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Hierarchical nature of spin glasses

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Résumé. — Nous avons analysé un modèle tridimensionnel de verre de spin du type Edwards-Anderson avec interaction de portée finie. Nous montrons que le verre de spin représente une hiérarchie superparamagnétique. Il est à noter que le nombre de niveaux hiérarchiques augmente infiniment quand la température T s'approche de la température de congélation T_f .

Abstract. — The 3D spin glass model of the Edwards-Anderson type with finite range interaction is studied. The spin glass is a superparamagnetic hierarchy, the number of hierarchy levels increasing infinitely with $T \rightarrow T_f$ from above (T_f is the freezing temperature).

1. We consider a 3D Ising spin glass with long-(but finite) range interaction at temperatures near the phase transition point. The spin interaction energy is :

$$H = \frac{1}{2} \sum_{ij} \mathfrak{J}_{ij} \sigma_i \sigma_j. \quad (1)$$

Summation in this formula is performed over sites (i) with spins (σ_i). The sites (i) are supposed to be distributed randomly with concentration c , $\mathfrak{J}_{ij}$ are independent random variables, $\overline{\mathfrak{J}_{ij}} = 0$, $\mathfrak{J}_{ij}^2 = K(r_i - r_j)$. The function K obeys the conditions :

$$\int K(r) d^3r = 1 \quad (2)$$

$$Z = c \int K(r) r^3 d^3r \gg 1. \quad (3)$$

The first condition fixes the temperature scale (e.g. $T_c \simeq 1$), the second condition means that every spin interacts with a large number (Z) of spins. The exact form of $K(r)$ and the distribution $p(\mathfrak{J}_{ij})$ are irrelevant at $Z \gg 1$.

It is well known [1, 2], that at $Z = \infty$ the phase transition at $T = 1$ is due to the appearance (at $T = 1$) of an unstable mode belonging to the maximum ($E^{\max} = 2$) eigenvalue of the matrix $\mathfrak{J}_{ij}$. The system's behaviour is also governed by modes with $|E - E^{\max}| \ll 1$ at finite Z in the temperature range near the transition point.

Hence, we must sum the partition function over almost all modes and obtain the effective free energy which depends on the amplitudes (a_i) of « slow » modes (with $|E - E^{\max}| \ll E^{\max}$). For an ordinary ferromagnetic phase transition these « slow » modes would be fluctuations with small momenta. First of all we sum over thermal fluctuations of the « fast » modes (with energies far from $E^{\max}$) keeping fixed mean spin values : $m_i = \langle \sigma_i \rangle$. Fast fluctuation properties are the same in cases of large finite, and infinite Z , therefore the result of summation is the free energy, obtained in [1] (for $Z = \infty$) :

$$F \{ m_i \} = -\frac{1}{2} \sum_{ij} \mathfrak{J}_{ij} m_i m_j - \frac{1}{4T} \sum_{ij} \mathfrak{J}_{ij}^2 (1 - m_i^2) (1 - m_j^2) + \frac{T}{2} \sum_i \left[m_i^2 + \frac{1}{6} m_i^4 + \frac{1}{15} m_i^6 \right]. \quad (4)$$

Here we use the fact that $m_i^2 \ll 1$ near the transition point and confine ourselves to necessary expansion terms. Minima of the free energy $F \{ m_i \}$ have been studied in papers [1, 2].

2. Next we want to obtain the free energy functional which depends only on the « slow » variables (the amplitudes a_λ with the energies $|E_\lambda - E^{\max}| \ll E^{\max}$). The function m_i can be presented as

$$m_i = \sum_{\lambda} a_{\lambda} \psi_{\lambda}(i) + \sum_{\alpha} b_{\alpha} \phi_{\alpha}(i) \quad (5)$$

where a_{λ} are « slow » and b_{α} are « fast » variables, $\psi_{\lambda}(i)$ and $\phi_{\alpha}(i)$ are eigenfunctions of the matrix $\mathfrak{J}_{ij}$. When deriving the free energy (4) we sum over thermal fluctuations of the « fast » variables, hence b_{α} in (5) are their mean values and can be found from the equation $\delta F / \delta b_{\alpha} = 0$. After inserting (5) into (4) we obtained with the required accuracy :

$$b_{\alpha} = + \frac{T}{3} \frac{1}{E_{\alpha} - 2} \sum_i \left[\sum_{\lambda} a_{\lambda} \psi_{\lambda}(i) \right]^3 \phi_{\alpha}(i). \quad (6)$$

