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The growth resistance of ^4He solid superfluid interfaces

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Résumé. — Les expériences de cinétique de croissance de cristaux de ^4He depuis le superfluide montrent qu'un processus unique en est responsable. Cela contredit les théories existantes qui prédisent un comportement totalement différent dans les régimes balistiques et hydrodynamiques, et les idées admises sur la réflexion des rotons qui conduisent au mieux à des valeurs beaucoup plus basses. Nous levons ces deux paradoxes en remarquant que l'ordre du solide doit se prolonger dans le liquide. Les caractéristiques importantes du potentiel qui en résulte pour les rotons sont d'une part sa longue portée, d'autre part, le fait qu'il n'est indépendant du temps dans aucun référentiel.

Abstract. — The three available sets of data on the growth resistance of ^4He solid-superfluid interface show that a unique process has to be invoked. However, the available theories predict strong differences between hydrodynamic and ballistic regimes, and the accepted ideas on roton scattering lead to much lower values of the growth resistance. We solve these two paradoxes by remarking that the solid order must propagate in the liquid. The important features of the resulting scattering potential for rotons are first its long range and second that it cannot be reduced to a static one by any of the frame of reference.

1. Introduction.

The first measurements of the growth resistance of hcp ^4He crystals [1, 2] have shown the rapid decrease of this resistance when the temperature is lowered. This is in qualitative agreement with the ideas of Andreev and Parshin [3] according to which the mobility of the rough solid-liquid interfaces of ^4He is limited by the bulk thermal excitations, that is, by the rotons of the liquid and by the phonons of both liquid and solid.

Recently, Bodensohn, Leiderer and Savignac [4] (B. L. S.) have measured this growth resistance at much higher temperatures with either hcp or bcc phase for the solid. Their results are the perfect extrapolation of the growth resistance roton part for both the melting freezing wave experiment of Keshishev, Parshin and Babkin (K. P. B.) [2] and the sound wave transmission experiment of Castaing, Balibar and Laroche (C. B. L.) [1].

This agreement is striking because, following the two available detailed theories [5, 6], one would expect a strong difference in behaviour between the pure hydrodynamic case of B. L. S. and the ballistic case of C. B. L. The amplitude of this roton term is also puzzling, as it corresponds, following Bowley and Edwards [5], to rotons giving their whole momentum to the interface.

However, Andreev and Knizhnick [6] (A. K.) have shown that, in the low temperature limit, the reflection channel, which corresponds to the minimum change in momenta, is preponderant. Their prediction is then one order of magnitude lower than the experimental result.

In this paper, we first reanalyse the published data of C. B. L. in order to extract the roton part of the growth resistance. Then we discuss the A. K. argument both on experimental and theoretical grounds. We shall show that, taking into account the probable structure of the interface, this argument fails at temperatures higher than 0.5 K that is in the whole experimental range. We also explain why the dissipation remains large even in the hydrodynamic regime [7].

2. Reanalysing the acoustic transmission experiments.

We shall use only the published data of C. B. L. summarized on their figure 6.

Following the paper of Castaing and Nozières [8] (C. N.), the coefficient of sound transmission τ from the liquid to the solid is given by :

$$\tau = \frac{2 Z_s}{Z_s + Z_L + \xi Z_s Z_L}$$

where Z_s and Z_L are the acoustic impedances of the solid and the liquid and ξ a « mobility » of the interface taking into account all the possible damping phenomena. In figure 1, we have plotted the quantity :

$$\frac{2}{\tau} = \frac{Z_s + Z_L}{Z_s} + \xi Z_L = \frac{2}{\tau_0} + \xi Z_L.$$

Assuming that ξ^{-1} is the sum of a roton part proportional to $\exp - \frac{\Delta}{k_B T}$ (Δ is the roton gap), and a phonon part proportional to T^4 , we obtain :

$$\frac{2}{\tau} = \frac{2}{\tau_0} \left(1 + \frac{A}{BT^4 + \exp - \frac{\Delta}{k_B T}} \right).$$

