than those corresponding to perfectly conducting horizontal walls.

Figure 1 shows the experimental apparatus we used : a layer of mercury (heat conductivity $k_F = 6.18 \times 10^{-2} \text{ W K}^{-1} \text{ cm}^{-1}$) of depth $d = 0.8 \text{ cm}$ is bound by two horizontal layers of thickness d made of a resin (stycast 2850 FT : heat conductivity $k_s = k_F/\beta_1$ with $\beta_1 = 4.35$). These two walls are limited by two plates of copper (heat conductivity $k_c = k_F/\beta_c$ with $\beta_c = 1.5 \times 10^{-2}$); the upper one is maintained at a constant temperature $T_0 = 25^\circ\text{C} \pm 5 \times 10^{-3}^\circ\text{C}$. The lower one is heated by a thermal flux P to an adjustable temperature $T_0 + \Delta T$. The mercury cell is a parallelepiped of square horizontal section $L \times L$ with an aspect ratio $\Gamma = L/d = 4$. The two resin plates can be either removed or replaced by stainless steel plates (heat conductivity $k'_s = k_F/\beta_M$ with $\beta_M = 0.44$) so that we can study the cases of « nearly insulating » ($\beta_1 = 4.35$), moderately conducting ($\beta_M = 0.44$), or « perfectly conducting » ($\beta_c = 0.015$) upper and lower boundaries for the same mercury cell. Two platinum linear temperature dependent resistors are flushed with the mercury, located on the same vertical in the stycast, stainless steel or copper plates (Fig. 1). They provide a measurement of the local temperature difference ΔT_v across the cell, with a sensitivity of 10^{-3} K .

Fig. 1. — Experimental apparatus. The mercury cell (illustrated by rolls) has dimensions $L \times L \times d$ with an aspect ratio $\Gamma = L/d = 4$ ($d = 0.8 \text{ cm}$). Heat conductivities : mercury, $k_F = 6.18 \times 10^{-2} \text{ W K}^{-1} \text{ cm}^{-1}$; stycast, $k_s = 1.42 \times 10^{-2} \text{ W K}^{-1} \text{ cm}^{-1}$; copper, $k_c = 4.18 \text{ W K}^{-1} \text{ cm}^{-1}$. The • indicate the loci of the platinum bolometers.

In these three experimental situations, we have studied $P/\Delta T_v$ as a function of P . Let us note that, as long as no convection has occurred and whatever the value of β , the ratio $P/\Delta T_v$ is proportional to the cell's heat conductivity; ΔT_v or P then represent the experimental external constraint applied to the fluid layer. But when motion has set in, and for $\beta \neq 0$, ΔT_v has a local significance and only P remains the experimental constraint on the system. So, up to the convection onset, we can use the Rayleigh number R and compare its experimental critical values R_c at onset with theoretical predictions in both cases ($\beta \neq 0$ and $\beta = 0$). R is then defined as usual :

$$R = \frac{g\alpha d^3}{\nu K_F} \Delta T_v$$

where g is the gravitational acceleration, α is the fluid's thermal expansion coefficient, K_F is its thermal diffusivity and ν is its kinematic viscosity.

Fig. 2. — Heat conductivity $P/\Delta T_v$ of the cell *versus* P . (●) nearly insulating boundaries ($\beta_I = 4.35$); (+) moderately conducting boundaries ($\beta_M = 0.44$); (□) perfectly conducting boundaries ($\beta_C = 0.015$). The critical heating powers P_c are $P_c(\beta_C) = 1.32$ W, $P_c(\beta_M) = 1.28$ W and $P_c(\beta_I) = 0.85$ W.

In figure 2 we have plotted $P/\Delta T_v$ *versus* P for the three values of β (copper : $\beta_C = 0.015$, stainless steel : $\beta_M = 0.44$, and stycast : $\beta_I = 4.35$). The most striking result is the observed difference in the convection thresholds :

$$P_c(\beta_I) = 0.85 \pm 0.05 \text{ W},$$

$$P_c(\beta_M) = 1.28 \pm 0.04 \text{ W},$$

$$P_c(\beta_C) = 1.32 \pm 0.05 \text{ W}.$$

These values correspond to the following Rayleigh numbers : $R_c(\beta_I) = 1\,640 \pm 80$, $R_c(\beta_M) = 2\,400 \pm 100$ and $R_c(\beta_C) = 2\,500 \pm 100$, so that convection sets in for a much smaller R for nearly insulating boundaries than for moderately or perfectly conducting ones. The theoretical calculation, *via* the assumption of an infinite horizontal extent of the fluid layer, leads to values of R_c of 1 180, 1 500 and 1 708 for the insulating, moderately, or perfectly conducting boundaries, respectively [4]; the observed differences arise from sidewall effects which are known to stabilize the system [5].