Insertion of (6) into (4) and (5) yields the effective Hamiltonian of the « slow » variables :

$$\begin{aligned} H \{ a_{\lambda} \} = & \frac{1}{2} \sum_{\lambda} (\tau^2 - \varepsilon_{\lambda}) a_{\lambda}^2 + \frac{1}{4} \sum_{\lambda_1 \dots \lambda_4} a_{\lambda_1} a_{\lambda_2} a_{\lambda_3} a_{\lambda_4} \times \sum_{ij} \psi_{\lambda_1}(i) \times \\ & \times \psi_{\lambda_2}(i) \psi_{\lambda_3}(j) \psi_{\lambda_4}(j) \left[\frac{T}{3} \delta_{ij} - \frac{1}{T} \mathfrak{J}_{ij}^2 \right] + \frac{1}{30} \sum_{\lambda_1 \dots \lambda_6} a_{\lambda_1} \dots a_{\lambda_6} \times \\ & \times \sum_i \psi_{\lambda_1}(i) \dots \psi_{\lambda_6}(i) - \frac{1}{18} \sum_{ij} \left[\sum_{\lambda} a_{\lambda} \psi_{\lambda}(i) \right]^3 \sum_{\alpha} \frac{\phi_{\alpha}(i) \phi_{\alpha}(j)}{-\varepsilon_{\alpha}} \times \left[\sum_{\lambda} a_{\lambda} \psi_{\lambda}(j) \right]^3 \end{aligned} \quad (7)$$

where $\tau = T - 1$, $\varepsilon_{\lambda} = E_{\lambda} - 2$.

$E_{\lambda} = 2$ is not the maximum of the eigenvalues of $\mathfrak{J}_{ij}$ at finite Z , so even if $\tau > 0$ there exist eigenfunctions ψ_{λ} , such that $\varepsilon_{\lambda} - \tau^2 > 0$, i.e. the mean field approximation (MFA) leads to $a_{\lambda}^2 \neq 0$. In what follows we shall require some information about the eigenfunctions and eigenvalues of the matrix $\mathfrak{J}_{ij}$. At $Z = \infty$ the density of states is : $\rho(E) = \sqrt{4 - E^2}/2\pi \simeq \sqrt{-\varepsilon}/\pi$ and all ψ_{λ} are delocalized. At finite Z the singularity in $\rho(\varepsilon)$ at $\varepsilon = 0$ is smeared over the region $|\varepsilon| \sim \varepsilon_0 = Z^{-4/3}$. At $\varepsilon \gg \varepsilon_0$ $\rho(\varepsilon) \sim \exp[-\alpha \times (\varepsilon/\varepsilon_0)^{3/4}]$ (with exponential accuracy : $\alpha \sim 1$), all eigenfunctions are localized and have the mean scale $l(\varepsilon) \sim Z^{1/3} \varepsilon^{-1/4}$. At $|\varepsilon| \lesssim \varepsilon_0$ $\rho(\varepsilon)$ varies slightly with ε , whereas the localization length $l(\varepsilon)$ increases rapidly near the mobility

edge $\varepsilon_c \lesssim Z^{-4/3}$, i.e. $l(\varepsilon) \sim Z^{2/3}[(\varepsilon - \varepsilon_c)/\varepsilon_c]^{-\nu}$. These results follow from perturbation theory, analysis of the saddle-point equations at $\varepsilon \gg \varepsilon_0$ and the scaling hypothesis in localization theory. We suppose, following [3], that the exponent $\nu = 1$. The Hamiltonian (7) can be divided into two parts: $H\{a_\lambda\} = H_0\{a_\lambda^2\} + H_1\{a_\lambda^2, \sigma_\lambda\}$ where $\sigma_\lambda = \pm 1$ are the signs of amplitudes a_λ . Below we shall show that $H_1 \ll H_0$, therefore H_0 is sufficient to determine a_λ^2 :

$$H_0\{a_\lambda^2\} = \frac{1}{2} \sum_\lambda (\tau^2 - \varepsilon_\lambda) a_\lambda^2 + \frac{\tau}{2} \sum_{\lambda_1, \lambda_2} a_{\lambda_1}^2 a_{\lambda_2}^2 p(\lambda_1, \lambda_2) \sum_i \psi_{\lambda_1}^2(i) \psi_{\lambda_2}^2(i) +$$