The reason for writing it in this way is the following : due to the closeness of Z_L and Z_s , the theoretical τ_0 is nearly 1 whatever the crystalline orientation and the roton contribution is expected to be isotropic. Thus A has to be independent of the sample studied. On the other hand, macroscopic defects of the surface result in a temperature independent reduction factor for τ . This is the most probable explanation for the hysteresis effect described in the C. B. L. paper [1] after the optical observations of Keshishev *et al.* [2]. The phonon contribution, due to the solid, is also sample dependent. Thus $2/\tau_0$ and B have to be adjusted for each sample.

Let us finally indicate that we have chosen $\Delta = 7.8$ K for consistency with the K. P. B. results. The difference with the real roton gap (7.2 K) probably comes, as they suggest, from a temperature dependence of the preexponential factor.

The results of our fits are presented in figure 1a. For two of the three samples (○, ●) the value obtained for τ_0 is close to 1. From the above considerations we estimate our precision on A to be better than 20 %, that is of the same order as the precision of the K. P. B. results. Our value is :

$$A = (3.5 \pm 0.7) \cdot 10^{-4}.$$

We have now to relate it to the A. P. quantity : $(Km_4)^{-1}$.

Fig. 1a. — The dependence of the sound transmission factor τ *versus* the temperature for three different samples in the C.B.L. experiment. The lines correspond to the fit which is described in the text.

Fig. 1b. — Evolution of $(Km_4)^{-1}$ *versus* the temperature following the three sets of experiments. In the case of B.L.S. (●, ○) and K.P.B. (□) the points correspond to typical experimental data. In the case of C.L.B. they are deduced from the fits of figure 1a.

In the K. P. B. [2] and B. L. S. [4] experiments, the low frequency ensures that the difference in temperature between liquid and solid is zero : $\Delta T = 0$. Using the C. N. [8] notations and the A. P. [3] definition of K gives :

$$(Km_4)^{-1} = \left(a - \frac{b^2}{c}\right) T \rho_s$$

where ρ_s is the density of the solid.

On the other hand, let us assume that the Kapitza resistance (cT^2) between the two phases is significantly higher than their thermal impedances : λ_i^{-1} , and let us neglect the damping due to the second sound emission. In this limit :

$$\xi^{-1} = \left(\frac{\rho_s \rho_L}{\rho_s - \rho_L}\right)^2 \left(a - \frac{b^2}{c}\right) T.$$

In the case of rotons the former statement assumes that their transmission coefficient across the interface is significantly smaller than 1. The later assumption is justified by the low velocity of the second sound. Then :

$$(Km_4)^{-1} = \frac{(\rho_s - \rho_L)^2}{\rho_s \rho_L} (\xi Z_L)^{-1} C_L$$

where C_L is the liquid sound velocity.

For the roton part of the growth resistance we thus find :

$$(Km_4)^{-1} = (4.6 \pm 0.9) \cdot 10^3 \exp\left(-\frac{7.8 K}{T}\right).$$

The K. P. B. result is :

$$(Km_4)^{-1} = 3.3 \times 10^3 \exp\left(-\frac{7.8 K}{T}\right).$$

The difference could be attributed to the second sound emission. However it is not really significant, corresponding to only 2 % of difference between the temperature scales.

The results of three different determinations of the roton part of the growth resistance are summarized on figure 1b. The agreement is obvious. Note that such a figure, with approximate values as for the C. B. L. experiment, was first presented by P. Leiderer in his talk at the Fourth Conference on Phonon Scattering recently held at Stuttgart [9]. The dashed line gives the B. E. result (Eq. (33) of their paper [5] with $\xi = 0$). We recall that their theory assumes a ballistic regime.

