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Horizontal magnetic field and the oscillatory instability onset

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Résumé. — Etude expérimentale du seuil de l'instabilité oscillatoire dans une expérience de Rayleigh-Bénard. Le fluide utilisé est le mercure en présence d'un champ magnétique horizontal parallèle à l'axe des rouleaux convectifs. Le seuil de l'instabilité est mesuré en fonction du nombre de Chandrasekhar. La loi de puissance observée correspond à l'analyse numérique. L'effet du rapport d'aspect est discuté.

Abstract. — Experimental study of the oscillatory instability onset in a Rayleigh-Bénard experiment. The fluid is mercury, in the presence of a horizontal magnetic field parallel to the convective roll axis. The oscillatory instability onset is measured as a function of the Chandrasekhar number. The observed power law corresponds to the numerical analysis. We discuss the effect of the aspect ratio.

This letter reports new experimental results on the effect of a horizontal magnetic field on thermal convection of an electrically conducting fluid, mercury. The steady two-dimensional convective flow is not affected by a horizontal magnetic field [1], but the stability problem is strongly influenced. Mercury being a low Prandtl number fluid, the first three-dimensional instability is a time dependent one, called by Busse the oscillatory instability (O.I.) [2]. It is a wavelike deformation of the convective rolls, leading to a transverse time-dependent oscillation of the cells. The longitudinal field inhibits this wavelike motion in an electrically conducting fluid like mercury [1, 3]. Thus the stability region is greatly enhanced.

In this letter we study the dependence of the onset of the O.I. on the magnetic field strength, which compares well with a recent numerical analysis by Busse and Clever [3].

The three dimensionless numbers pertaining to our problem are the Rayleigh number, the Prandtl number, and the Chandrasekhar number

$$R = \frac{g\gamma d^3 \Delta T}{\nu K} \quad P = \frac{\nu}{K} \quad Q = \frac{\sigma B^2 d^2}{\rho \nu}$$

where γ is the isobaric thermal expansion coefficient, ν is the kinematic viscosity, K is the heat diffusivity, ρ is the density, σ is the electrical conductivity of the fluid. The layer thickness is d and B is the amplitude of the applied D.C. magnetic field. Let us call $R_Q^{(0)}$ the onset value of the O.I. for a given field (for a given Q). Busse and Clever have found a power law dependence

$$\frac{R_Q^{(0)} - R_O^{(0)}}{P} \propto Q^\alpha \quad \text{where} \quad \alpha \simeq 1.2 \quad [3].$$

This result is plotted in figure 1 for two Prandtl number values. This calculation was done in the limit where the magnetic diffusivity λ is much larger than the heat diffusivity K . This is clearly the case for mercury, a poor electrical conductor, where $\lambda/K \simeq 10^5$. It says that the magnetic diffusion time d^2/λ is much shorter than the heat diffusion time d^2/K . In the wavelike deformation of the convective rolls the characteristic time constant is d^2/K . Thus the magnetic lines of forces are not entrained by the fluid motion, but diffuse, introducing a type of anisotropic dynamic viscosity that damps the O.I. motion and thus inhibits its onset.

In an experiment the cell is always of finite lateral extent, defined by what is called the aspect ratio. We have used cells of rectangular shape and will report here results on two cells, one of small aspect ratio, and one of moderate aspect ratio. As known from previous results the aspect ratio has a strong influence on the onset of convection [4]. It also affects the onset of O.I. : the results of our experiments are that the lateral boundaries do not affect the power law dependence, but increase the stability region as the aspect ratio decreases. The lateral boundaries have an effect similar to the magnetic field one in increasing the domain of stability of the steady two-dimensional convective solution.

Small aspect ratio cell : $L_x = 2.8$ cm, $L_y = 0.7$ cm, $d = 0.7$ cm. The magnetic field direction is the y axis. The results are shown in figure 2 where $R_Q^{(0)} - R_0^{(0)}$ is plotted as a function of Q . One gets

$$R_Q^{(0)} - R_0^{(0)} \propto Q^\alpha \quad \text{where} \quad \alpha = 1.2$$

thus a power law close to the numerical one. The main effect of the lateral walls is that the stability region is enhanced as compared to the numerical analysis which relates to a sample of infinite extent.