$$+ \frac{1}{2} \sum_{\lambda_1 \lambda_2 \lambda_3} a_{\lambda_1}^2 a_{\lambda_2}^2 a_{\lambda_3}^2 \left[q(\lambda_1, \lambda_2, \lambda_3) \sum_i \psi_{\lambda_1}^2(i) \psi_{\lambda_2}^2(i) \psi_{\lambda_3}^2(i) \right.$$

$$\left. - \frac{2}{3} r(\lambda_1, \lambda_2, \lambda_3) \sum_{ij} \psi_{\lambda_1}(i) \psi_{\lambda_2}(i) \psi_{\lambda_3}(i) G(i, j) \psi_{\lambda_1}(j) \psi_{\lambda_2}(j) \psi_{\lambda_3}(j) \right]. \quad (8)$$

Here $G(i, j) = \sum_\alpha \phi_\alpha(i) \phi_\alpha(j) (-\varepsilon_\alpha)^{-1}$ is the Green function of the « fast » modes (the sum over the spectrum is determined by the region $-\varepsilon_\alpha \sim 1$). Other types of contraction are absent in the last term in (8) due to the orthogonality of $G(i, j)$ and the « slow » eigenfunctions $\psi_\lambda(i)$. The small factor τ in the second term of (8) comes from the partial cancellation of the two sums in the second term of (7). This cancellation is due to the Gaussian character of the $\psi_\lambda(i)$ distribution in small volumes ($\sim Z$).

At high temperatures $\tau \gg \sqrt{\varepsilon_0}$ the main contribution to the sum over λ comes from the terms with $\lambda_1 = \lambda_2 = \lambda_3$. At $\tau \sim \sqrt{\varepsilon_0} = \tau_0$ terms with $\lambda_1 \neq \lambda_2 \neq \lambda_3$ and terms with $\lambda_1 = \lambda_2 = \lambda_3$ give approximately equal contributions to the sum, thermal fluctuations of a_λ^2 are of the order of their mean values at these temperatures. At lower temperatures ($-\tau \gg \tau_0$) a_λ^2 can be calculated by the mean field approximation: let us introduce quantities $B_\lambda(i) = \sum_{\mu \neq \lambda} a_\mu^2 \psi_\mu^2(i)$ which are molecular fields conjugated to $a_\lambda^2 \psi_\lambda^2(i)$. In the temperature range $-\tau \gg \tau_0$ thermal fluctuations of a_λ^2 are small, and the main contribution to the sum over μ comes from a large number of terms, hence we can use the mean field approximation, i.e. replace $B_\lambda(i)$ by

$$B = \overline{\sum_\mu a_\mu^2 \psi_\mu^2(i)} = \frac{1}{N} \sum_\lambda a_\lambda^2 = \int \rho(\varepsilon) a^2(\varepsilon) d\varepsilon$$

(N is the total number of spins). Minimization of $H_0\{a_\lambda^2\}$ with respect to a_λ^2 yields:

$$(\tau + B)^2 - \varepsilon_\lambda + \frac{2}{3} [\tau + (\gamma_1 + 1) B] a_\lambda^2 \sum_i \psi_\lambda^4(i) + \gamma_2 a_\lambda^4 \sum_i \psi_\lambda^6(i) = 0 \quad (9)$$

where $\gamma_1, \gamma_2 \sim 1$. At $Z = \infty$, $\psi_\lambda(i)$ are distributed randomly according to the Gaussian law and therefore $\gamma_1 = 0$. We deal with $\psi_\lambda(i)$ near the mobility edge where the distribution is non-Gaussian and, as it can be shown $0 < \gamma_1 \sim 1$. Therefore the self-consistency equation leads to $\xi = (\tau + B)^2 \ll \tau^2$ and so the last term in (9) can be omitted:

$$a_\lambda^2 = \frac{3}{2\gamma_1} \frac{\varepsilon_\lambda - \xi}{|\tau|} \left[\sum_i \psi_\lambda^4(i) \right]^{-1} = \frac{\varepsilon_\lambda - \xi}{|\tau|} l^3(\varepsilon_\lambda) \quad (10)$$

where $l(\varepsilon)$ is the localization length at energy ε . From the definitions of ξ and B it follows that

$$|\tau| = B \simeq \frac{Z^2}{|\tau|} \int_\xi^\infty \rho(\varepsilon) (\varepsilon - \xi) \left(\frac{\varepsilon - \varepsilon_c}{\varepsilon_0} \right)^{-3\nu} d\varepsilon. \quad (11)$$

