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Low-temperature state of Ising spin glass

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Résumé. — Le modèle de verre de spin à interaction à longue portée proposé récemment [1, 2] est examiné pour le système à 3 dimensions en utilisant la méthode du groupe de renormalisation analytique. Il existe une phase de basse température, à stabilité marginale, qui diffère considérablement de la phase d'Edwards-Anderson. La susceptibilité à l'équilibre obéit à la loi de Curie : $\chi = c/T$ dans cette phase.

Abstract. — A recently proposed [1, 2] model for a spin glass with long-range interaction is considered for a 3D system using the analytic renormalization group technique. There exists a marginally stable low-temperature state which is rather different from the Edwards-Anderson state. The equilibrium susceptibility is paramagnetic : $\chi = c/T$ in this state.

Recently [1, 2] we have proposed a model for a spin glass with long-range interaction which is rather different from the model of Sherrington and Kirkpatrick [3]. In [1] we have considered a one-dimensional version of the model and shown that its phase space consists of a great number of metastable states separated from each other by high but finite barriers — a result that corresponds to a qualitative picture of real spin glasses. In [2] one of the authors has considered a three-dimensional version — a model with the interaction :

$$F = \frac{1}{2} \sum_{i,j} S_i S_j V(\mathbf{r}_i - \mathbf{r}_j), \quad V(\mathbf{r}) = \frac{xp_0}{2\pi r} \sin p_0 r \exp(-xr). \quad (1)$$

The low-temperature state of this model is rather different from the Edwards-Anderson state [8]. In our model this state (Ψ -state) is described in terms of a complex field ψ , whose real part is a molecular field : $\langle S \rangle = \text{th}(\text{Re } \psi/T)$. The field ψ is nearly periodic along one direction of space : $\psi = \rho \exp[ip_0 \mathbf{n} \mathbf{r} + i\theta] = \rho \exp(i\varphi)$. Thus there is a spontaneous breaking of orientational and translational symmetries in the MFA for the Ψ -state. If $\gamma = xp_0^2/4\pi c \ll 1$ (c is the spin concentration) then the MFA is valid and there appear certain effects, inconceivable in a real spin glass e.g. : the transition into the Ψ -state is of first order. In the present letter we remove the con-

dition $\gamma \ll 1$, leaving only the condition $p_0 x^2 \lesssim c$ and study the properties of the Ψ -state in a renormalization group (RG) manner. We prove that there are field variables in the Ψ -state whose correlation functions are decreasing according to a power-law, hence the Ψ -state is a genuine thermodynamic spin glass state, separated from the paramagnetic phase by a phase transition. However, $\langle \cos \varphi \rangle_T = 0$ in the Ψ -state, so that the equilibrium magnetic susceptibility obeys the Curie law $\chi = c/T$ and translational symmetry is restored.

The free energy density H (« Hamiltonian ») in terms of the field ψ is [2] :

$$H[\psi] = \frac{1}{T_0} \left[\frac{1}{4 x^2 p_0^2} |(\nabla^2 + p_0^2) \psi|^2 - \tau |\psi|^2 + \frac{1}{2 c^2} |\psi|^4 + \frac{1}{2} \sum_i \delta(\mathbf{x} - \mathbf{x}_i) (\psi^2 + (\psi^*)^2) \right]. \quad (2)$$

Here $\tau = (T_0 - T)/T_0 \ll 1$, $T_0 = c$ is the transition temperature in the MFA, the summation in the last term is performed over the random points $\mathbf{x}_i$ where the spins are situated.

The Hamiltonian [2] is obtained from the initial Hamiltonian (1) by using the Hubbard-Stratonovich transformation. We have replaced $\sum_i \delta(\mathbf{x} - \mathbf{x}_i)$ by its mean value c in the second and third terms of $H[\psi]$ and retained it in the last phase-dependent term. The representation of the Ising system partition function in terms of the complex field ψ is quite natural in our case due to the fact that the Fourier-transform of $V(\mathbf{r})$ approaches its maximum value at $p = p_0 \neq 0$ and so the first unstable mode is a plane wave, as in the Ising model with the interaction of next-nearest neighbours [11]. If $\gamma \ll 1$ then in the temperature range $\tau \gg \gamma^{2/3}$ the fluctuations of the amplitude ρ of ψ may be neglected and only the phase Hamiltonian is left :

