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Flicker noise near the percolation threshold

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Résumé. — Une théorie de milieu effectif pour l'amplitude du bruit $1/f$ est formulée pour le problème du réseau aléatoire de résistances. Au voisinage du seuil de percolation p_c , l'amplitude du bruit diverge comme $(p - p_c)^{-\kappa}$, avec $\kappa = 1$ pour le modèle de sites ainsi que le modèle de liaisons. L'exposant κ est calculé dans le cadre de deux transformations de renormalisation. L'observation possible du comportement prédit est discutée dans le cas des mélanges isolant-métal et des cermets.

Abstract. — An effective-medium theory, for the flicker ($1/f$) noise amplitude, is formulated for random resistor networks. Close to the percolation threshold p_c , the magnitude of the noise is shown to diverge as $(p - p_c)^{-\kappa}$, with $\kappa = 1$ for both site and bond percolation models. The exponent κ is also calculated in the framework of two different renormalization group transformations. The possible observation of the predicted behaviour in metal-insulator mixtures and cermets is discussed.

1. — Flicker ($1/f$) noise [1] refers to the frequency spectrum of excess voltage fluctuations measured when a constant current is applied to a resistor. That spectrum almost always has a power law form $\omega^{-\alpha}$ with α close to unity. The origin of this power law has been the subject of innumerable controversies and remains not very well established. Recently [2], a simple model for the magnitude of this noise in random resistor networks has been introduced. This model is based on two well established properties of $1/f$ noise : it is resistance noise [3] and the resistance fluctuations are correlated over microscopic distances only [4]. The simplest version of the model can be formulated as follows. The resistances, all assumed to have the same value r , fluctuate independently in time on each bond with a correlation function $\langle \delta r_\alpha(\omega) \delta r_\beta(-\omega) \rangle = \rho^2(\omega) \delta_{\alpha\beta}$. As long as each bond resistance fluctuates independently with the same spectrum, the explicit frequency dependence can be removed and will not be considered anymore.

The magnitude of the relative noise for the overall resistor network is defined by $S_R = \langle \delta R \cdot \delta R \rangle / R^2$, where R and δR are respectively the overall resistance and its time fluctuation.

A general expression for S_R has been obtained [2, 5] :

$$S_R = \frac{\rho^2}{r^2} \left(\sum_{\alpha} i_{\alpha}^4 \right) / \left(\sum_{\alpha} i_{\alpha}^2 \right)^2. \quad (1)$$

Here i_{α} denotes the current that flows in branch α in the steady-state without resistance fluctuations. Using equation (1), lower and upper bounds for S_R may be found (2) :

$$1/N_b \leq S_R/s \leq r/R \quad (2)$$

where $s \equiv \rho^2/r^2$ is the relative noise for the individual resistances and N_b is the total number of conducting branches in the system. The lower bound is obviously reached for regular Euclidean networks : $S_R \simeq s \cdot L^{-d}$ where L denotes the length scale of the network. However, for a self-similar network, it was shown that $S_R \simeq s \cdot L^{-b}$, where b denotes a new exponent. This exponent b appears as a member of a new hierarchy of exponents describing the cumulants of the resistance fluctuations and is related to the scaling of the quantities $\sum_{\alpha} i_{\alpha}^{2n}$ (n = positive integer). The

hierarchy includes the fractal dimension $\bar{d}(n=0)$ and the resistance exponent [6] $\beta_L(n=1)$ of the structure (i.e. $R(L) \sim L^{-\beta_L}$) as special cases. Using equation (2), one deduces : $-\beta_L \leq b \leq \bar{d}_B$ where $\bar{d}_B$ is the fractal dimension of the backbone. The value of the exponent b for randomly diluted resistor networks is $b=1$ at $d=1$ and $b=2$ at $d \geq 6$.