Inserting values of $\nu = 1$ and $\rho(\varepsilon) = \sqrt{\varepsilon_0}$ (as has been explained above) into (11) we get

$$\zeta = \frac{\xi - \varepsilon_c}{\varepsilon_0} \simeq \frac{\varepsilon_0}{\tau^2}. \quad (12)$$

It follows from (12), that with a temperature decrease a larger number of modes (i.e. those closer to the mobility edge) acquire non-vanishing amplitudes (in the MFA), however the mobility edge is unattainable at any finite τ . The remarkable feature of equation (11) is the existence of the solution type-(12) only in the dimensions $2 < d \leq 4$ (if we believe [3] that $\nu = [d - 2]^{-1}$).

Now we estimate the number of neighbours of a given typical mode (with $\varepsilon_\lambda - \varepsilon_0 \sim \zeta \varepsilon_0$), i.e. the number of typical modes which interact strongly with it. This number is the same order of magnitude as the number of modes of the same type situated in the same volume :

$$Z_1(\zeta) = \rho(\varepsilon_0) (\xi - \varepsilon_c) l^3(\zeta) = \zeta^{-2} = \tau^4/\varepsilon_0^2 \gg 1.$$

The total number of typical modes is $N_1(\zeta) \simeq \rho(\varepsilon_0) (\xi - \varepsilon_c) N \simeq Z^{-2} \zeta N$.

At relatively low temperature — $\tau \gg \tau_0$ the thermal fluctuations of a_λ^2 are small whereas sign variables (σ_λ) fluctuate strongly. The system as a whole looks like a superparamagnet in this temperature range ; note, however, that this superparamagnet is built not from rigid clusters but from overlapping eigenfunctions $\psi_\lambda(i)$. Let us find interactions of σ_λ using (10), (12). We can rewrite $H_1 \{ \sigma_\lambda \}$ as :

$$H_1 \{ \sigma_\lambda \} = \sum_{\mu \neq \lambda} \sigma_\mu \sigma_\lambda I_{\mu\lambda} + \sum_{\mu \neq \lambda \neq \rho \neq \nu} \sigma_\mu \sigma_\lambda \sigma_\rho \sigma_\nu \tilde{I}_{\mu\lambda\rho\nu}. \quad (13)$$

The main contribution to $I_{\mu\lambda}$ comes from the terms of the following type :

$$I_{\mu\lambda} \simeq |\tau| a_\mu a_\lambda \sum_i \psi_\mu(i) \psi_\lambda(i) \sum_\nu a_\nu^2 \psi_\nu^2(i). \quad (14)$$

It can be shown that the four-spin interaction (the second term in (13)) is irrelevant. Let us estimate the sum over i in (14). First of all note that if we replace $\sum_\nu a_\nu^2 \psi_\nu^2(i)$ by its mean value B , the sum over i will be zero due to orthogonality of the μ, λ eigenfunctions. The main contribution to the spatial variation of B , $\delta B_i = \sum_\nu a_\nu^2 \psi_\nu^2(i) - B$ comes from modes (ν) with $\varepsilon_\nu - \varepsilon_c \sim \varepsilon_0$. The number of modes of this type overlapping with a given mode is of the order of unity, hence the spatial variation $\delta B \sim B(\tau \sim \tau_0) \sim \tau_0$. The average $\langle I_{\mu\lambda} \rangle = 0$, $I_{\mu\lambda}$ with different pairs of indices do not correlate with each other. Now we calculate $\langle I_{\mu\lambda}^2 \rangle$:

$$\begin{aligned} I_1 = Z_1(\zeta) \langle I_{\mu\lambda}^2 \rangle &= \frac{\tau^2}{N_1} \sum_{\mu\lambda} a_\mu^2 a_\lambda^2 \sum_{ij} \psi_\mu(i) \psi_\mu(j) \psi_\lambda(i) \psi_\lambda(j) \times \delta B(i) \delta B(j) \\ &\simeq \frac{\tau^2}{N_1} N B^2 (\delta B)^2 \simeq (\tau/\tau_0)^6 Z^{-2}. \end{aligned} \quad (15)$$

It immediately follows from (15) that the sign variable interaction is small when compared with the $a_\lambda^2 a_\mu^2$ -interaction, as has been supposed above.