$$H[\varphi] = H_0[\varphi] + H_r[\varphi]; \quad H_0[\varphi] = \frac{1}{2t} \left\{ \frac{1}{4} A[(\nabla\varphi)^2 - 1]^2 + B(\nabla^2\varphi)^2 \right\} \\ H_r[\varphi] = V^{1/2} \cos \varphi \sum_i \delta(\mathbf{x} - \mathbf{x}_i) \quad (3)$$

$$t = p_0 x^2 / 2 c \tau, \quad A = B = 1, \quad V^{1/2} = c \tau. \quad (4)$$

(We have rescaled the variables $x \rightarrow x/2 p_0$, $\varphi \rightarrow \varphi/2$). The fluctuations of ρ cannot be neglected if the condition $\gamma \ll 1$ is violated. It means that on a small length scale ρ fluctuates strongly but at scales exceeding the ρ -correlation length l only phase correlations survive. At these scales the Hamiltonian (3) is again valid but its parameters depend differently on τ

$$t \approx p_0 x^2 / c \tau^\mu, \quad V^{1/2} = c \tau^\sigma, \quad l \approx p_0 x^{-1} \tau^{-\nu}. \quad (5)$$

If we believed that the random part H_r of the Hamiltonian (2) did not affect the small scale behaviour then we could use the result by Toner [9], who claims that the phase transition in systems with Hamiltonian H_0 belongs to the same universality class as the XY model. In that case $\nu = \mu = 0.67$, $\sigma = 0.87$. The dimensionless parameter describing the random potential is $VI^4 \sim \tau^{2\sigma-4\nu} \sim \tau^{-0.8}$, so we see that the random potential is a relevant variable i.e. the XY fixed point is unstable and the exponents (μ, σ, ν) are unknown. At $p_0 x^2 \lesssim c$ and $\tau \sim 1$ we have $t \lesssim 1$, $VI^4 \sim 1$.

Below we shall perform calculations as if $t \lesssim 1$, $VI^4 \ll 1$ hoping that the qualitative results are applicable also at $VI^4 \sim 1$. (Besides, a Hamiltonian of type (3) also describes other systems where VI^4 is an independent parameter which may be set small).

In any case at large scales we have the phase Hamiltonian (3) and we wish to investigate whether it describes a low-temperature state which is different from the paramagnetic state.

The « pure » part $H_0[\varphi]$ of the Hamiltonian (3) has been investigated by Grinstein and Pelcovits in [4], they have shown that H_0 is renormalizable in a three-dimensional space, and that A and B depend slowly on x :

$$A = (1 + 5 t \xi / 64 \pi)^{-4/5}, \quad B = (1 + 5 t \xi / 64 \pi)^{2/5}, \quad \xi = \ln x. \quad (6)$$

The random part of the Hamiltonian (3) ($H_r[\varphi]$) resembles that of the Hamiltonian describing a 2D XY magnet with random anisotropy [5, 6]. After averaging over disorder with the help of the replica trick we obtain the effective Hamiltonian :

$$H_{\text{eff}} = \sum_a H_0[\varphi_a] + \sum_{a,b} V \cos \varphi_a \cos \varphi_b. \quad (7)$$

To derive RG equations for the Hamiltonian (7) we divide φ into two parts : $\varphi = \varphi_0 + \tilde{\varphi}$, where φ_0 is a slow and $\tilde{\varphi}$ is a fast field variable, integrate the partition function over $\tilde{\varphi}$ and rescale the variables. (A similar procedure has been used in [7]). Hence the RG equations for V are :

$$\frac{dV_R}{d\xi} = V_R \left(4 - \frac{d \langle \tilde{\varphi}^2 \rangle}{d\xi} \right) = 4 V_R [1 - \tilde{t}(AB)^{-1/2}]; \quad \tilde{t} = t/16 \pi. \quad (8)$$