Close to the percolation threshold, the ratio $s(p) \equiv S_R(p)/S_R(p=1)$ is predicted to diverge as $(p-p_c)^{-\kappa}$ where $\kappa = v_p(d-b)$ and v_p is the percolation correlation length exponent : $\xi_p \simeq |p-p_c|^{-v_p}$. Accordingly, one deduces [2] the following bounds for κ :

$$\beta_B \leq \kappa \leq 2 v_p(d-1) - t \quad (3)$$

where β_B is the backbone density exponent $B(p) \simeq (p-p_c)^{\beta_B}$ and t that of the macroscopic conductivity. Equation (3) implies $\kappa=2$ at $d \geq 6$ and, using known estimates of various exponents, one obtains

$$\begin{aligned} 0.51 &\leq \kappa \leq 1.40 & \text{at } d=2 \\ 1.01 &\leq \kappa \leq 1.56 & \text{at } d=3. \end{aligned} \quad (4)$$

The exponent κ and the new hierarchy of exponents appear as new numbers providing a way of characterizing the percolation clusters in finer detail with a large set of independent measurable quantities. In this Letter we focus our attention on the quantity $s(p)$. $s(p)$ is first calculated in the framework of the effective-medium theory. The exponent $\kappa(=1)$ so obtained is compared with the results of two different real space renormalization transformations, previously introduced for the percolation problem. The possible observation of the predicted behaviour in real systems which can be modelled by a random resistor network is discussed in the concluding section.

2 — The effective-medium theory (EMT) of a random lattice of resistors has received considerable attention largely because of its simplicity and its accuracy. For instance, close to $p=1$ on a randomly diluted resistor network, EMT reproduces very well the exact behaviour of the macroscopic conductivity. In what follows we shall consider separately bond and site models.

2.1 BOND MODELS. — Let us assume that the net effect of the random conductances is represented by a homogeneous effective medium with equal conductances g_m , and equal relative noise s_m , connecting all the nearest-neighbour sites of the lattice. Consider a bond l selected at random in the lattice and change its characteristics from (g_m, s_m) to (g_l, s_l) , where g_l and s_l are random. The EMT equations are obtained by equating the average resistance g^{-1} and the average relative noise s measured across this branch l , to the corresponding values when $g_l = g_m$ and $s_l = s_m$.

In the configuration considered, the whole network can be viewed as an equivalent circuit containing two parallel branches. The first is represented by (g_l, s_l) and the second corresponds to the infinite effective network where the branch l has been deleted. Let us denote by $g' = \alpha g_m, s' = \beta s_m$ the second branch parameters, where α and β are numerical factors given by the coordination number z of the lattice. According to the composition rules for conductances and [2] relative noises, the conductance g_{tot} of the equivalent circuit and its relative noise s_{tot} are given by

$$\begin{aligned} g_{\text{tot}} &= g_l + \alpha g_m \\ s_{\text{tot}} &= (g_l^2 \cdot s_l + \alpha^2 g_m^2 \cdot \beta s_m) / g_{\text{tot}}^2. \end{aligned} \quad (5)$$

Therefore, g_m and s_m are given by the solution of the EMT equations

$$\int p(g_l, s_l) dg_l ds_l \frac{1}{g_l + \alpha g_m} = \frac{1}{(1 + \alpha) g_m} \quad (6)$$

$$\int p(g_l, s_l) dg_l ds_l \frac{g_l^2 s_l + \alpha^2 g_m^2 \beta s_m}{(g_l + \alpha g_m)^2} = s_m \frac{1 + \alpha^2 \beta}{(1 + \alpha)^2}. \quad (7)$$

Here, $p(g_l, s_l)$ denotes the joint probability distribution of g_l and s_l for an individual bond l . As it should be, equation (6) does not involve s_l and reduces to the known result [7]. Equations (6) and (7) hold in general for an arbitrary distribution $p(g_l, s_l)$. In the following we shall consider the case of randomly diluted networks where $g_l = g, s_l = s$ with probability p and $g_l = 0, s_l = 0$ with probability $1 - p$. Equation (6) gives the following expression for g_m :

$$\begin{aligned} g_m/g &= 1 - (1 - p) \frac{\alpha + 1}{\alpha}, \quad p > p_c \\ &= 0, \quad p < p_c. \end{aligned} \quad (8)$$

Using the known value $\alpha = \frac{z-2}{2}$ for regular lattices of coordination number z , we recover the EMT threshold $p_c = 2/z$ and the corresponding slope $z/(z-2)$ at $p = 1$.