Another remarkable feature of figure 2 is that the slopes of the conductivity curves just above the convective threshold, say $S(\beta)$, are very different :

$$S(\beta_I) = 0.10 \text{ K}^{-1};$$

$$S(\beta_M) = 0.11 \text{ K}^{-1};$$

$$S(\beta_C) = 0.30 \text{ K}^{-1}.$$

Let us emphasize that these values are provided by local measurements and so merely represent the local convective heat transport for $\beta \neq \beta_C$.

It is possible, with the experimental arrangement of figure 1 to measure the global heat conductivity $P/\Delta T$, and the slope $\bar{S}(\beta)$ for $P > P_c$, deduced from the curve $P/\Delta T = f(P)$, is related to the global convective heat flux across the mercury cell. In the case of stainless steel plates, we found $\bar{S}(\beta_M) = 0.12 \text{ K}^{-1}$, which is close to the local value $S(\beta_M)$. The ratio $\frac{\bar{S}(\beta_M)}{S(\beta_C)} \simeq 0.4$ measures the ratio of the convective heat fluxes in both cases and shows that the convective heat transport, as convection has set in, is far less important for moderately conducting boundaries than for

almost perfectly conducting ones. For stycast plates, we can obtain no information from this study since the global heat conductivity $P/\Delta T$ of the apparatus is hardly affected by convection.

Apart from the drop of R_c , the linear theory also predicts an increase of the convective structure wavelength $\lambda(\beta)$ with β [2]. In particular, for $\beta_1 = 4.35$ (stycast plates), $\frac{\lambda(\beta_1)}{\lambda(\beta_c)} = 1.85$ is predicted, so that a structure whose wavelength is $3.75 d$ is predicted (whereas $\lambda(\beta_c) \simeq 2 d$). To test this result, we have replaced the lowest bolometer (stycast plates) in the experimental system of figure 1 by a lattice of 25 probes, in the bottom of the cell (x - y plane in Fig. 3), and we have

Fig. 3. — Deviation $\theta(x, y)$ from the diffusive linear temperature profile in the lower plane of the cell, for $P/P_c(\beta_1) = 1.45$ (nearly insulating boundaries).

measured the spatial temperature distribution $T(x, y)$. Figure 3 shows the deviation $\theta(x, y)$ from the conductive linear temperature profiles deduced from these measurements.

A three-dimensional pattern is associated with this thermal distribution, but rolls parallel to the x axis can be seen for $0 \leq x \leq 2d$. The essential fact is that their reduced wavelength is 4, which is the nearest integer allowed by the boundary conditions to the theoretical value 3.75. Figure 4 shows the deviation $\theta(x_0, y)$ for one line of bolometers ($x_0 = 1.35 d$) and for three different values of P larger than the critical value P_c . It is seen that the spatial periodicity still holds for various constraints P , and becomes even more obvious for increasing P .

Furthermore, a comparison of time-dependent effects for $\beta_1 = 4.35$ and $\beta_c = 0.015$ confirms these results. For $1.7 \leq P(\beta_1)/P_c \leq 2.0$, oscillations are observed; their onset corresponds to the second slope discontinuity in the curve $P/\Delta T_v = f(\Delta T_v)$. At $P > 2 P_c$, chaotic motion took place. The frequencies of these oscillations are in the range 0.5 to 2 mHz and so are found to be much lower than those for $\beta_c \simeq 0$ (3 to 10 mHz). This can be understood as follows : the

order of magnitude of the oscillations frequency is $f(\beta) \sim \frac{K_F}{\lambda^2(\beta)}$ [6]. So, we found $\frac{\lambda(\beta_1)}{\lambda(\beta_c)} \sim \left(\frac{f(\beta_c)}{f(\beta_1)} \right)^{1/2} \sim 2$, which is compatible with the previous results.

This experimental study confirms the results of the linear theory : (i) the increase of the convective structure wavelength at onset and (ii) the decrease of the critical Rayleigh number when the

Fig. 4. — Evolution of $\theta(x_0, y)$ for $x_0 = 1.35 d$ with the heating power P (nearly insulating boundaries).

upper and lower boundary heat conductivity is smaller than the fluid heat conductivity. Moreover, in the weakly non-linear range, we have shown that the convection effectiveness on the heat flux decreases when the boundaries are no longer perfect heat conductors.

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