From equations 6 we see that $(AB)^{-1/2} \sim \xi^{1/5}$ provided we neglect the influence of V on the renormalization of A, B (that influence may only increase $\langle \tilde{\varphi}^2 \rangle$). It follows from equation 7 that $V(\xi \rightarrow \infty) = 0$. However, new relevant random terms are generated by the V -term until V disappears. If after the renormalization the terms of the second order in V are retained we obtain the new terms in the effective Hamiltonian :

$$H_1 = \sum_{a,b} \frac{V^2}{4} \int \cos \mathbf{Q}_a \cdot \mathbf{x} \cos \mathbf{Q}_b \cdot \mathbf{x} \{ \exp[G_{aa}(\mathbf{x}) + G_{bb}(\mathbf{x})] - 1 \} \times \\ \times \exp[-G_{aa}(0) - G_{bb}(0)] d^3x \quad (9)$$

$$\mathbf{Q}_a = \nabla \varphi_{0a}, \quad G_{ab}(x) = \langle \tilde{\varphi}_a(0) \tilde{\varphi}_b(x) \rangle.$$

This term depends on the angle between $\mathbf{Q}_a, \mathbf{Q}_b$, hence the Green function acquires a new term $p^2 G_0^2(p)$, which has a strong singularity at small momentum. The dependence H_1 on Q_a^2, Q_b^2 leads to less singular terms in the Green function of the type $(\mathbf{p} \cdot \mathbf{Q})^2 G_0^2(p)$. We shall neglect these corrections and keep only the singular ones arising from $(\mathbf{Q}_a \cdot \mathbf{Q}_b)$. Moreover, we can expand H_1 in powers of $(\mathbf{Q}_a - \mathbf{Q}_b)^2$ and retain only the first term, the only one giving important corrections to the Green function. Finally, we arrive at the Hamiltonian :

$$H = \frac{1}{2t} \left\{ \sum_a \frac{A}{4} [(\nabla \varphi_a)^2 - 1]^2 + B(\nabla^2 \varphi_a)^2 \right\} + \frac{h^2}{4} \sum_{a,b} (\nabla \varphi_a - \nabla \varphi_b)^2 \quad (10)$$

where

$$h^2 = \frac{V^2}{8} \int [x^2 - (\mathbf{x} \cdot \mathbf{Q})^2] \{ \exp(2 G(x)) - 1 \} \exp(-2 G(0)) d^3x.$$

At $t \geq 3/4$ the main contribution to h^2 comes from small $x \sim l$:

$$h^2 \sim V^2 l^6 A^{1/2} B^{-1/2} (\tilde{t} - 3/4)^{-1}$$

whereas at $t \lesssim 3/4$ the main contribution to h^2 comes from large scales where the renormalization of $G(\mathbf{x})$ is important (see below), but the qualitative description of the large scale behaviour with Hamiltonian (10) is true at any $\tilde{t}$.

The thermal fluctuations that lead to the logarithmic renormalization (4), (5) may also be neglected at these scales. The theory with the Hamiltonian (10) is marginal in dimension $d = 5$. We perform the renormalization of the Hamiltonian (10), using the ε -expansion near $d = 5$ and arrive at the RG equations :

$$\frac{d \ln A}{d\xi} = -6w, \quad \frac{d \ln B}{d\xi} = w, \quad \frac{dh^2}{d\xi} = 0, \quad \xi = \ln \frac{1}{q} \quad (11)$$

$$\frac{dw}{d\xi} = \varepsilon w - \frac{11}{2} w^2, \quad w = \frac{th^2}{128 \pi^2} A^{1/2} B^{-5/2}. \quad (12)$$

There are no diagrams in the low orders of the perturbation theory in w that renormalize the value of h^2 , therefore we suppose that $dh^2/d\xi \equiv 0$.

At the fixed point ($w^* = 2\varepsilon/11$) we have $B \sim q^{-2\varepsilon/11}$, $A \sim K^{6\varepsilon/11}$ (K is a longitudinal and $\mathbf{q}$ is a transverse component of $\mathbf{p}$ with respect to $\mathbf{Q}$). We see that the random field leads to partial isotropization of the fluctuation spectrum : the bare correlation function $G_0^{-1} = K^2 + q^4$ becomes renormalized : $G_R^{-1}(\mathbf{p}) = \bar{A}K^{2+6\varepsilon/11} + \bar{B}q^{4-2\varepsilon/11}$. This form of $G(\mathbf{p})$ implies that at $\varepsilon = \varepsilon^* = 11/4$ the spectrum becomes isotropic : $G^{-1}(\mathbf{p}) = \bar{A}K^{7/2} + \bar{B}q^{7/2}$. This specific value of ε^* cannot be taken seriously because it was obtained in the first order of the ε -expansion, but a certain value of $\varepsilon^*(\varepsilon^* \sim 2)$, where the spectrum becomes isotropic, undoubtedly exists. We shall argue that $\varepsilon^* \leq 2$ so that at $d = 3$ the spectrum is isotropic. Let us derive general scaling relations without resorting to the ε -expansion. We look for A, B in the form $A \sim \bar{A}K^{\alpha-2}$, $B \sim \bar{B}q^{\beta-4}$, so that $G^{-1}(\mathbf{p}) = \bar{A}K^{\alpha} + \bar{B}q^{\beta}$. Then the scaling analysis of the divergence of diagrams renormalizing A, B leads to the exact relation :