For the same distribution $p(g_l, s_l)$, equation (7) reduces to

$$s_m/s = \left[p \left(1 + \left(\frac{z-2}{2} \right)^2 \beta \right) - p \frac{z^2}{4} \beta - \beta(1 - zp) \right]^{-1}.$$

The numerical value of β can be obtained either directly (as α) or more simply by imposing the divergence of s_m/s at the EMT threshold. This gives $\beta = 1/\alpha$ and leads to the simplified form $s_m/s = [p(1 + \beta) - \beta]^{-1}$, and then:

$$\begin{aligned} s_m/s &= \frac{z-2}{z} (p - p_c)^{-1}, \quad p > p_c \\ &= 0, \quad p < p_c. \end{aligned} \quad (9)$$

This result shows that $\kappa = 1$ in the framework of EMT. Furthermore, for the particular distribution used, $g_m s_m = gs$ holds for all $p > p_c$. In the case of the square lattice ($z = 4$), equation (9) implies $s_m(p) = (2p - 1)^{-1}$ which agrees surprisingly well with the results of Monte Carlo calculations [8]: the EMT result is well followed up to $p \gtrsim 0.6$. As expected, strong deviations appear near the threshold $p_c = 1/2$.

It should be noted that the naive picture where the system is assumed to be homogeneous near $p = 1$, leads to the erroneous result $s(p) = 1/pB(p) \simeq 1/p$. The breakdown of this picture occurs also for the conductance $g(p)$: even at $p \leq 1$, the inhomogeneities of the current pattern are enhanced and lead to non-trivial behaviour for both s_m and g_m .

2.2 SITE MODELS. — Let us consider the simple percolation site model, where a fraction $1 - x$ of nodes are selected at random from an infinite lattice of conductances g (and relative noise s) and the entire cluster of conductances about each site, are changed to zero. The fraction of conducting bonds is therefore $b = x^2$. Following the procedure as above, the EMT equations yield :

$$g_m/g = 1 - (1 - b) \frac{2\alpha' + 1}{2\alpha'}$$

and ($\beta' = 1/4 \alpha'$) : $s_m/s = g/g_m$. Here, α' and β' denote numerical factors which depend on the lattice structure. For instance [9], $\alpha' = 1/(\pi - 2)$ for the square lattice and $\alpha' \simeq 1.88$ for the simple cubic lattice.

As expected, both models (bond, site) lead to similar results : $s_m/s \simeq (p - p_c)^{-1}$, i.e. $\kappa = 1$. In addition, as for $g(p)$, the EMT equations reproduce the correct qualitative behaviour close to p_c , become exact for small dilution ($p \simeq 1$ or $x \simeq 0$) but fail to give the exact value of the exponent κ .

3. — In order to go beyond the EMT approximation, in particular to have reliable estimates for the exponent κ (e.g. its variation with d), more elaborate methods must be used. In what follows, two real space renormalization group (RSRG) transformations will be used. Such methods have been introduced in the past for the calculation of the percolation exponents. We direct the reader to reference [10] for a very clear exposition. We shall adopt here the same notation and only the bond percolation problem on cubic lattices will be considered.

