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The effect of different similarity transformations on ensemble mean particle paths in turbulent spots

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Résumé. — Nous présentons quelques résultats de calculs effectués d'après des mesures faites dans le plan de symétrie de spots turbulents dans une couche limite laminaire. Plusieurs régimes similaires sont comparés. Les lignes de courant et les trajectoires instantanées sont faiblement affectées par le changement de la loi de similarité verticale et ne procurent pas d'explication aux contradictions observées entre les champs thermiques et cinématiques. Ce résultat suggère que le champ moyen représente la dynamique des spots turbulents de façon incorrecte.

Abstract. — Computations from measurements taken on the plane of symmetry of turbulent spots in a laminar boundary layer are presented here. Different types of similarity growths are compared and discussed. The streamline pattern and the instantaneous particle trajectories are weakly affected by the change of vertical growth law, providing no explanation for the apparent contradictions observed in temperature and velocity fields. This result suggests that the ensemble averaged field improperly represents the dynamics of the spot.

1. Introduction.

Recent visual studies of the turbulent boundary layer [1], confirmed the existence of Reynolds number dependent hair-pin vortices originating from the wall. Such structures also appear to be present in a transitional turbulent spot artificially generated by a spark or a speaker [2, 3]. This similarity has stimulated extensive studies of spots, especially in the last decade. In early measurements, Schubauer and Klebanoff [4] determined the general shape of the spot and its normal (y) and lateral (z) growth rates as a function of downstream distance x . More recently, Wygnanski *et al.* [5] (hereinafter referred to as WSF) found experimental evidence for two-dimensionality of the ensemble averaged flow in the plane of symmetry of the spot.

Cantwell *et al.* [6] (later referred to as CCD) were the first to attempt to characterize the growth of turbulent spots in a laminar boundary layer using a space-time similarity transformation. They found experimentally that the spot was growing nearly linearly in space and time and introduced conical coordinates :

$$\xi = \frac{x - x_0}{U_\infty(t - t_0)}, \quad \eta = \frac{y}{U_\infty(t - t_0)} \quad (1)$$

where (x_0, t_0) defines an effective origin. Then they defined a suitable two-dimensional stream function and computed instantaneous particle paths in the (ξ, η) plane. The interpretation of these particle paths leads to contradictions when compared with ensemble mean temperature contours measured by Van Atta and Helland [7], i.e., the hot core region is associated with fluid coming from cold regions away from the wall. Others [8, 9], later proposed alternative viscosity dependent similarity growth laws with $\eta \propto y/t^{4/5}$ and $\eta \propto y/x^{4/5}$, respectively, and concluded that ensemble mean perturbation velocity contours were more accurately described by such viscous transformations. The present investigation seeks to determine the nature and magnitude of the effects of the suggested alternate transformations on the instantaneous particle paths and to see if such effects could be responsible for the fundamental differences observed between ensemble mean velocity and temperature measurements [7].

2. Analytical derivations of the instantaneous particle paths.

Let us consider a two-dimensional, unsteady flow for which we can define an unsteady stream function $\psi(x, y, t)$ as follows :

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \quad (2)$$

Since the flow is unsteady the total differential is :

$$d\psi = -v dx + u dy + \frac{\partial \psi}{\partial t} dt. \quad (3)$$

Several experiments, in air [7, 10] and water [6], showed a linear growth rate in time along the longitudinal direction x . This suggests a new variable :

$$\xi = X/U_\infty T, \quad \text{with } X = x - x_0 \quad \text{and} \quad T = t - t_0 \quad (4)$$

where x_0 and t_0 are virtual origins to be experimentally determined.