The sign variable interaction is the same as the Ising spin interaction (1) from which we start, whereas the effective interaction I_1 is given by (15). At relatively low temperatures $Z^{-2/3} \ll -\tau \ll \tau_1 \simeq Z^{-1/3}$ the system of signs behaves as a paramagnet which corresponds to the superparamagnetic behaviour of real spins. At $|\tau| = \tau_1$, $I_1 \simeq 1$ and the sign variables σ_λ are described by the Ising spin glass Hamiltonian (1) near the transition point but with a new number of neighbours : $Z_1 \simeq Z^{4/3}$ and reduced temperature : $\tilde{\tau} = 3(1 - |\tau|/\tau_1)$, so all the procedure described above can be repeated once more and so on...

3. In short the spin glass is a hierarchy of superparamagnets. At the n th level of the hierarchy we denote the number of interacting neighbours by Z_n and interactions by I_n , the reduced temperature at which $I_n = 1$ by τ_n . Z_n increases rapidly with n :

$$\ln Z_n = (4/3)^n \ln Z \quad (16)$$

so the accuracy of the $1/Z$ approximation improves with n . The sequence τ_n converges very rapidly because $|\tau_{n+1} - \tau_n| \simeq Z_n^{-1/3} |\tau_n - \tau_{n-1}|$. Hence there is a finite T_f so that at $T < T_f$, $I_n \rightarrow \infty$ with n whereas at $T > T_f$, I_n decreases with n for large enough n . Hence, T_f is the transition temperature from a natural point of view. We shall show later that the low-temperature phase behaviour is strongly non-ergodic therefore the usual question about the existence of some order parameter (e.g. of the Edwards-Anderson parameter) is not easy to formulate, and we shall not deal with this problem here.

To understand the ergodicity loss we consider a system of finite size N for finite time t . There are two scales of relaxation times in ordinary (regular) finite systems, one scale being independent of $N \gg 1$, and the other increasing with N . The equilibrium thermodynamics is easily obtained in this case, if at first we set $N = \infty$, i.e. the second time scale becomes infinitely remote, and then $t = \infty$. Our case is essentially different : at low temperatures ($T \leq T_f$) the spins of all hierarchy levels strongly interact so there is a continuous spectrum of relaxation times, from microscopic up to macroscopic, the macroscopic times increasing rapidly (at least exponentially) with N .

In some sense ergodicity is broken even in ordinary phase transitions because a system travels over some hypersurface in phase space at low temperatures. However, the codimension of this hyperspace is finite, it is simply the number of order parameters. In our case the codimension (at $N = \infty$) is infinite at all times, because there is an infinite number of hierarchy levels with relaxation times larger than t . Above we have called this property a strong non-ergodicity. (If $Z \rightarrow \infty$ the relaxation times of all hierarchy levels tend to infinity and we obtain the non-ergodicity of the Sherrington-Kirkpatrick model [5, 6].) Since the limit $N \rightarrow \infty$ implies non-equilibrium at $T \leq T_f$, the equilibrium thermodynamics which we are to study exists at $T > T_f$ only.

4. Let us calculate the nonlinear susceptibility $\tilde{\chi} = -\partial^2 \chi / \partial n^2$ at $T > T_f$. The quantity $\tilde{\chi}$ can be expressed in terms of the spin correlation function (see e.g. [7]) :

$$\tilde{\chi} = \frac{2}{N} \sum_{ij} \langle\langle \sigma_i \sigma_j \rangle\rangle^2. \quad (17)$$

Near the transition point (i.e. at $\tilde{\chi} \gg 1$) σ_i in (17) can be replaced by m_i and $\tilde{\chi}$ can be expressed by the correlation function of the amplitudes a_λ :

$$\tilde{\chi} = \frac{2}{N} \sum_{\lambda\mu} \langle\langle a_\lambda a_\mu \rangle\rangle^2. \quad (18)$$