Medium aspect ratio cell : $L_x = 3.55$ cm, $L_y = 2$ cm, $d = 0.6$ cm. There we get $\alpha \simeq 1.15$ also close to the numerical estimate. One also notice that the stability region is reduced and that one moves closer to the numerical estimate. Let us remark that, as the aspect ratio increases, the experiment becomes more and more difficult. At zero field it is difficult to keep a stable pattern with the rolls parallel to the y axis. Various possible patterns come in competition cor-

Fig. 1. — Analysis of Busse and Clever for the increase of O.I. onset as a function of Q .

Fig. 2. — Experimental results for two different aspect ratio cells : $2.8 \times 0.7 \times 0.7$ (○); $3.55 \times 2 \times 0.6$ (●); and numerical results of Busse and Clever (solid curve).

responding to different values for $R_O^{(0)}$. $R_O^{(0)}$ is a very critical number, as one plots $R_Q^{(0)} - R_O^{(0)}$. It is thus not clear that one could go much farther in aspect ratio and have a precise measurement.

The two main problems we are confronted to, when comparing the numerical analysis and the experimental results, are i) is the value of α universal and ii) can we explain the surprising large effect of the lateral boundaries.

Let us present a simple model to qualitatively answer those questions. The O.I. onset is related to a critical Reynolds number for the flow circulation velocity

$$Re = \frac{d^2/v}{d/v} = \frac{vd}{v}.$$

When a magnetic field is applied a new dissipation process is introduced related to the magnetic diffusion-like term in the Navier-Stokes equation

$$- \frac{\sigma B^2}{\rho} \Delta^{-1} \partial_{yy}^2 \mathbf{v}.$$

For a large enough field this term becomes dominant as compared to the viscous diffusion one. We define a new dimensionless number

$$Rm = \frac{\rho/\sigma B^2}{d/v} = \frac{Re}{Q}.$$

The analysis now goes as follow : there is a critical Rm for the O.I. onset. Thus the Reynolds number for the O.I. onset increases linearly with Q

$$Re_Q^{(0)} - Re_O^{(0)} \propto Q.$$

This is of course valid only for Q not too small. Moreover the O.I. period is related to the circulation time of a convective roll, thus the frequency is proportional to the Reynolds number and

$$f_Q^{(0)} - f_O^{(0)} \propto Q.$$

This law is in good agreement with the observed frequency dependence of the O.I. onset *versus* Q , as shown in figure 3.

Fig. 3. — Frequency at the O.I. onset *versus* Q , for two different aspect ratio cells : $2.8 \times 0.7 \times 0.7$ (○); $3.55 \times 2 \times 0.6$ (●).

Can we from this simple argument derive the Rayleigh number for the O.I. onset and its dependence with the magnetic field ?

Very close to the onset of convection one can apply the Landau criteria for the circulation velocity, which would lead to $\alpha = 2$. But we are in an intermediate regime, not close to the convection threshold, and not far enough in Rayleigh number, where other simple asymptotic laws exist. It seems thus difficult to derive the Q^α law from an asymptotic calculation, and the value $\alpha = 1.2$ does not seem universal.

Let us note that the experimental results show an almost constant value for α , as we change the aspect ratio which, in our simple model, would indicate that the power-law dependence of the convective circulation velocity on the Rayleigh number is almost independent of the aspect ratio.

Let us try now to answer the second question related to the anomalously large stability region introduced by the lateral boundaries. In an infinite aspect ratio cell the O.I. is a long wavelength instability, and moreover the wavelength varies with the Rayleigh number, as shown by Busse and Clever [3]. In a finite aspect ratio cell the boundaries perpendicular to the rolls axis shorten the wavelength, and may even select a particular wavenumber. This increases the magnetic diffusion-like term, and thus the critical Rayleigh number $R_Q^{(0)}$. Let us note in conclusion that our simple model does not take into account the change in wavelength of the O.I. with the Rayleigh number. It should therefore apply to finite aspect ratio cell where the wavelength change is inhibited by the space quantification.

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