The vertical growth is thought to be at most linear in time or slightly slower, like the fully turbulent boundary layer growth. Three different vertical similarity variables can then be considered :

$$\text{Case 1} \quad \eta = \frac{Y}{U_\infty(t - t_0)}, \quad Y = y - y_0 \quad (5)$$

$$\text{Case 2} \quad \eta = \frac{Y}{U_\infty^{3/5} \nu^{1/5} T^{4/5}}, \quad Y = y - y_0 \quad (6)$$

$$\text{Case 3} \quad \eta = \frac{y}{\delta(x)}, \quad \delta(x) = 0.37(x - x_1)^{4/5} \left(\frac{\nu}{U_\infty} \right)^{1/5} \quad (7)$$

(x_1, y_0, t_0) is the virtual origin, $\delta(x)$ is a hypothetical turbulent boundary layer thickness, U_∞ is the free stream velocity and ν the kinematic viscosity. Cases 1 and 2 are similar in nature and can be combined as $\eta = Y/(K_2 T^m)$ for the analytical derivations. The other variable can be generalized as $\xi = X/(K_1 T^n)$. The corresponding general form for the stream function is $\psi = K_3 T^l g(\xi, \eta)$, where K_3 has suitable dimensions so that g is a dimensionless function of ξ and η . After reduction the local rates of displacement in the new coordinates are :

$$\begin{aligned} \frac{d\xi}{d\tau} &= -n\xi + \frac{K_3}{K_1 K_2} T^{l-m-n+1} g(\xi, \eta) \\ \frac{d\eta}{d\tau} &= -m\eta - \frac{K_3}{K_1 K_2} T^{l-m-n+1} g(\xi, \eta) \end{aligned} \quad \tau = \ln T. \quad (8)$$

Then it follows that, in order to have an autonomous system, the conditions

$$\begin{cases} K_3 = K_1 K_2 \\ -m - n = -1 \end{cases} \quad (9)$$

must be satisfied. Thus, for any growth rate in x and y , it is possible to define a stream function satisfying the above conditions. For the cases of interest, corresponding particle path equations and stream functions are :

$$\text{Case 1} \quad \frac{d\eta}{d\xi} = \frac{-\eta - g_\xi}{-\xi + g_\eta}, \quad \psi = U_\infty^2 T g(\xi, \eta) \quad (10)$$

$$\text{Case 2} \quad \frac{d\eta}{d\xi} = \frac{-\frac{4}{5}\eta - g_\xi}{-\xi + g_\eta}, \quad \psi = v^{1/5} U_\infty^{8/5} T^{4/5} g(\xi, \eta) \quad (11)$$

where g_ξ and g_η are the partial derivatives of g with respect to ξ and η , respectively. The very weak change in the particle path equations suggests only a slight distortion of the trajectories between these two cases.

For case 3, the virtual origin x_1 is, *a priori*, arbitrary and different authors [8, 9] suggest it should be chosen to match laminar and turbulent boundary layer thickness at the spot generator location x_s , i.e. :

$$\delta_t(x_s) = \delta_l(x_s) \quad \text{or} \quad 5.2 \sqrt{\frac{\nu x_s}{U_\infty}} = 0.37 \left(\frac{\nu}{U_\infty} \right)^{1/5} (x_s - x_1)^{4/5}. \quad (12)$$

But, in order to have a consistent similarity regime and an autonomous system for the particle paths, x_1 has to be equal to x_0 the virtual origin for the x - t growth. By dimensional analysis it is possible to define a stream function ψ as :

$$\psi = U_\infty \delta g(\xi, \eta). \quad (13)$$

The velocity field is :

$$\begin{aligned} u &= U_\infty g_\eta \\ v &= -\frac{\delta(x)}{T} \left[g_\xi + \frac{4}{5} \frac{1}{\xi} \frac{x - x_0}{x - x_1} (g + \eta g_\eta) \right]. \end{aligned} \quad (14)$$

At this point, we need to require that x_1 be equal to x_0 ; otherwise, the vertical local displacement $d\eta/d\tau$ will contain the ratio $(x - x_0)/(x - x_1)$, generating a non autonomous system. Then :

$$\begin{cases} \frac{d\xi}{d\tau} = -\xi + g_\eta \\ \frac{d\eta}{d\tau} = -g_\xi - \frac{4}{5} \frac{g}{\xi} \end{cases} \quad \text{with } \tau = \ln T \quad (15)$$

and the particle path equation is

$$\frac{d\eta}{d\xi} = \frac{-\frac{4}{5}\frac{g}{\xi} - g_\xi}{-\xi + g_\eta}. \quad (16)$$