3.1 DIFFERENTIAL RSRG. — The central idea [11] is that of modifying the lattice by moving interactions (i.e. bonds) in order to facilitate the decimation procedure. To rescale one coordinate by b , we first move the bonds which are perpendicular to that axis, grouping then b bonds at a time, separated by 1D chains of b bonds. The chains can be replaced by single effective bonds using the composition rules. The effect of grouping b bonds is given by the second transformation. After this has been repeated for each of the d axes, each set of bonds has been subjected to one 1D transformation, and $(d - 1)$ parallel transformations. In order to remove anisotropy, resulting from the order in which the transformations are applied, one may take the limits $b = 1 + \zeta$, $\zeta \ll 1$. To obtain the exponent κ , one needs only the effect of the renormalization on $\langle s \rangle$:

$$\mathcal{R}[s] = b^d \{ \mathcal{R}_{\parallel}[s] \}^{d-1} \mathcal{R}_{1D}[s]. \quad (10)$$

As usual, for $\zeta \ll 1$, one needs the expression of the infinitesimal generators $\mathcal{L}_{\parallel}$ and $\mathcal{L}_{1D}$ associated with $\mathcal{R}_{\parallel}$ and $\mathcal{R}_{1D}$. Using the composition rules, $\mathcal{R}_{1D}$ takes $\langle s \rangle$ into $\langle s \rangle/b$ and then

$$\mathcal{L}_{1D}[\langle s \rangle] = - \langle s \rangle. \quad (11)$$

The expression of $\mathcal{L}_{\parallel}$ is obtained by taking the average noise of 1 to b bonds in parallel, weighted by the probability of occurrence

$$\sum_{n=1}^b \frac{1}{n} \binom{b}{n} p^n (1-p)^{b-n} \langle s \rangle$$

where $\binom{b}{n}$ is the binomial coefficient. In order to preserve the normalization a factor $[1 - (1-p)^b]$ is required and

$$\mathcal{R}_{\parallel}[s] = \sum_{n=1}^b n^{-1} \binom{b}{n} p^n (1-p)^{b-n} \langle s \rangle / [1 - (1-p)^b] \equiv f(p) \langle s \rangle. \quad (12)$$

The function $f(p)$ which appears in equation (12) can also be written as

$$f(p) = \frac{(1-p)^b}{1-(1-p)^b} \int_0^1 dt \frac{(1+at)^b - 1}{t}, \quad a = p/(1-p). \quad (13)$$

A straightforward calculation leads to ($\zeta \ll 1$)

$$f(p) \simeq 1 + \zeta [I(a)/a - 1]$$

and

$$\mathcal{L}_{||}[\langle s \rangle] = \langle s \rangle [I(a)/a - 1]. \quad (14)$$

In equation (14), $I(a)$ refers to the integral $I(a) = \int_0^1 \frac{dx}{x} \ln(1+ax)$. Adding a trivial rescaling (Eq. (10)) factor $d \langle s \rangle$ to obtain $\mathcal{R}'$ and identifying the scaling factor λ of $\langle s \rangle$: $\lambda = b^{\kappa/\nu} \simeq 1 + \zeta \left(\frac{\kappa}{\nu} \right)$, one obtains

$$\kappa/\nu = d - 1 + (d - 1) \left[\frac{I(a)}{a} - 1 \right] = \frac{d - 1}{a} I(a). \quad (15)$$

The numerical values of κ/ν are obtained from equation (15) using the corresponding values [11] of p_c and ν :

$$2D : p_c = 1/2, \quad I(a=1) = \pi^2/12, \quad \nu = 1.629 \quad \text{and} \quad \kappa/\nu = 0.822, \quad \kappa = 1.339$$

$$3D : p_c = 0.16, \quad I(a) = 0.182, \quad \nu = 1.22 \quad \text{and} \quad \kappa/\nu = 1.911, \quad \kappa = 2.332.$$