$$5 + d = 2\beta + \beta/\alpha. \quad (13)$$

We consider also the renormalization of the structure wave vector $\mathbf{Q}$ which is equal to $\langle (\nabla \tilde{\varphi}_a)^2 \rangle$. This average value is finite if $3 + d > 2\beta - \beta/\alpha$ and diverges logarithmically if $d = d^*$, where d^* satisfies the equation :

$$3 + d^* = 2\beta - \beta/\alpha. \quad (14)$$

If (13) is inserted into (14) then at $d = d^*$ (and at $d < d^*$) $\alpha = \beta = 2 + d/2$, thus the isotropization of the spectrum and the divergence of the $\mathbf{Q}$ -corrections take place in the same dimension d^* — an appealing result ! Note that $\langle (\nabla \varphi)^2 \rangle$ diverges logarithmically in three-dimensions even if calculated with the unrenormalized Green function $G_0(\mathbf{p})$. It seems natural that this divergence is not removed by spectrum renormalization, we verified this assumption directly in three-dimensions by first order perturbation theory calculations. It means that $d^* \geq 3$, and at $d = 3$, $\alpha = \beta = 7/2$. In that case $C(\mathbf{x}) = \langle \mathbf{Q}(0) \mathbf{Q}(\mathbf{x}) \rangle$ obeys the RG equation :

$$\frac{dC}{d\xi} = -C\eta t^2 h^2 \bar{A}^{-2/7} \bar{B}^{-12/7}, \quad \xi = \ln x, \quad \eta^{-1} \approx 25$$

$$\bar{A} \approx q_1^{-3}, \quad \bar{B} \approx q_1^{1/2}, \quad q_1^2 \approx \frac{3}{64\pi} t^2 h^2. \quad (15)$$

Hence $C(\mathbf{x})$ decreases slowly with $\mathbf{x}$: $C(\mathbf{x}) \sim x^{-4}$, $\Delta \approx 2.5 q_1^2$ while in the paramagnetic state $C(\mathbf{x})$ decreases exponentially. Therefore the low-temperature state is a genuine thermodynamic state different from the paramagnetic state. Note that the thermal fluctuations with renormalized spectrum $G(p) \sim p^{-7/2}$ destroy the correlation function $\langle \cos [\tilde{\varphi}(0) - \tilde{\varphi}(x)] \rangle \sim \exp[-3\tilde{t}\sqrt{q_1 x}]$. ($\tilde{\varphi}$ is the fluctuating part of φ).

So $\langle \sigma_i \rangle = \langle \cos \varphi_i \rangle = 0$ in the low-temperature state and the equilibrium susceptibility is the same as in the paramagnetic state. Thus we can produce a paradoxical answer to the question whether the spin glass is an equilibrium state or is it only a dynamic phenomenon. The spin glass is a genuine thermodynamic state, though revealing itself in dynamic effects only.

The Ψ -state-paramagnetic phase transition looks like dislocation melting which was assumed [9] to describe the smectic A-nematic transition. At $T > T_c$ the equilibrium properties of the system considered does not differ from properties of a paramagnetic. However, there may exist some time scale t_0 such that $\langle \sigma_i(0) \sigma_i(t) \rangle \approx \text{const.}$ at $t < t_0$ in the temperature range $T_c < T < T'_c$. This phase is analogous to the low-temperature phase of the Edwards-Anderson model and is not a genuine thermodynamic state.

We note, in conclusion, that some structure which looks like a Ψ -state was observed recently by Cable *et al.* [10].

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