At $\tau \gg \tau_0 = Z^{-2/3}$ we get the perturbation theory result, i.e. $\tilde{\chi} = 3/\tau$. At $\tau \lesssim Z^{-2/3}$, $\tilde{\chi} \simeq Z^{2/3}$. At $Z^{-2/3} \ll |\tau| \ll \tau_1$, the quantities a_λ^2 do not fluctuate and are determined by (10) and (12). The different spins σ_λ of the next level do not correlate, so :

$$\tilde{\chi} = \frac{2}{N} \sum_{\lambda} a_\lambda^4 = \int_{\xi}^{\infty} \rho(\varepsilon) \left(\frac{\varepsilon - \xi}{\tau} \right)^2 l^6(\varepsilon) d\varepsilon \simeq Z^{10/3} \tau^4. \quad (19)$$

At $|\tau| \gtrsim \tau_1$, $\tilde{\tau} \sim 1$ the susceptibility increases up to the value of Z^2 , at lower temperatures correlation appears between spins σ_λ of the first level, hence, at $|\tau| = \tau_1$ ($\tilde{\tau} \lesssim Z_1^{-2/3}$, this temperature corresponds to $\tau = \tau_0$ of the zero level) $\tilde{\chi}(\tau_1) = \tilde{\chi}_1 = Z^{20/9} \tilde{\chi}_0$, where $\tilde{\chi}_0 = \tilde{\chi}(\tau = 0) =$

$Z^{2/3}$. The susceptibility $\tilde{\chi}$ increases rapidly with appearance of correlation between spins of higher levels of hierarchy : $\tilde{\chi}_n = Z_{n-1}^{20/9} \tilde{\chi}_{n-1}$. The comparison of this formula with the equation for τ_n :

$$\tau_n - \tau_f \simeq \tau_{n+1} \simeq Z_n^{-1/3} \tau_n = Z_{n-1}^{-4/9} (\tau_{n-1} - \tau_f)$$

yields $\tilde{\chi}_n \sim (\tau_n - \tau_f)^{-5}$. Note, however, that the formula $\tilde{\chi}_+(\tau) \sim (\tau_n - \tau_f)^{-5}$ is a poor approximation for $\tilde{\chi}(\tau)$ in the whole range τ , it is rather the upper envelope of $\tilde{\chi}(\tau)$. To obtain the lower envelope we proceed similarly : we calculate $\tilde{\chi}$ at $\tau = 2\tau_n - \tau_f$ and obtain : $\tilde{\chi}_-(\tau) \sim (\tau - \tau_f)^{-19/4}$. At $\tau \rightarrow \tau_f$ the $\tilde{\chi}(\tau)$ function oscillates rapidly between the two envelopes $\tilde{\chi}_+(\tau)$ and $\tilde{\chi}_-(\tau)$. Hence, the $\tilde{\chi}(\tau)$ function can be approximated with equal success by any power functions with exponents in the range 4.75-5.

The volume of the n th level eigenstate increases rapidly with n : $V_n = Z_0 \exp \{ 9 \ln Z_0 \times [(4/3)^n - 1] \}$. Hence, systems should be large so that interesting physical phenomena associated with many hierarchy levels could be observed. It appears to be unlikely that such phenomena can be discovered by numerical simulation. In real systems the maximum number of thermalized levels is determined by the time of the experiment.

5. In conclusion we show that the phase transition into a strongly non-ergodic state occurs at finite temperature in a 3D Ising spin glass. This transition does not have anything in common with macroscopic condensation into one delocalized mode and it is not governed by any scaling laws. Near T_f the spin glass is the hierarchy of superparamagnets and the number of its levels increases infinitely as $T \rightarrow T_f$. We have used the large Z_1 approximation, however, the number Z_n of neighbours at the n th hierarchy level increases with n , so the accuracy of the $1/Z_1$ approximation is also improved. Therefore we hope that the results obtained hold for real spin glasses with $Z = 6-10$.

The approach to the spin glass theory based on the consideration of eigenfunctions of the J_{ij} matrix has been proposed by Anderson [8] and developed in papers [9-12]. It has been assumed in [9-12] that at sufficiently low temperatures condensation into a delocalized mode occurs which implies the existence of a phase transition. We show this holds for the space dimension $d > 4$, in that case the integral on the right-hand side of equation (11) (after replacing $3\nu \rightarrow d/d - 2$) is finite at $\xi = \varepsilon_c$. Nevertheless the qualitative picture of dynamics at $T > T_f$ developed in [11, 12] seems very plausible.

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