In the limiting case, $d = 1 + \varepsilon$ ($\varepsilon \ll 1$) one obtains ($p_c = 1 - e^{-1/\varepsilon}$, $\nu = 1/\varepsilon$) : $I(a) \simeq \frac{1}{2} \ln^2 a$

and $\kappa/\nu \simeq \frac{1}{2\varepsilon} e^{-1/\varepsilon}$ (in agreement with $\kappa/\nu = 0$ at $d = 1$). On the contrary, for $\varepsilon \gg 1$ where $p_c = e^{-\varepsilon}$ and $\nu = 1 + \varepsilon e^{-\varepsilon}$ one obtains $\kappa/\nu \simeq \varepsilon$ in disagreement with the large d result $\kappa/\nu = 4$. As expected the differential RSRG is useful only at $d = 2$ or 3 but fails at large d .

3.2 CELL TRANSFORMATION. — This is essentially a discrete transformation : each cell of 2^d sites and the associated bonds are replaced by one site and d bonds by requiring each of the new bonds to express the possibility that opposite sides of the cell are connected in that direction. In the following we shall limit our discussion to 2D lattices and $b = 2$. The construction requires that we combine probabilities in the « Wheatstone bridge » arrangement. If p denotes the probability of presence of each bond, then p renormalizes [12] as $p' = p^5 - 5p^4q + 8p^3q^2 + 2p^2q^3$ ($q = 1 - p$), leading to the fixed point $p_c = 0.5$ and $\nu = 1.428$. Assuming a binary distribution of conductances g_l and relative noise s_l ($g_l = g_0$, $s_l = s_0$ with probability p , and zero with probability $1 - p$) one can follow the renormalized probability distribution $\rho_n(g)$ and $\tilde{\rho}_n(s)$ associated with the « iterated Wheatstone bridge ».

Let us consider first the limit $q \sim 0$ of small dilution and calculate $\langle g \rangle_n$ and $\langle s \rangle_n$ to first order in q : $\langle g \rangle_n = \int g \rho_n(g) dg$, $\langle s \rangle_n = \int s \tilde{\rho}_n(s) ds$. Using the composition rules, one can show that, to first order in q , ρ_n and $\tilde{\rho}_n$ are binary distributions :

$$\begin{aligned} \rho_n(g) &= p_n \delta(g - g_0) + q_n \delta(g - g_n) \\ \tilde{\rho}_n(s) &= p_n \delta(s - s_n) + q_n \delta(s - s'_n) \end{aligned}$$

where p_n , q_n , s_n and s'_n are the solutions of the recursion equations

$$p_n = p_{n-1}^5 + q_{n-1} p_{n-1}^4, \quad q_n = 4 q_{n-1} p_{n-1}^4 \quad (16.a)$$

$$(p_0 = p, q_0 = q)$$

$$g_n = g_0 \frac{3 g_0 + 5 g_{n-1}}{5 g_0 + 3 g_{n-1}} \quad (16.b)$$

$$s_n = \frac{1}{4} s_{n-1} \quad (16.c)$$

and

$$s'_n = s'_{n-1} \left(\frac{4^{2n-1} - 4}{4^{2n} - 1} \right)^2 + 3 s_n \frac{1 + 16^{2n-1} + 16^n}{(4^{2n} - 1)^2} \quad (16.d)$$

$$(s'_0 = 0).$$

These equations lead to $g_n/g_0 = \frac{4^n - 1}{4^n + 1}$, $p_n = 1 - q_n = 1 - 4^n q$ and $s_n/s_0 = 4^{-n}$ for $n \geq 1$.

For $n \geq 1$, equation (16.d) reduces to $s'_n = \frac{3}{16} 4^{-n} s_0 + \frac{1}{16} s'_{n-1}$ and $s'_n = 4^{-n} \left(\frac{1}{4} + 3 \cdot 4^{-n} \right) s_0$.

Therefore,

$$\langle g \rangle / g_0 = \lim_{n \rightarrow \infty} \langle g \rangle_n / g_0 = 1 - 2q + \mathcal{O}(q^2) \quad (17)$$

which coincides with the exact result [12], to first order in q .

