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Spectral dimension and conductivity exponent of the percolating cluster

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Résumé. — Nous discutons une conjecture récente d'Alexander et Orbach sur la dimension spectrale $\tilde{d}$ d'un amas percolant. Nous envisageons la transformation permettant de passer de la percolation au « squelette » par suppression des bras morts. Certains exposants sont invariants par cette transformation. Nous suggérons $\tilde{d} = \frac{4(d+2) - 4\eta}{3(d+2) - 2\eta}$, en accord avec la conjecture citée si on suppose $\eta = 0$. Ceci nous permet de calculer l'exposant de conductivité t . Les résultats sont en accord raisonnable avec les simulations numériques.

Abstract. — We discuss a recent conjecture by Alexander and Orbach about the spectral dimension $\tilde{d}$ of a percolating cluster. We consider the transformation from percolation to backbone by removing the dangling ends, and those exponents which remain invariant in this transformation. We suggest $\tilde{d} = \frac{4(d+2) - 4\eta}{3(d+2) - 2\eta}$ where d is the dimension of space, leading to the former conjecture if one supposes $\eta = 0$. This allows for an evaluation of the conductivity exponent t .

Very recently, Alexander and Orbach [1] and Rammal and Toulouse [2] introduced the notion of spectral dimensionality $\tilde{d}$ [3] of a fractal [4] : the density of states $\rho(\omega)$ varies the frequency ω of the excitations as

$$\rho(\omega) \sim \omega^{\tilde{d}-1}. \quad (1)$$

At the percolation [5] threshold, the infinite cluster is a fractal, and relation 1 may be used. The spectral dimensionality may be related to the usual percolation exponents [1, 5]

$$\tilde{d} = 2 \frac{vd - \beta}{t - \beta + 2v}, \quad (2)$$

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where t is the conductivity exponent [6], β and ν those of the order parameter and connectivity length. It was conjectured in references 1 and 2 that $\tilde{d}$ could be a constant, related to an intrinsic dimension of the fractal (for instance $\tilde{d} = 1$ for linear polymer chains [1]). For the percolation problem, the conjecture is [1, 2]

$$\tilde{d} = \frac{4}{3}. \quad (3)$$

Combining relations 3 and 2 leads to an expression of the conductivity exponent t in terms of the « static » exponents :

$$t = \frac{1}{2} [\nu(3d - 4) - \beta] \quad (4)$$

$$= \nu(d - 2) + \frac{1}{2}(\beta + \gamma). \quad (5)$$

In this note we would like to criticize relation 5. Let us accept the idea that the exponent t is indeed a function of the « static » exponents. We know that the dangling ends do not contribute to the conductivity of the infinite cluster, and thus that only the backbone [9] does contribute. Then the exponent t is the same for the percolation and the backbone problems. Let us then consider the transformation from the percolation to the backbone problems. In this transformation, t and ν are invariant. We argue that $I \equiv t - \nu(d - 2)$ should also be an invariant in the transformation. Obviously this is not realized by relation 5. Coming back to relation 2, we express I in terms of $\tilde{d}$:

$$I = \frac{2 - \tilde{d}}{\tilde{d}}(\beta + \gamma) \quad (6)$$

$$I = \frac{\nu}{2} \frac{2 - \tilde{d}}{\tilde{d}}(d + 2 - \eta) \quad (6')$$

where ν is invariant in the transformation. A simple conjecture may be made for I , if we suppose that it is a linear function of the invariants : the only invariant we know is the exponent ν . The simplest suggestion then is a linear interpolation [10] between $d = 2$ and 6

$$\frac{I}{\nu} = (Ad + B).$$

The unknown constants A and B are determined by the following conditions :

(i) For $d = 6$

$$\frac{I}{\nu} = 2.$$

(ii) For $d = 2$, we suppose that $I = \nu$, consistent with numerical simulations [11-14] and dual symmetry [15]. This hypothesis is extensively discussed by Straley [16]. We find

$$I = \frac{\nu}{4}(d + 2) \quad (7)$$

$$t = \frac{\nu}{4}(5d - 6). \quad (8)$$

Table I shows that the numerical values of t from relation 8 are in good agreement with the numerical estimates. Note however that relation 8 is in disagreement with Stephen's expansion [17].

Combining (8) and (2), we find

$$\tilde{d} = \frac{4(d + 2) - 4 \eta}{3(d + 2) - 2 \eta} \tag{9}$$

which is also given in table I, and is close to 4/3. (Note that η is very small and changes sign near $d = 3$ in the percolation problem.)

Table I.

d	2	3	4	5	6
ν (a)	4/3	0.88 (f)	0.7	0.6	0.5
t (b)	4/3	1.89	2.45	2.85	3
$t_{\text{exp.}}$ (a)	1.3	1.7	2.4	2.7	3
$\tilde{d}$ (c)	1.31	1.33	1.36	1.36	1.33
$\tilde{d}_{\text{exp.}}$ (e)	1.36	1.42	1.39	1.44	4/3
$(5d - 6)/4$	1	2.25	3.5	4.5	6
$t/\nu_{\text{exp.}}$	0.95	2.15	3.43	4.5	6

(a) Ref. 4a; (b) Eq. 8; (c) Eq. 9; (e) Ref. 1; (f) Ref. 18.

We conclude by reiterating our doubts about the recent conjecture concerning the spectral dimension $\tilde{d} = 4/3$ in the percolation problem. These arise from a critical analysis of the invariants in the percolation to backbone transformation, especially for the resulting conductivity exponent t . We suggest a relation for t in terms of these invariants (ν and d) leading to a reasonable numerical agreement with both t and $\tilde{d}$ for space dimensions ranging from 2 to 6. The 4/3 conjecture is recovered if one takes the exponent η to be zero.

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