3. Discussion of the A. K. argument.

As an introduction, let us summarize our point. The A. K. argument lies on the fact that there exists a reflexion channel for rotons for which the change q in wave vector is such that $ql \ll 1$ where l is the « thickness » of the interface. If l is equal to k_0^{-1} , where k_0 is the roton wave vector, their argument applies on the whole temperature range. We have, however, reasons to think that l is much bigger.

Let us now recall the A. K. argument in the simple case of normal incidence. Figure 2 shows the phonon roton dispersion curve. Owing to the energy conservation and to the fact that the group velocity must be reversed on reflection, an incident roton (A) can be reflected into three

Fig. 2. — Possible reflection channels for a roton.

different states (B, C, D). B corresponds to a rather small change in the wave vector of order :

$$q = 2 \left(\frac{2 \mu k_B T}{\pi \hbar^2} \right)^{1/2}.$$

Following A. K. the probability p of the reflection $A \rightarrow B$ goes like :

$$p = 1 - 2 ql$$

when ql goes to zero. A. K. refer to the reflection of glancing electrons on rough surfaces. It can be understood also with light. Consider the reflection of a beam of light on a surface which is rough on a characteristic depth l (Fig. 3a) the angle with the surface being θ . The specular reflection channel will be depleted if the difference of phase between the two paths : (a, b, c) and (a, d) is comparable to 1. This difference is :

$$k \left| \frac{2l}{\sin \theta} (1 - \cos^2 \theta) \right| = (2k \sin \theta) l$$

k being the wave number of light, $2k \sin \theta$ is the change of wave vector on reflection.

Now, arguments similar to those which lead to the estimation of the roton-roton interaction potential [10] show that the crystalline potential creates in the liquid static density oscillations decaying on a depth $l = \frac{\hbar}{(\mu \Delta)^{1/2}} \cong 3.6 \text{ \AA}$ where μ is the effective mass of rotons (see Fig. 3b).

The reason is the closeness of the reciprocal lattice vector to the roton wave vector k_0 . The physical origin of the depth l is the same as for the range of a Yukawa potential : $l' = \frac{\hbar}{\mu' c}$, where μ' is the mass of the particle which is responsible for the interaction and c the speed of light (in this case $\Delta' = \mu' c^2$). Thus, the « small » parameter : $2ql$ becomes : $4(2k_B T / \pi \Delta)^{1/2}$ which is equal to 0.85 for $T = 0.5 \text{ K}$. The consequence is that the A. K. argument applies only at temperatures much lower than 0.5 K.

We want now to emphasize another important consequence of this structure for the interface. The density oscillations are prolongations of the solid order. Their maxima are then static compared to the solid. When the interface is moving, the potentiel seen by the rotons thus cannot be reduced to a fixed potential moving at some velocity v . Put another way, there is no reference frame where this potential is static. The consequence is that the dissipation against the roton gas never cancels, whatever the drift velocity of this gas is. This explains why the growth resistance remains always close to the Bowley-Edwards formula, even in the hydrodynamic regime.

Fig. 3a. — Reflection of a light beam on a rough surface : specular channel.

Fig. 3b. — Sketch of the density oscillations induced in the liquid by the solid order.

4. Conclusion.

The strong dissipation of the solid-liquid interface of ^4He against rotons, which is clearly and coherently proved by three available sets of experiments, shows that the roton gas is strongly coupled with the solid lattice. Moreover, the agreement between the experimentally observed growth resistance and the Bowley-Edwards [5] formula shows that the Andreev-Knizhnik [6] argument, according to which the anomalous reflection for rotons has overwhelming probability, fails on the whole experimentally spanned temperature range. We have proposed an explanation, in a quantum language which is appropriate for ^4He . Let us remark however that the classical analog exists : it is well known [11] from molecular dynamics simulations, that on growing, the solid order is prolonged far into the liquid. In this respect, it would be of prime interest to measure the growth resistance when the liquid is in the normal state and to see in what measure it is the extrapolation of the solid-superfluid system. Indeed, if the interaction of the excitations with this structure is the dominant process, the state of the bulk liquid (normal or superfluid) would not have a great importance.

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