Similarly, one obtains $\langle s \rangle_n / s_0 = 1 + q \left(\frac{s'_n}{s_n} - \frac{1}{4} \right) 4^n$, leading to ($n \geq 1$):

$$\langle s \rangle / s_0 = 1 + 3q + \mathcal{O}(q^2). \quad (18)$$

This result for $\langle s \rangle$ disagrees with that from EMT which is $1 + 2q$. The « iterated Wheatstone bridge » ($b = 2$) reproduces therefore the correct qualitative behaviour but fails quantitatively. The agreement for $\langle g \rangle$ is probably accidental.

We have applied the above ($b = 2$) procedure, to a binary distribution and evaluated $g(p) = \lim_{n \rightarrow \infty} \langle g \rangle_n / g_0$ and $s(p) = \lim_{n \rightarrow \infty} \langle s \rangle_n / s_0$ as defined above by Monte Carlo technique. We have performed such calculations for $n \leq 8$, which corresponds to $\simeq 10^6$ bonds. The fluctuations in the Monte Carlo data are very small and the resulting approximations for $g(p)$ and $s(p)$ are very accurate. The results we obtained for $g(p)$ and $s(p)$ are shown in figure 1. Note that close to $p = 1$, $s(p)$ as well as $g(p)$ follow the expected behaviour (Eqs. (17) and (18)). Near $p_c = 1/2$, $s^{-1}(p)$ decreases sharply as does $g(p)$ and deviates from the EMT predictions. The critical exponents t and κ have been extracted from the data near p_c . We obtained $t = 1.32 \pm 0.02$ and $\kappa = 1.70 \pm 0.02$. Note that this value of t compares very well with that of reference [12]. However the value thus obtained for κ lies outside the exact bounds given in the introduction for 2D lattices. This overestimation of κ must be compared with the similar behaviour of t and ν at $b = 2$. Indeed, recent accurate estimates of t by various authors give $t = 1.30 \pm 0.01$. Accordingly, the iterated Wheatstone bridge, viewed as a hierarchical structure may reproduce the 2D values of exponents only in the limit $b = \infty$.

4. — It is clear that the hierarchy of exponents (Ref. [2]) pertaining to the scaling behaviour of the quantities $G_{2n} = \sum_{\alpha} i_{\alpha}^{2n}$ can be calculated following the same line of ideas outlined in this paper.

Fig. 1. — Relative noise $s(p) = S(p)/S(1)$ for the bond-percolation problem on the two-dimensional square lattice. The solid line refers to the real space renormalization group (RSRG) results ($b = 2$). The dashed line shows the effective-medium theory result. The conductance $g(p)$ obtained with RSRG is shown for comparison.

Our results have implications for ongoing experiments in real materials such as metal-insulator mixtures [13] or $\text{Ni-Al}_2\text{O}_3$ cermets [14], where the measurement of the flicker noise may be useful. First, the measurement of $s(p)$ as function of the metallic filling fraction will provide a fine tool for the geometry of the structure. In fact, as shown in equation (1), the magnitude of the relative noise is more strongly affected by the current pattern than by the squared conductance. The inhomogeneities of the current distribution are more and more enhanced as n increases. In this respect, not only the critical region ($p \sim p_c$) but already the region of small disorder ($p \sim 1$) is of great interest. Simultaneous measurement of both $g(p)$ and $s(p)$ is certainly of some interest. Secondly, the measurement of $s(p)$ will shed some light on the microscopic origin of the flicker noise : does it originate from the contacts between grains (e.g. in carbon films) or from intragrain fluctuations ? Note that in addition to real materials, experimental simulations can also be performed. For instance such measurements can be carried out on graphite-coated paper [15] on which calibrated holes are randomly cut out (site problem).