This expression is very similar to the one obtained for the previous growth rate law, with g/ξ substituted for η . All three dimensionless stream functions g obey the same relationship :

$$\frac{g(\xi, \eta)}{\eta} = \frac{\psi}{U_\infty Y}. \quad (17)$$

3. Results.

The measured longitudinal velocity field for the data of CCD was vertically integrated as in CCD to obtain the unsteady stream function ψ . Figures 1(a), (b), (c) show contours of constant dimensionless stream function $g(\xi, \eta)$ for the different similarity growths. The streamlines are plotted in a reference frame moving at the horizontal velocity of the main vortex (referred to as II in table I), $0.78 U_\infty$. The new coordinates are defined as :

$$\xi' = \xi - 0.78 U_\infty \quad \text{and} \quad \eta' = \eta. \quad (18)$$

The solid lines represent characteristic points of the turbulent spot, i.e. the first change in velocity and local velocity extrema of the ensemble averaged data using CCD's definitions [6]. This choice was made in order to stress the differences induced by the different similarity growths when compared with CCD's original conical similarity results. In figures 1 and 2, the vertical axis scales were chosen to cover the same physical domain in y' and t . Few differences are observed between figures 1(a), (b), (c) except that with « viscous » growth (case 3) the streamlines are more vertically constrained, limiting the influence of the spot on the outer laminar flow. The instantaneous particle paths are shown in figures 2(a), (b) and (c). In all cases, there are four singularities whose locations are presented in table I. The roman numeral label refers to their respective locations in figures 2. In case 3 the stable focus or centre IV has a stronger influence on the outer laminar flow than in cases 1 and 2, but the topology remains identical.

Table I. — *Loci of singularities*

Singularity	Coordinate	Case 1	Case 2	Case 3
I (Saddle)	$\begin{cases} \xi \\ \eta \end{cases}$	0.600 0.000 35	0.598 0.005 9	0.604 0.061
II (Stable focus)	$\begin{cases} \xi \\ \eta \end{cases}$	0.640 0.000 53	0.645 0.009 5	0.635 0.065
III (Saddle)	$\begin{cases} \xi \\ \eta \end{cases}$	0.736 0.002 44	0.732 0.036 2	0.732 0.100
IV (Stable focus)	$\begin{cases} \xi \\ \eta \end{cases}$	0.767 0.003 87	0.769 0.064 8	0.769 0.132

Fig. 1. — Contours of constants $g(\xi, \eta) = \frac{U_\infty \eta \psi}{Y}$ in moving similarity coordinates. (a) Case 1 : $\eta = \frac{Y}{U_\infty T}$; (b) Case 2 : $\eta = \frac{Y}{U_\infty^{3/5} \nu^{1/5} T^{4/5}}$; (c) Case 3 : $\eta = \frac{Y}{\delta(x)}$.

Fig. 2. — Instantaneous particle paths in similarity coordinates. (a) Case 1; (b) Case 2; (c) Case 3.

4. Conclusions.

The main result of these computations is that the essential nature of the particle trajectories for a turbulent spot does not depend on the choice of similarity law from those variously suggested for the vertical coordinate. Only the viscous growth allows for Reynolds number dependence and experimental evidence [7-10] suggests that it produces the best fit to experimental data in terms of velocity and temperature perturbation contours. Cantwell [11] has suggested that similarity in time could provide a framework for the study of coherent structures in various turbulent flows. Turbulent spots are known to contain such coherent structures, but their number increases [2] while the spot is growing. This phenomenon as well as molecular diffusion are not included in any of the existing similarity laws. Also, the ensemble average over many spots could smear out the singularities due to smaller coherent structures. Both of these considerations might help to explain the observed differences between velocity and temperature perturbation contours.

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