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References

- [1] DUTTA, P. and HORN, P. M., *Rev. Mod. Phys.* **53** (1981) 497 and references therein.
HOOGE, F. N., KLEINPENNING, T. G. M. and VANDAMME, L. K. J., *Rep. Prog. Phys.* **44** (1981) 479.
- [2] RAMMAL, R., TANNOUS, C. and TREMBLAY, A.-M. S., *Phys. Rev. A* (1984), in press.
- [3] VOSS, R. F. and CLARKE, J., *Phys. Rev. B* **13** (1976) 556 ;
CLARKE, J. and VOSS, R. F., *Phys. Rev. Lett.* **33** (1974) 24 ;
BECK, H. G. E. and SPRUIT, W. P., *J. Appl. Phys.* **49** (1978) 3384 ;
TREMBLAY, A.-M. S. and NELKIN, M., *Phys. Rev. B* **24** (1981) 2551.

- [4] BLACK, R. D., WEISSMAN, M. B. and FLIGEL, F. M., *Phys. Rev. B* **24** (1981) 7454 and references therein.
SCOFIELD, J. H., DARLING, D. H. and WEBB, W. W., *Phys. Rev. B* **24** (1981) 7450.
- [5] This result was derived in reference [2] with the help of Tellegen's theorem. A simple proof is provided by the following remark. The dissipated power P in the network can be written as $P = I^2 R = \sum_{\alpha} r_{\alpha} i_{\alpha}^2$. Then $I^2 \delta R = \sum_{\alpha} i_{\alpha}^2 \delta r_{\alpha} + \sum_{\alpha} r_{\alpha} \delta i_{\alpha}^2$. Using the principle of the minimum rate of entropy growth, at constant temperature and under the corresponding boundary conditions one can show that the second term vanishes and therefore $\delta R = \sum_{\alpha} \delta r_{\alpha} (i_{\alpha}/I)^2$ to first order in δr_{α} 's. Equation (1) follows immediately from this expression for δR . From the definition of $\mathcal{S}_R$, one deduces (Ref. [2]) the following composition rules : $\mathcal{S}_R = \sum_i (R_i/R)^2 \mathcal{S}_{R_i}$, for uncorrelated resistance fluctuations (serie resistances) and $\mathcal{S}_R = \sum_i (R/R_i)^2 \mathcal{S}_{R_i}$, $R^{-1} = \sum_i R_i^{-1}$ for parallel resistances.
- [6] RAMMAL, R. and TOULOUSE, G., *J. Physique Lett.* **44** (1983) L-13.
- [7] KIRKPATRICK, S., *Rev. Mod. Phys.* **45** (1973) 574.
- [8] RAMMAL, R., BRETON, P. and TREMBLAY, A.-M. S. (1984), to be published.
- [9] WATSON, B. P. and LEATH, P. O., *Phys. Rev. B* **9** (1974) 4893.
BUTCHER, P. N., *J. Phys. C* **8** (1975) L324.
JOY, T. and STRIEDER, W., *J. Phys. C* **11** (1978) L867.
- [10] KIRKPATRICK, S., in *Ill Condensed Matter*, Les Houches Summer School, ed. R. Balian, R. Maynard and G. Toulouse (North Holland, Amsterdam) 1979.
- [11] KIRKPATRICK, S., *Phys. Rev. B* **15** (1977) 1533.
- [12] BERNASCONI, J., *Phys. Rev. B* **18** (1978) 2185.
- [13] RUDMAN, D. A. and GARLAND, J. C., *Bull. Am. Phys. Soc.* **29** (1984) 352.
- [14] MANTESE, J. W., GOLDBURG, W. I., DARLING, D. H., CRAIGHEAD, H. G., GIBSON, U. J., BUHRMAN, R. A. and WEBB, W. W., *Solid State Commun.* **37** (1981) 353.
- [15] LAST, B. J. and THOULESS, D. J., *Phys. Rev. Lett.* **27** (1971) 1719.
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